Name: ....

Day: ....

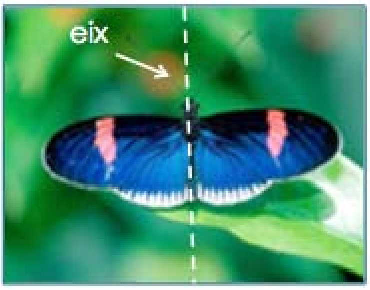
ImaginaryBCN

Monitoring notebook

Activity: Symmetry and coordinates

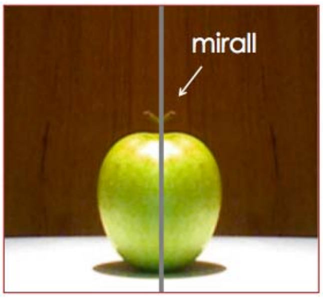
◆ The concept of symmetry

Axial symmetry



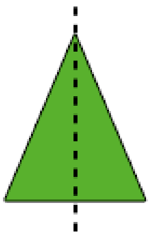
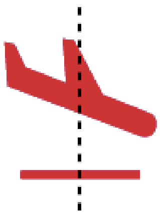
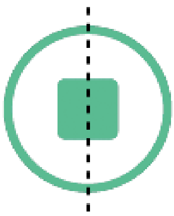
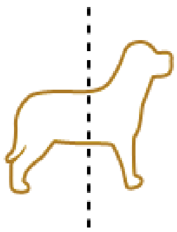
A planar figure has an **axial symmetry** when it can be folded in half along a line so that the two halves match up exactly. The folding line is named the **axis of symmetry**. For example, the butterfly has an axial symmetry around the dotted axis.

Mirror symmetry

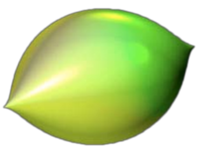
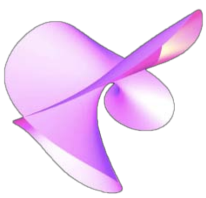
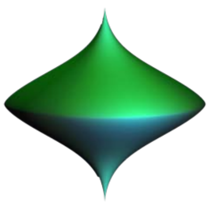
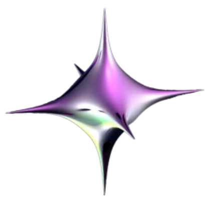


A three-dimensional figure has a **mirror symmetry** when it can be cut in two parts so that one is the reflection of the other in a mirror. The mirror is named the **symmetry plane**. For example, if you cut an apple in half and if you put one half in front a mirror, you will have the whole apple again.

1. Encircle the figures that have an axial symmetry.

2. Encircle the figures that have a mirror symmetry.

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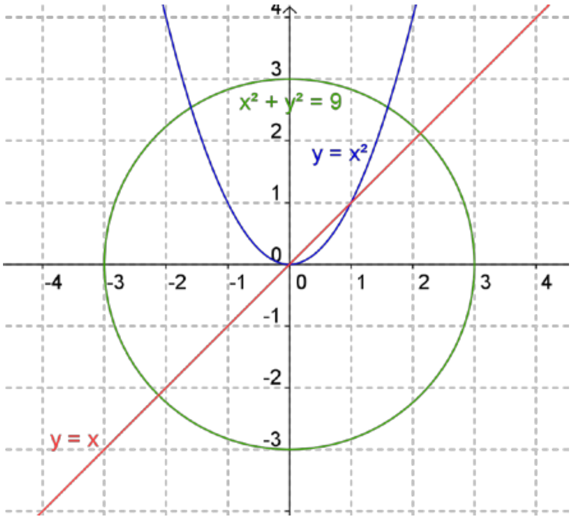
◆ Identify symmetry by the equation

The solutions of an equation determine a set of points in the space, which make a figure.



The equations, besides helping to distinguish the different figures, they also show some properties of the figure like axial symmetry. Look the figures on the left.

Which is the equation of each figure? Identify some points like $(3{,}0), (1{,}1), (0{,}0), (-1{,}1)$; which figures contain these points? Check that these points are solutions of the corresponding equations.

Figure and equation	$(3{,}0)$	$(1{,}1)$	$(0{,}0)$	$(-1{,}1)$
Line: $y = x$	No: $0 \neq 3$	Sí: $1 = 1$		
Circumference:				
Parabola:				

Now look at the symmetries around the vertical coordinated axis and fill the gaps:
the symmetric point of (3,0) is (,); the symmetric point of (1,1) is (,);
the symmetric point of (0,0) is (,); and the symmetric point of (-1,1) is (,).
Draw the symmetric points and discover the rule: the symmetric point of a point (x,y) is (,).
Choose a point of a figure, for example (1,1) on the line or on the parabola, and check if their symmetric point is also a point of the figure. In which figures does it happen?

A figure that has symmetry around the vertical coordinated axis $x = 0$ is a figure which equation does not change when you modify (x, y) by $(-x, y)$. For example, in the case of the parabola, we can see that $y = (-x)^2 = x^2$, and this demonstrates that the parabola has this type of symmetry. Otherwise, in the case of the line, the equation after interchanging (x, y) by $(-x, y)$. turns out to be $y = -x$; draw this new line. What will it occur to the circumference? Argue it.

Now look at the symmetries around the horizontal coordinated axis. A figure that has symmetry around the horizontal coordinated axis $y = 0$ is a figure which equation does not change when you modify (x, y) by $(x, -y)$. Which figures do you think that don't change? Which figures do you think that do change? Write down the equations and then try to draw them.

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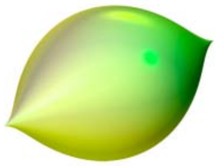
◆ The equation's alphabet

Algebraic equations are special formulas; they can only have elementary operations: multiplications and sums. The equations can be discomposed as a sum of monomials. A monomial has the following elements: the sign, the coefficient, the variables, the exponents and the degree:

$$2xy^2z \;=\; +2x^1y^2z^1$$

The monomial's degree is the sum of the exponents of its variables:

$$degree = 1 + 2 + 1 = 4$$

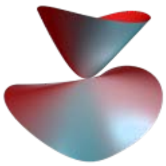


The degree of the equation is the degree of the monomial of the highest degree. For example, the equation of Zitrus $x^2 + z^2 = y^3(1 - y)^3$ can be discomposed as $x^2 + z^2 - y^3 + 3y^4 - 3y^5 + y^6 = 0$. It has two monomial of degree 2: x^2 and z^2; a monomial of degree 3: $-y^3$; a monomial of degree 4: $3y^4$; a monomial of degree 5: $-3y^5$; and a monomial of degree 6: y^6. In conclusion, *Zitrus* has degree 6.

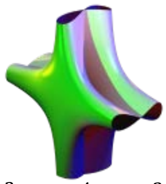
Practice the algebraic language answering the following questions:

1. Which of these figures has the equation of lowest degree?

$(xy - z^3 - 1)^2 = (1 - x^2 - y^2)^3$



$x^2 + y^2z = z^2$



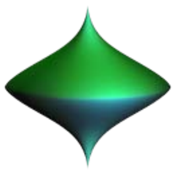
$6x^2 = 2x^4 + y^2z^2$

2. Which is the coefficient of the monomial of highest degree of *Seepferdchen*?

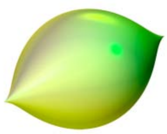
$(x^2 - y^3)^2 = (x + y^2)z^3$

3. Which of these figures do not pass through the origin of coordinates (0,0,0)?

$(xy - z^3 - 1)^2 = (1 - x^2 - y^2)^3$



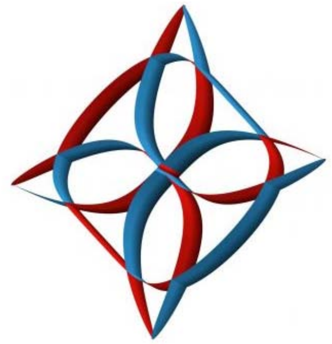
$60\,(x^2 + y^2)\,z^4 = (60 - x^2 - y^2 - z^2)^3$



$x^2 + z^2 = y^3\,(1 - y)^3$

◆ Create a surface with SURFER!

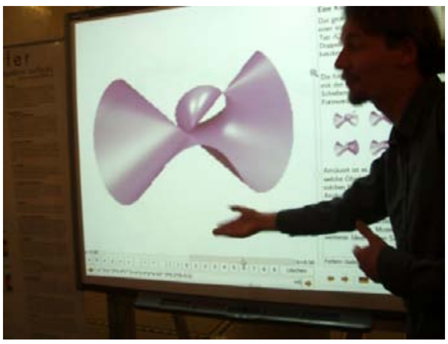
Invent an equation that has a low degree, for example 2, and another that has a high degree, for example 5. Check how are the surfaces that you have invented with SURFER. It is true that the equation of higher degree gives you a more complicated surface?



$$0 = (x^2 + y^6 - 1) \cdot (2x^3 + 4y) \cdot (2y^3 + 2x) \cdot (2x^3 - 4y) \cdot (2y^3 - 2x)$$

Imagine that you want to make a surface with a spike like the spike of *Zitrus*, in other words a spike that delimits the surfaces only in one side (a spike that you can follow with your finger and hurt yourself!): which degree do you think the equation should have? Let's investigate: complete the following table displaying the figures with SURFER:

Equation	Degree	Spike as *Zitrus*	Another type of spike
$x^2 + z^2 - y^3\,(1 - y)^3 = 0$			
$x^2 + z^2 - y^3 = 0$			
$x^2 + z^2 - y^2 = 0$			
$x^2 + z^2 - 100y^2 = 0$			
$100x^2 + 100z^2 - y^2 = 0$			
$x^2 + z^2 - y^2 + xz = 0$			
$x^2 + z^2 - y^2 + y = 0$			



Which is your conclusion? You dare to conjecture that the minimum degree that it is needed to obtain a spike like the one of *Ziturs* is $\boxed{}$

Create your own surface and participate to the contest! www.imaginary-exhibition.com/concurso

You can download (for free!) SURFER from the website: www.imaginary-exhibition.com/surfer?lang=es

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