The mathematics of tides

Spending some time by the sea leads one to become aware of tides as a natural cyclic phenomenon.

One rapidly notices that there are two high tides and two low tides every day, and that each of those is a little later than the corresponding tide of the previous day. A little more observation is required to notice that sometimes high tides are higher than at other times (and low tides lower), that this is also cyclic and that the times when the difference between high tides and low tides is greatest occur approximately twice a month (they are called spring tides; neap tides correspond to tides when that difference is smallest).

The idea that tides are related to the moon's position is well-known, and it is easy to observe that a high tide occurs at a place some time after the moon passes at its highest point over that place. This is usually explained by the moon's attraction of the ocean water, and this explanation is reinforced by the fact that after a new moon (when the sun and the moon are aligned on the same side of the earth) there is a spring tide ("stronger" tide), which seems to be naturally explained by the attraction of the sun and the moon acting in the same direction.

However there are several facts which not only remain unexplained but actually seem to contradict this explanation. For example, the moon only passes over each meridian once each day, so why are there two high tides every day? And there is only one new moon each month (when the moon and the sun are aligned on the same side of the earth) so why are there two spring tides every month? And why is the other spring tide at full moon, when the moon and the sun are aligned on opposite sides of the earth, and it would therefore seem that their attractions should partially cancel each other out, leading to weaker instead of stronger tides?

Another problem has to do with the usual explanation of tides mainly through the action of the moon's attraction. Why does the moon's attraction exert greater influence than the sun's? According to Newton's law, attraction is directly proportional to mass and inversely proportional to the distance squared. The sun's mass is very much larger than the moon's, but the sun is also very much farther away, so perhaps the smaller distance compensates for the smaller mass? However, actually doing the maths shows us that the attraction exerted by the sun on an object on the surface of the earth is approximately 180 times the one exerted by the moon, so the effect of the moon on tides should be insignificant compared to that of the sun.

One is almost led to think that Newton was wrong! Because if we worked under the hypothesis that attraction were inversely proportional to the distance cubed, then we would conclude that the attraction of the moon would be slightly more than twice the attraction of the sun, which is compatible with direct observations of the role of the sun on the tides.

This presentation explores in an interactive way some of the questions related with tides and tries to give some insight into the reasons for the observed behaviour.