The Use of SURFER Software as an Example of Integrating Information into Math Instruction

Wei-Chi Hsiao
Wen-Shan Junior High School, New Taipei City
Translated by Alexander Huang, Taipei American School

1. Origins

On December 6th, 2015, the author and 8th grade advanced mathematics students participated in the “IMAGINARY: the Application of SURFER Software” studies program, which was taught by German mathematician Bianca Violet. Violet introduced the evolution of “Imaginary” in Germany as well as the arts and mathematics exhibitions that they held in cities around the world. She also introduced the Surfer software and used the topic of “snowman” to inspire participating students and teachers to explore and create with the software.

After the studies program, the author wondered the possibility of incorporating this software in gifted education programs. The design of gifted education programs requires organizing learning experiences in a systematic way in order to arrange appropriate learning activities to help students learn. These learning experiences should match the level of the gifted students in order to provoke their interest in learning. The results of these learning programs should be able to demonstrate higher-order thinking, and creative or productive thinking. Therefore, the author wanted to progress in a step by step manner, by consolidating past experience, critical thinking, learning new knowledge, and applying this knowledge, in order to facilitate the goal of achieving “Symbolic-graphic Combination” and for students not only to understand the concept, but also to understand the reasoning behind these concepts.

Thus, the author designed a two-hour mathematics enrichment programs which require that students understand the Pythagorean Theorem and factorisation as a prerequisite.

2. The “Snowman” Curriculum

2.1 Revisiting past experiences: The Distance Formula

The distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is

$$\overline{PQ} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

2.2 Questioning and Thinking: The Equation of a Circle (New Knowledge)

Question 1: Find the point whose distance from the origin is 5.

Because the students have studied the Pythagorean Theorem, students quickly pointed out $(\pm 5, 0), (0, \pm 5), (\pm 3, \pm 4)$. However, once students pointed these eight points out, they quickly realized that instead of eight, there are probably many points that satisfy this requirement.

Question 2: What shape will the points that satisfy this requirement form?

Question 3: What is the equation of the circle?

Depending on the situation, the instructor can lead students to discuss how to use the Distance Formula to describe algebraically Question 1.

$$\sqrt{(x-0)^2+(y-0)^2}=5$$

$$x^2+y^2-25=0$$

Show the "Standard Form" for the equation of a circle.

$$(x-h)^2+(y-k)^2=r^2$$

Here, the center of the circle is (h,k), and the radius is r $(r>0)$.

Question 4: Given some equations of circles, ask students to find the center and radius of the circles. Also ask them to observe the relationship between the locations of the circles.

$$\Gamma_1: x^2+y^2-3=0$$

$$\Gamma_2: (x-3)^2+(y-1)^2-4=0$$

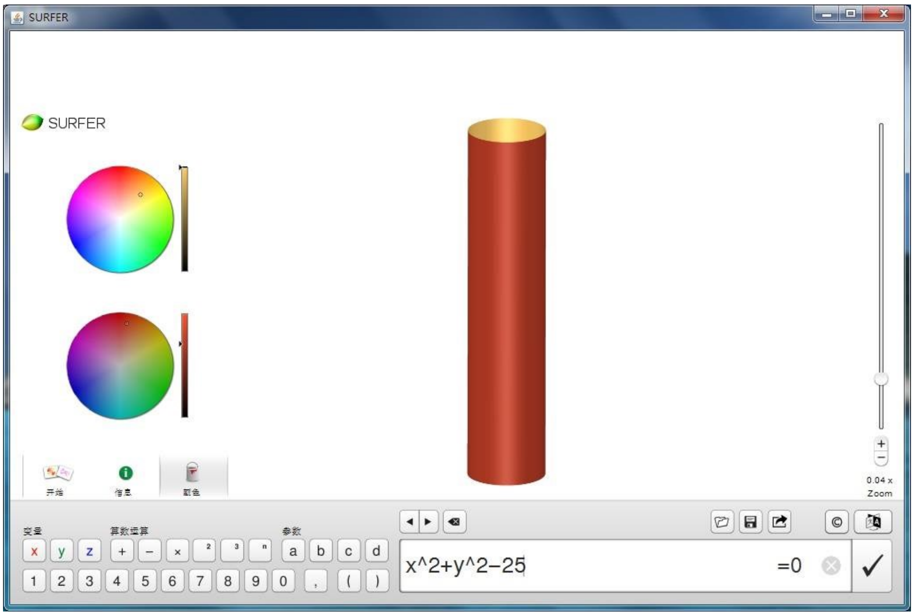
$$\Gamma_3: (x+2)^2+(y-1)^2-9=0$$

2.3 From 2D to 3D (New Knowledge)

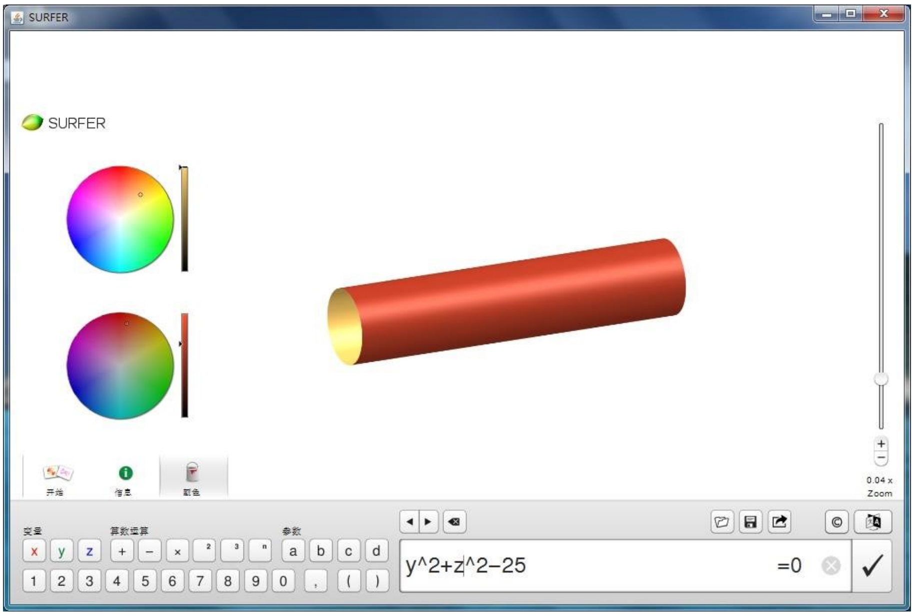
Question 5: The given equation $x^2+y^2-25=0$ is a circle on a plane, what would it be in three dimensional space?

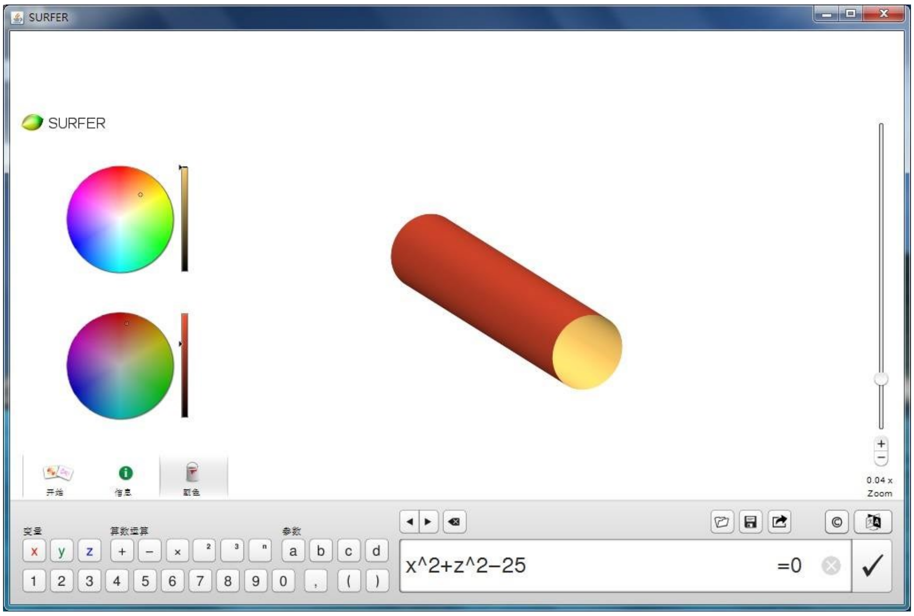
Using the walls of the classroom as an example, inform students that three-dimensional space will lead to an additional dimension, the z-axis.

Ask students to discuss the geometric figure of the equation. Usually one or more students will be able to guess that it is a cylinder. The instructor then uses SURFER for a demonstration and lead students to discuss and explain why such a shape emerges.



Then, ask students to enter the equations "$x^2 + z^2 - 25 = 0$" and "$y^2 + z^2 - 25 = 0$" and employ logical reasoning to understand where the x, y, z axis is in the 3-D Coordinate System.





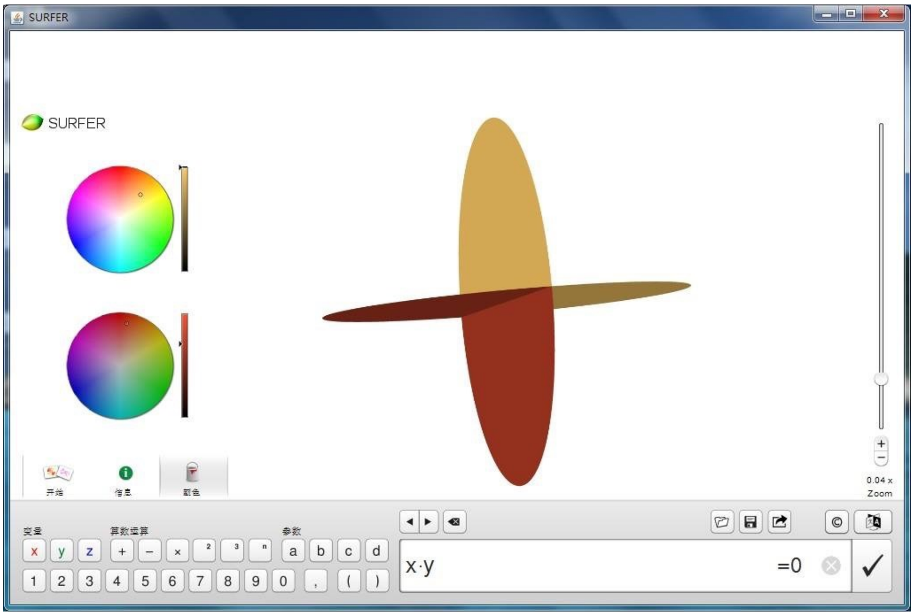
2.4 The Zero-product Property

2.4.1 Review the concept of factorization and rule of zero product

$$A \times B = 0 \;\Rightarrow\; A = 0 \text{ or } B = 0$$

2.4.2 Generalize

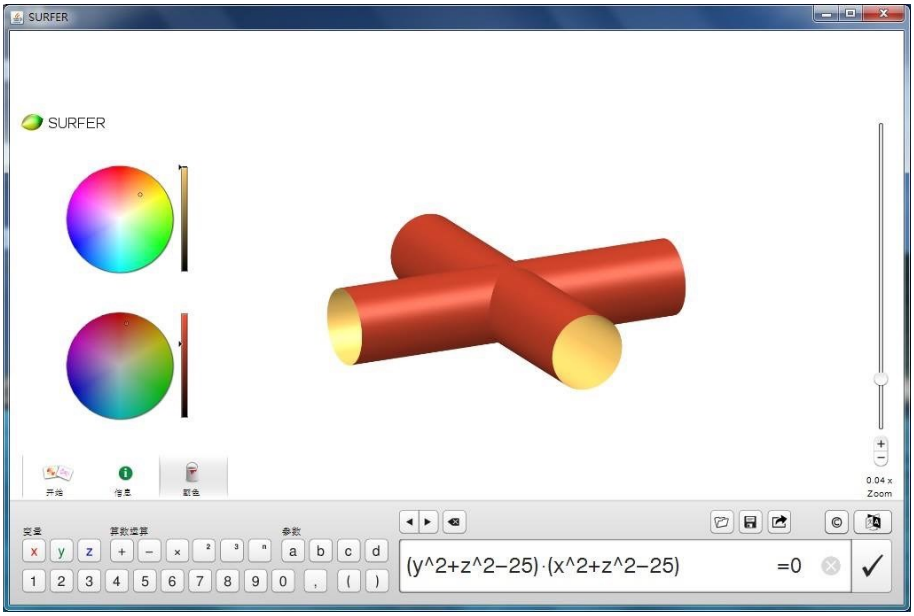
Use the shapes that come with SURFER or discuss the significance of the three dimensional figures as an example of $xy = 0$ （$x = 0$ or $y = 0$）.Also, ask students what the geometric significance of “$xy = 0$” on a plane (the x-axis or y-axis).



2.4.3 Drawing the Steinmetz solid

With the knowledge that students just acquired, given a “Steinmetz solid”, ask students to recreate the figure in SURFER.

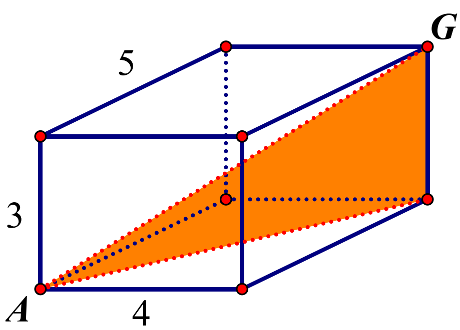
For example, $(y^2 + z^2 - 25) \times (x^2 + z^2 - 25) = 0$



2.5 Summarize: from circle to sphere

2.5.1 Prior knowledge: The “Bees Flying Problem” in a rectangular prism

Question 6: Inside a rectangular room, measuring 5 feet in length, 4 feet in width, and 3 feet in height, if a bee is to fly from point A to point G, what would the shortest distance be between the two points?



$$\overline{AG} = \sqrt{5^2 + 4^2 + 3^2}$$

2.5.2 New knowledge: the Distance Formula in Three Dimensional Space

With the experiences from 2.5.1 in mind, using the format of questions 1 through 3, attempt to recreate the Distance Formula in three dimensional space.

The distance between two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ is

$$\overline{PQ} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

2.5.3 New knowledge: the equation, center, and radius of a sphere

Using the format of question 4, attempt to analogize the equation of a sphere in three dimensional space.

Given some equations of spheres, ask students to find the center and the radius of the spheres, and look into the relationships between the locations of the spheres.

$$\pi_1: x^2 + (y-1)^2 + z^2 - 3 = 0$$

$$\pi_2: (x-3)^2 + (y-1)^2 + z^2 - 4 = 0$$

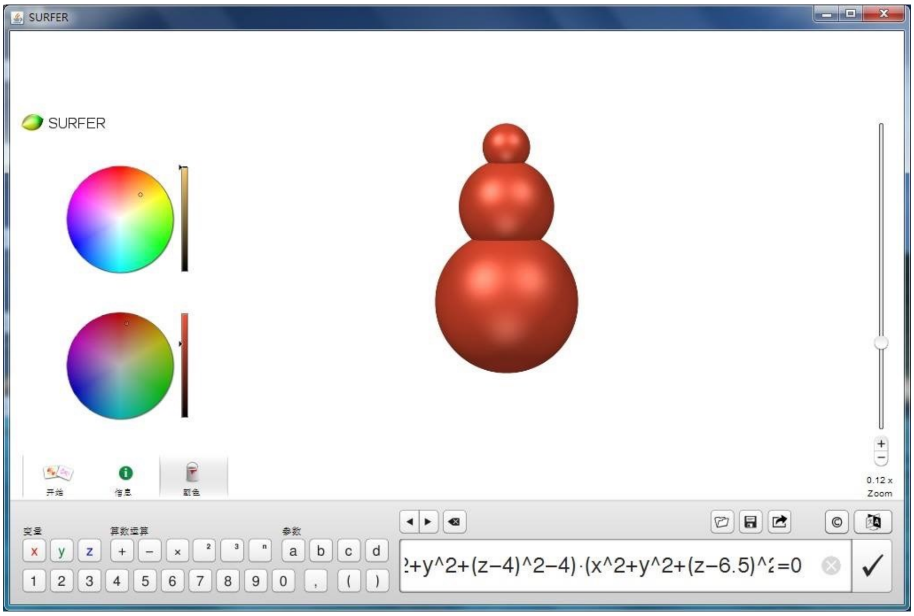
$$\pi_3: (x+2)^2 + (y-1)^2 + z^2 - 9 = 0$$

Show the “Standard Form” for the equation of a sphere.

$$(x-h)^2 + (y-k)^2 + (x-l)^2 = r^2$$

2.5.4 Snowman

Using SURFER, lead students to draw a snowman composed of big, medium and small spheres. Finally, encourage students to continue to explore on their own at home. If possible, draw eyes and clothing for the snowman.



3. Conclusion

The author thanks the main speaker Bianca Violet for bringing SURFER, a very interactive program. For the general public, it is an extremely valuable tool to advance scientific inquiry as well as art. For students who demonstrate strong potential in mathematics and the sciences, it is a valuable tool for exploring geometry and algebra.

The aforementioned lesson plan is the work of the author after participating in the SURFER studies program. In fact, the content of the middle school mathematics curriculum can be seen as material for developing critical thinking. The focus of learning should be to foster the ability to think critically and to understand the world through a mathematical lens, even if the student will not pursue a mathematics related career.

To the author , helping students understand and accept mathematical concepts, helping students acquire new knowledge by applying what they have already learned, and inspiring students to learn on their own at home are the goals that he has been dedicated to for a long time.

4. References

Official Notes on Snowman from Imaginary, available at
https://imaginary.org/sites/default/files/snowman_bridges2015_0.pdf