

Hands-on maths bag by mmaca

Materials and activities

All the hands-on materials here presented, except multilinks and dice, can be printed on paper or cardboard, and then coated with plastic.

Are easy to reproduce, adapt, transform, enrich,

Other simple material can be added to reinforce the project.

The suggested activities are supported by written instructions and/or specific boards and frames.

The first step of the learning process is the physical manipulation of materials, in order to involve pupils at the intuitive level, before leading them to abstraction.

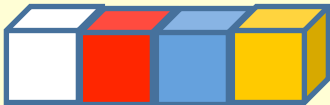
Virtual tools (as eg Geogebra or Excel) can be used to reinforce investigation .

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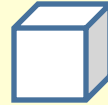
Materials and activities : calculation

2.9) Constructing numbers (with $1+2+3+4+5+6+7+8+9+10$ multilinks)

Constructing the DISCRETE MODEL:
composing numbers by increasing units.



Constructing the CONTINUOUS MODEL:
The value of the number is expressed by the length of the prism

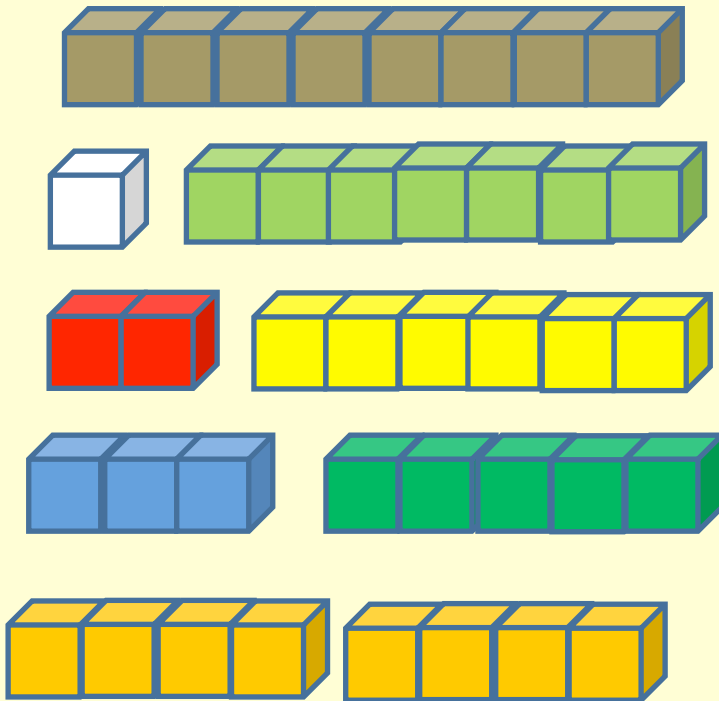


It is very important to present early the two models that must cooperate for a deeper understanding of mathematics

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Materials and activities : calculation

2.10) Composing and descomposing numbers (with $1+2+3+4+5+6+7+8+9+10$ multilinks)



Construction and deconstruction of numbers is one of the early and important manipulation to be done again and again

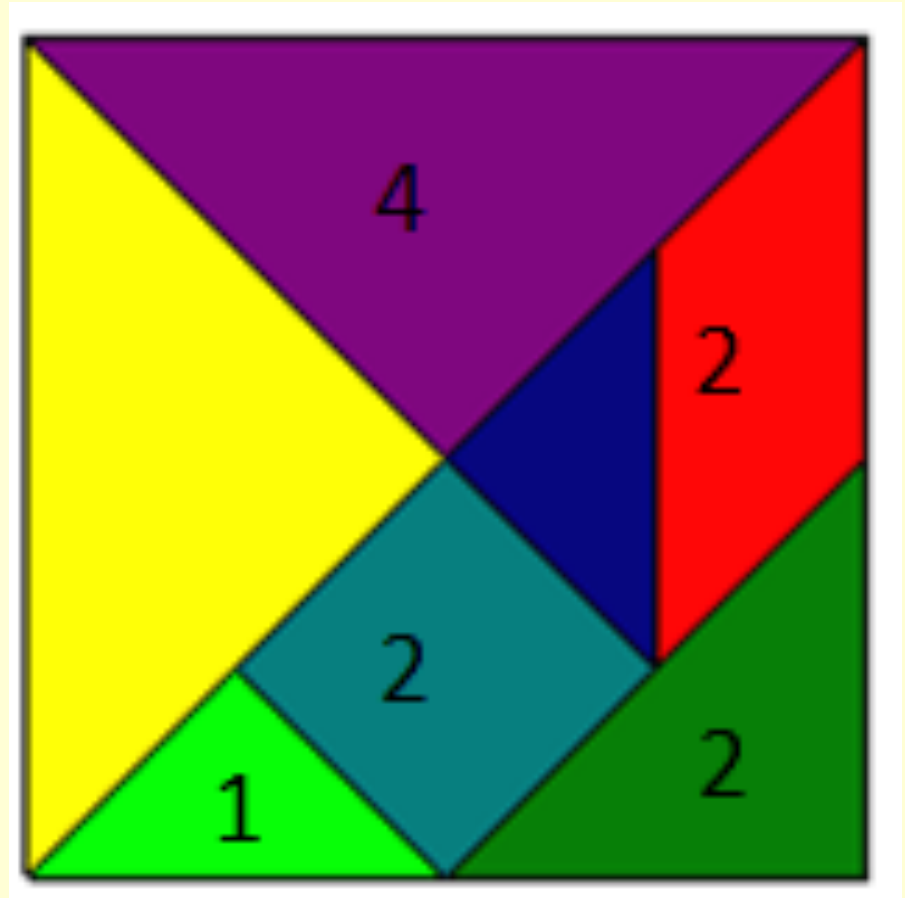
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Materials and activities: calculation

1.4) Increasing numbers (with 1 tangram)

Considering the little triangle as 1

- Construct other numbers: 5, 6 ...
- Is it possible to find different ways to compose the same number?
- Which is the biggest number that is possible to represent with all the tangram shapes?



Working on numbers composition

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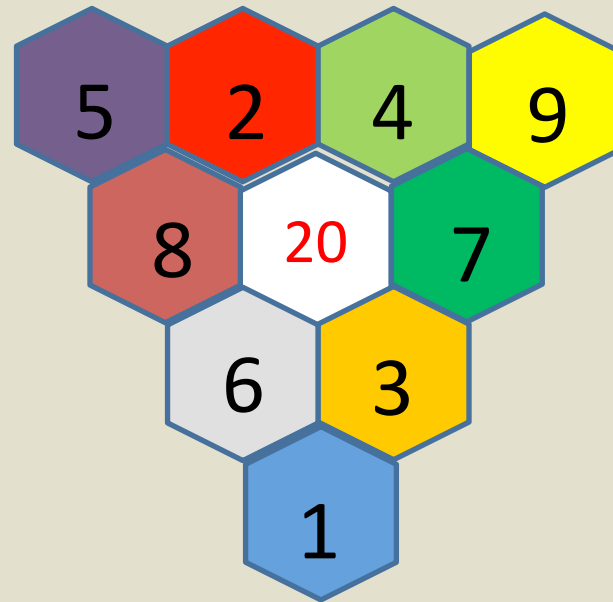
Materials and activities: calculation

5) Magic triangles (2 versions: 6 and 10 pieces - hexagons)



1) Sides can sum: 9, 10, 11 or 12

Reflect on strategy



2) Sides can sum 17, 19, 20, 21 or 23

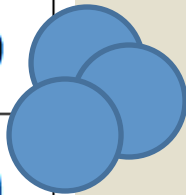
A charming version of an old challenge

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Materials and activities : calculation

11) Shut the box (a board, 9 cards, two dice)

1	2	3
4	5	6
7	8	9



It is a very good exercise on composition/ decomposition of numbers and mental calculation as well.

Erase from the board the same value you obtained by dice:

Eg: $6+5 = 11 = 9+2$; $8+3$; $7+4$; $6+5$.

Stop when you can't.

Different strategies are possible



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Materials and activities : calculation

12) Figures and numbers (a tens dice and 6 normal dices)



Use any mathematical operation on the numbers you get on the red dice, to obtain the same value of the sum of the other two dice.

$46 = (6+1) \times 6 + 4 \rightarrow 4 \text{ dices} = \text{you score 4 points}$

$46 = 6 \times 6 + (4+1) \times 2 \rightarrow 5 \text{ dices} = \text{you score 5 points}$

A great challenge of mental calculation and strategy.

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Materials and activities : calculation

6) Fractions disc



One of the simplest and more significant instrument to understand the meaning of fractions, compare and calculate with them

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Materials and activities : calculation

7) Fractions game



Fill up the circle with **equal** sectors (color=value).
How many sectors do you need for each circle?
Which is the value of each sector?
Which is the value of each circle?

It is simple, but it works!



Fill up the circle with sectors of **different** colors.
Describe the composition of each circle in terms of fractions.

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Materials and activities : calculation

8) Fractions sticks

A first attempt to pass from fractions to percentages



Compare the length of the sticks.

How many $\frac{1}{4}$ sticks do you need to cover a $\frac{1}{2}$ stick?

How many $\frac{1}{6}$ sticks do you need to cover a $\frac{1}{3}$ stick?

How many $\frac{1}{6}$ sticks do you need to cover a $\frac{1}{2}$ stick?

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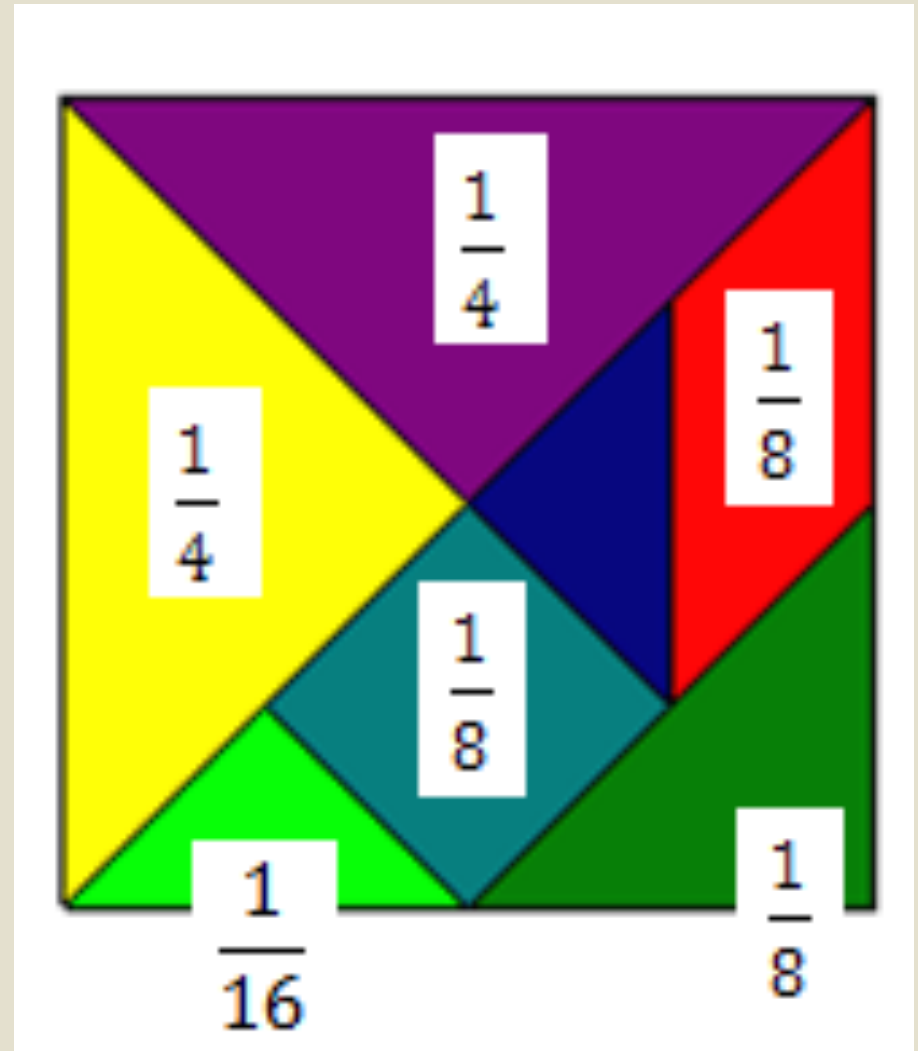
Materials and activities: calculation

1.5) Increasing fractions (with 1 tangram)

Considering the whole tangram as 1:

- Assign to each Tangram shape is fractionary value
- Construct more fractions.
- Is it possible to use different shapes to compose the same fraction?
- Is it possible to represent any fraction with the tangram shapes?

It is a very good activity to work on concepts and calculation of fractions



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Materials and activities : calculation

9) Who is who? of fractions (8 different boards)

$\frac{14}{7}$	$\frac{3}{3}$	$\frac{8}{3}$	$\frac{1}{2}$	$\frac{10}{6}$
$\frac{2}{3}$	$\frac{9}{4}$	$\frac{7}{2}$	$\frac{3}{4}$	$\frac{5}{3}$
$\frac{3}{5}$	$\frac{6}{4}$	$\frac{4}{6}$	$\frac{2}{8}$	$\frac{1}{3}$
$\frac{6}{8}$	$\frac{2}{4}$	$\frac{1}{4}$	$\frac{5}{8}$	$\frac{9}{2}$

$\frac{7}{4}$	$\frac{7}{2}$	$\frac{14}{7}$	$\frac{10}{6}$	$\frac{1}{3}$	$\frac{2}{2}$	$\frac{1}{4}$
$\frac{5}{3}$	$\frac{2}{4}$	$\frac{5}{8}$	$\frac{10}{5}$	$\frac{6}{2}$	$\frac{8}{3}$	$\frac{9}{2}$
$\frac{2}{3}$	$\frac{6}{4}$	$\frac{1}{2}$	$\frac{5}{2}$	$\frac{2}{8}$	$\frac{5}{5}$	$\frac{3}{4}$
$\frac{3}{5}$	$\frac{3}{3}$	$\frac{4}{6}$	$\frac{3}{2}$	$\frac{9}{4}$	$\frac{2}{6}$	$\frac{6}{8}$

$\frac{6}{6}$	$\frac{8}{8}$	$\frac{3}{3}$	$\frac{6}{6}$	$\frac{3}{3}$	$\frac{2}{2}$	$\frac{5}{8}$
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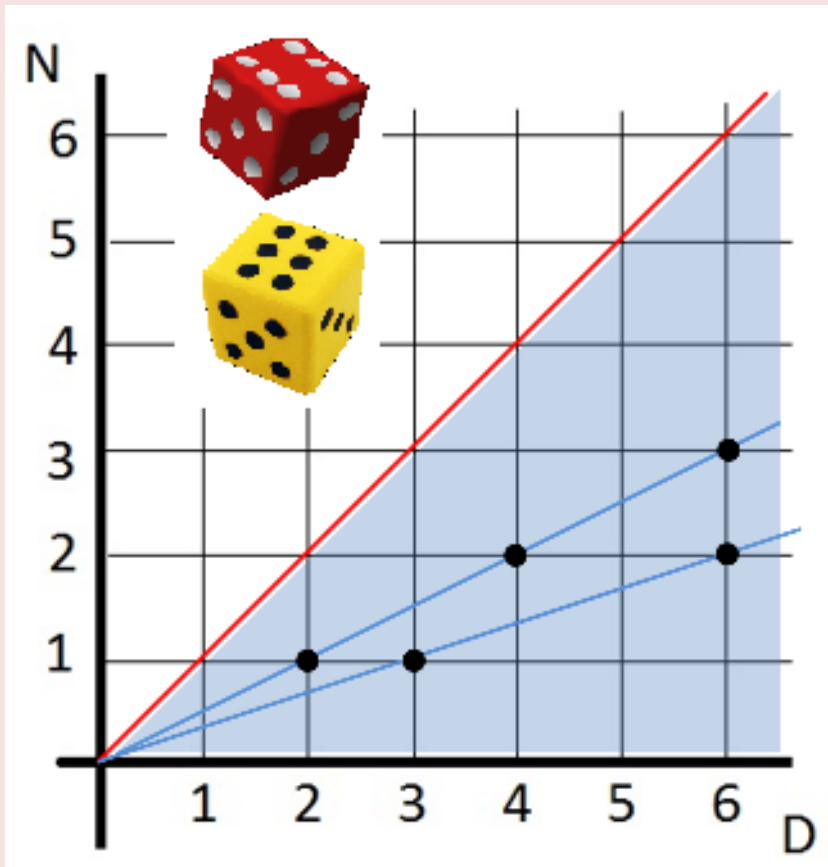
With a minimum number of true/false questions, try to discover the fraction that one of the player chose.

Strategy and language

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Materials and activities : calculation

10.5) Dice and fraction (a board, two dice)



It is a very good activity to work on concepts and calculation of fractions, introducing, for secondary school students, elements of analytical geometry

Discuss the meaning of:

Fractions above and below the red line

Fractions on the same line

Fractions on lines perpendicular to axes

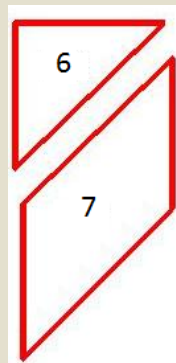
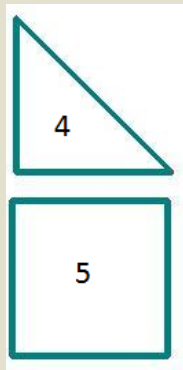
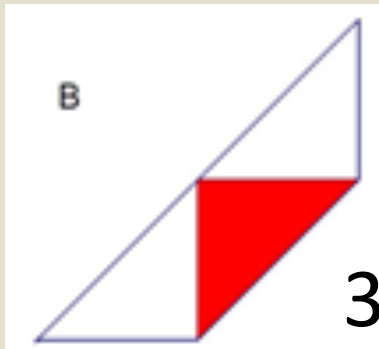
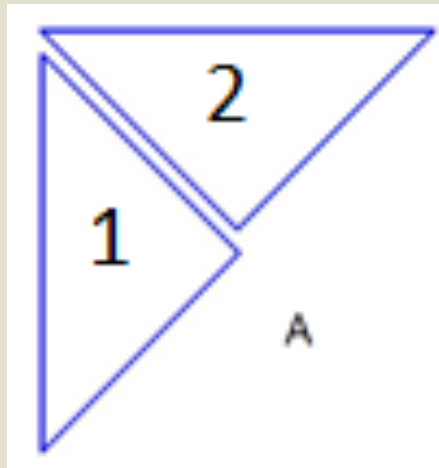
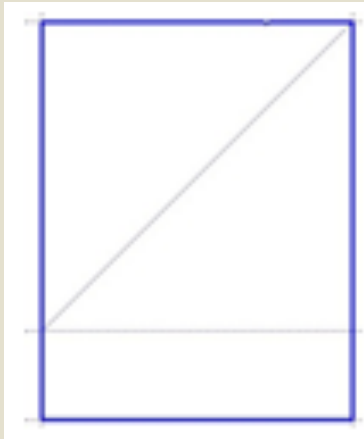
Position on the graph and value of a fraction

...

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Materials and activities: geometry

0) Preliminary activity: construction of the Chinese Tangram (origami)



CONTENTS OF GEOMETRY:

- Classification of polygons
- Regular and irregular polygons
- Congruent polygons
- Equivalent polygons
- Similar polygons
- Classification of triangles based on angles and sides
- Isometries and motions in the plane

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Materials and activities : geometry

1.1) Duplication of a square (with 2 tangrams)

- Concepts of area and perimeter:
- Comparison of areas
- Comparison of perimeters

Do we need formulas?

Introducing $\sqrt{2}$

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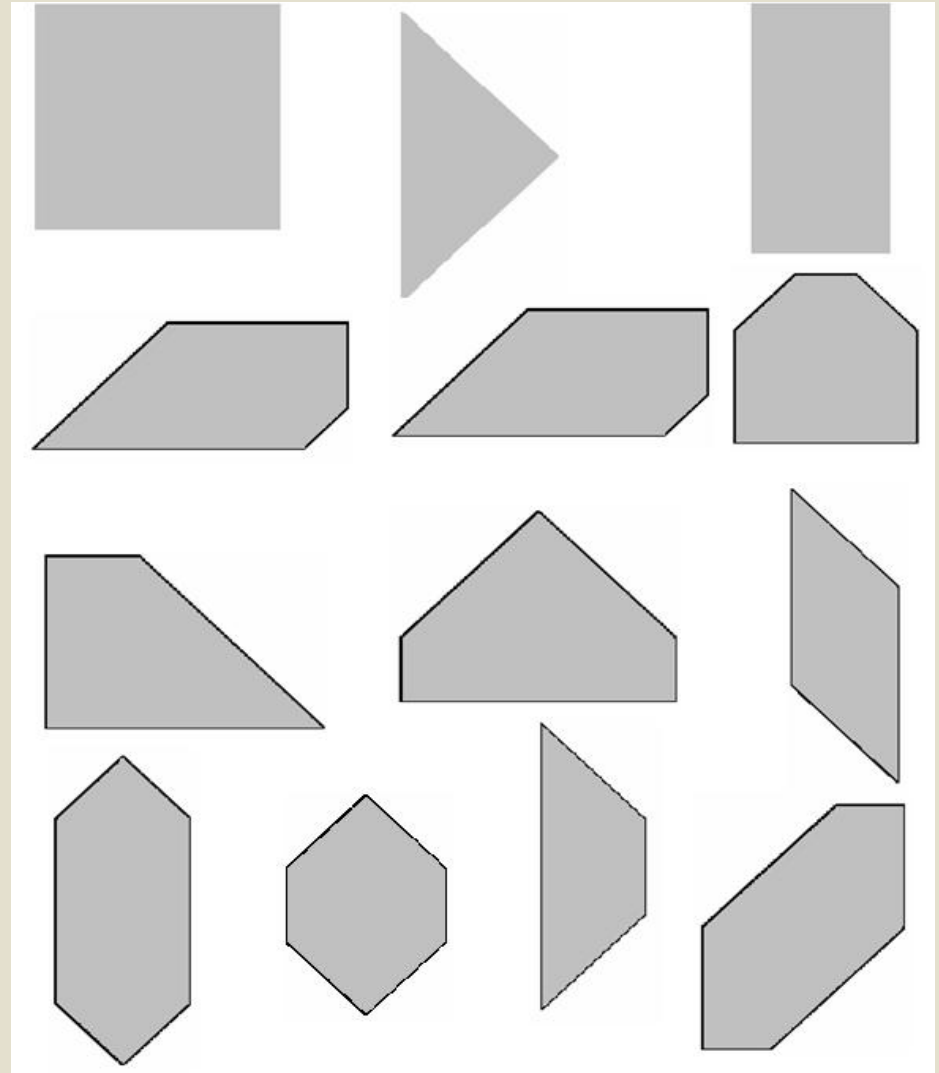
Materials and activities : geometry

1.2) Equivalent polygons (with 1 tangram)

All the polygons are composed by the 7 tangram shapes

- Concepts of area and perimeter:
- Comparison of areas
- Comparison of perimeters

Do we need formulas?



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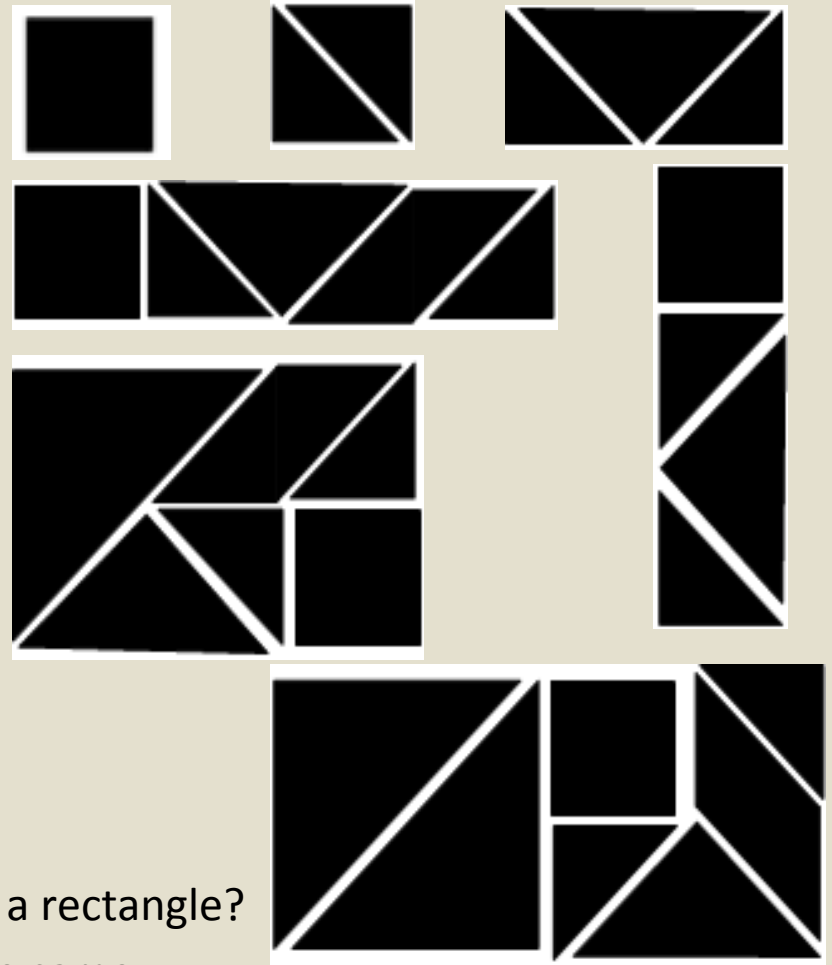
Materials and activities : geometry

1.3) Growing squares and/or rectangles
(with 1 tangram)

Make the figures with 1, 2, 3 ... 7
tangram shapes

A good exercise of figures composition

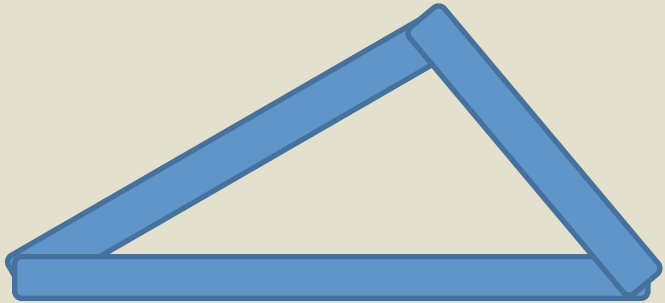
- Is it always possible to construct a square and a rectangle?
- Is it possible to find different ways to make the same figure?



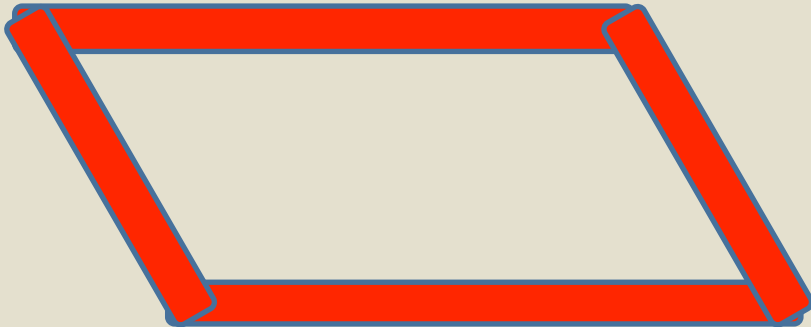
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Materials and activities : geometry

14) Constructing polygons (meccano)



Conditions to build a triangle
«Solidity» of the triangle
«Solidity» of the polygons
Relationships between parallelogram
and rectangle
How to «block» a polygon
Polygons and diagonals



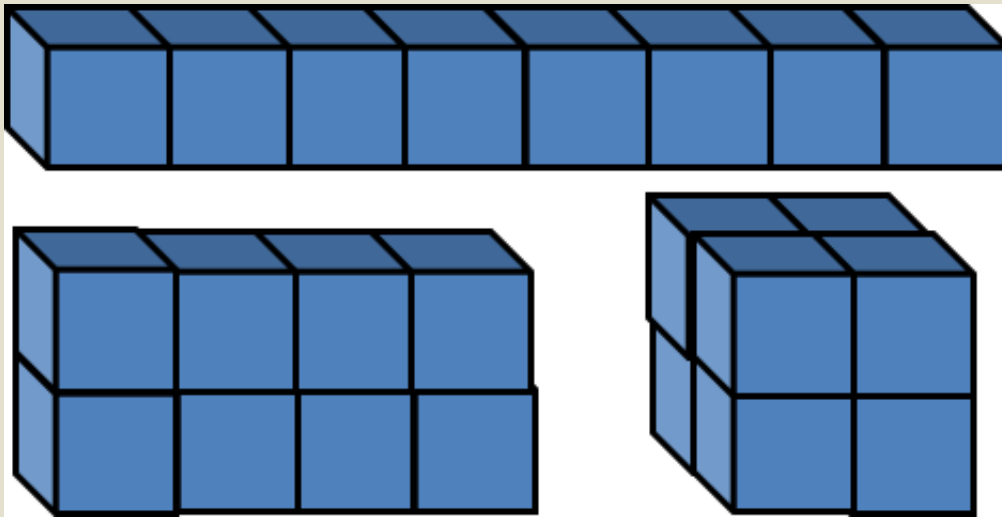
A simple but powerful laboratory of geometry

Hands-on maths bag by mmaca

Materials and activities: geometry

2.1) Equivalent prisms (with 8 multilinks)

Compose different prisms with 8 multilinks



- Concepts of area and volume:
- Comparison of volumes
- Comparison of areas

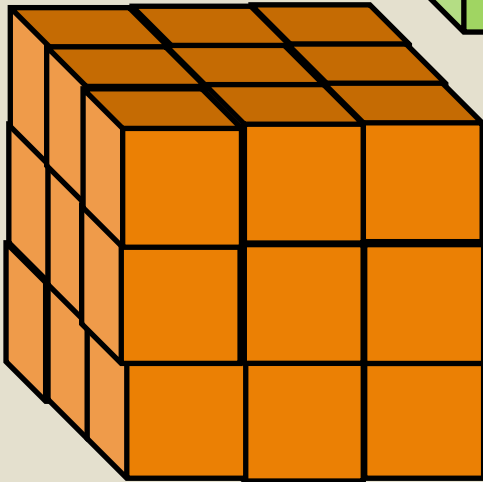
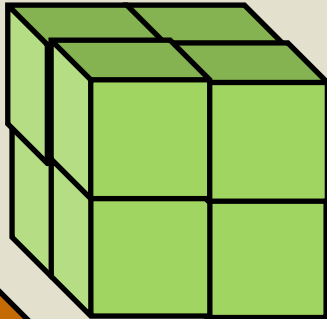
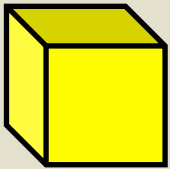
Do we need formulas?

Hands-on maths bag by mmaca

Materials and activities : geometry

2.2) Doubling the volume of a cube (with 36 multilinks – minimum.)

Compose different cubes with multilinks



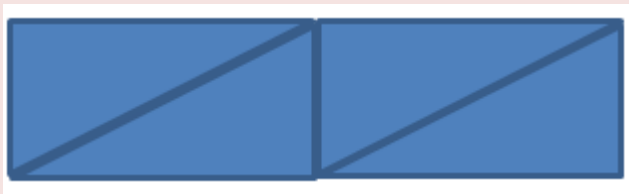
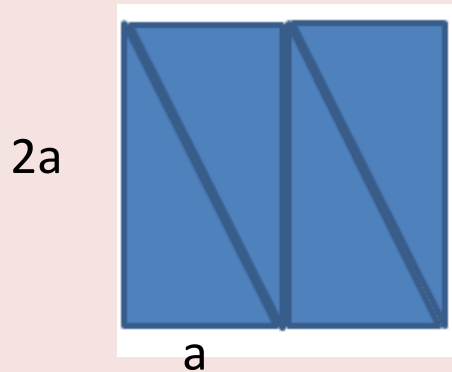
- Concepts of area and volume:
- Comparison of volumes
- Comparison of areas

How many unitary cubs do I need to build the cube with 4 units per side?

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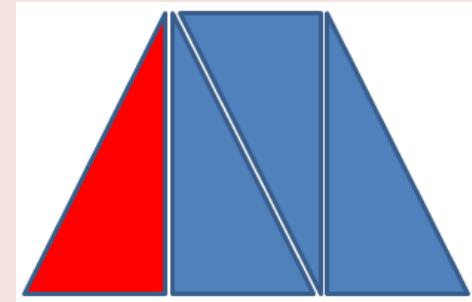
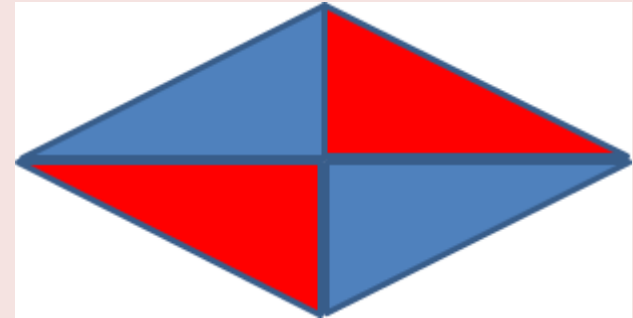
Materials and activities : geometry

3) Right-angle triangle (4 triangles)



...

- Comparison of areas
- Comparison of perimeters



Do we need formulas?

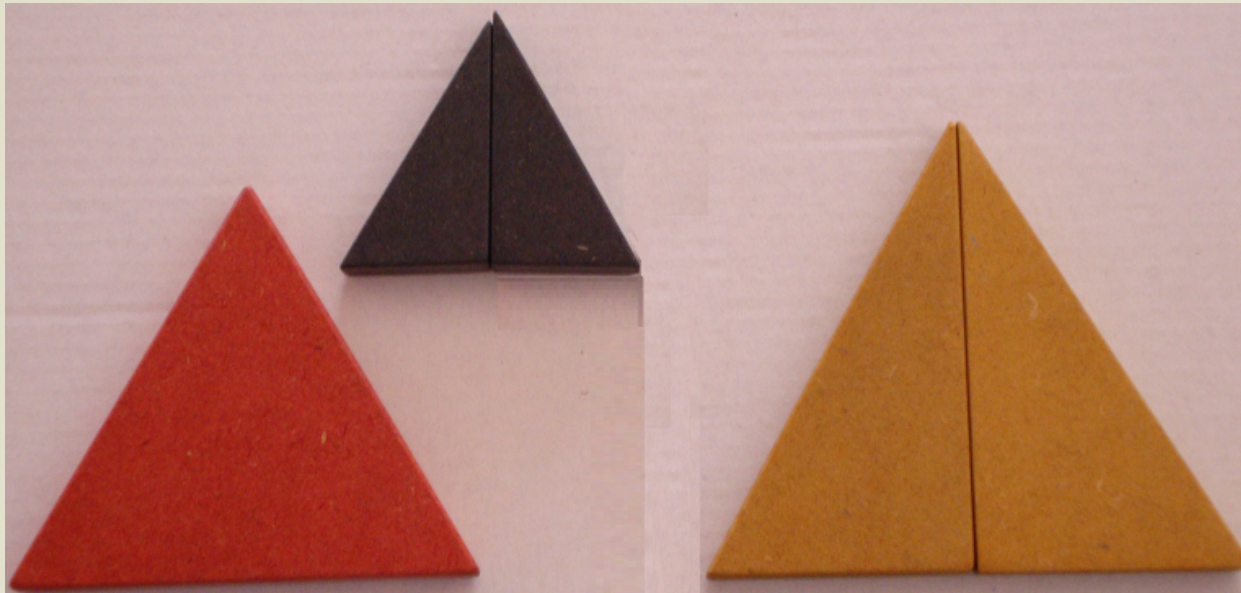
Some shapes need to move some triangle outside the plane: symmetry

A very easy material for a rich task

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Materials and activities : geometry

4) Equilateral triangles (5 triangles)



Two different activities
(attitudes, approaches,
knowlegment ...)

1) With all the pieces, build
1, 2 or 3 equilateral triangles

2) The side of the red triangle is 3 units.
Build the equilateral triangle whose side is
2, 4 or 5 units

Hands-on maths bag by mmaca

Materials and activities : geometry

2.3) Different cubesomes (with 27 multilinks – minimum.)



Find different ways to obtain the cube with the given shapes.

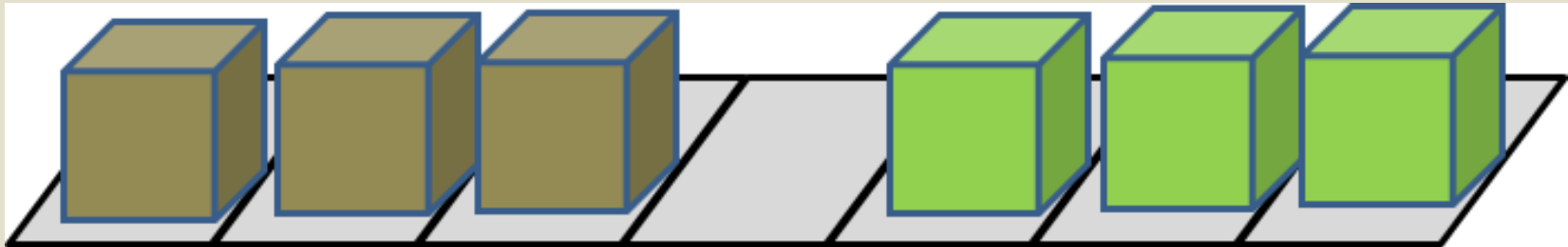
Invent your own cubesome

A good exercise of 3D figures composition

Hands-on maths bag by mmaca

Materials and activities: strategy

2.5) Frogs and Toads (with 3+3, 4+4 ... multilinks)



Advancing by one place or jumping just one piece, you must exchange the initial position of frogs and toads (no return allowed).

Try to do it with the minimum number of movements.

Can you foresee how many movements are necessary to exchange the position of 100 frogs and toads?

Introducing generalization and the meaning of using formulas

Hands-on maths bag by mmaca

Materials and activities : strategy

2.4) Skyscrapers challenge (with 4+8+12 +16 multilinks)



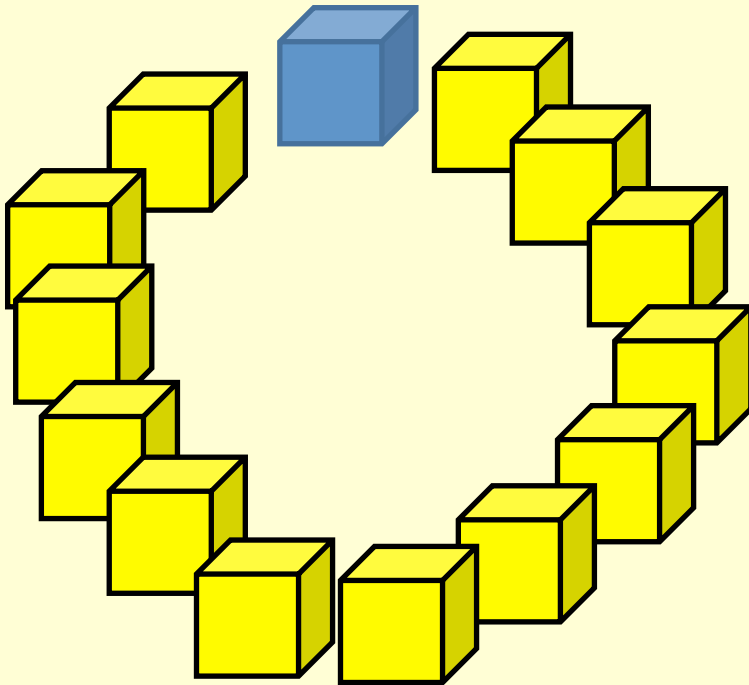
An amusing 3D sudoku to stimulate spatial vision and to train the change of the point of view

Invent your own challenge

Hands-on maths bag by mmaca

Materials and activities : strategy

2.7) Take up to 3 (with 14 multilinks)



Two player

One can take between 1 and 3 cubes

The player who takes the last cube is the loser

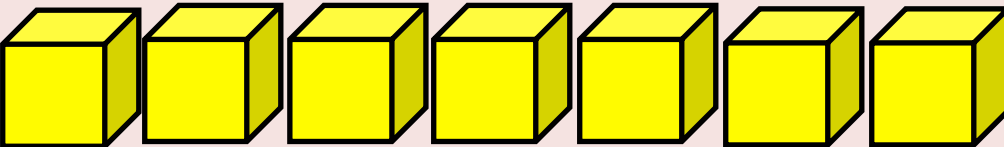
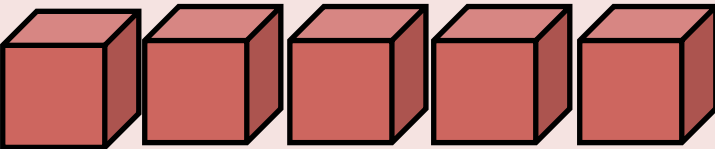
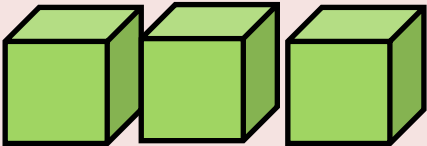
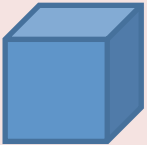
Have the two players the same chances of winning?

The winning strategy is simpler than Nim's one

Hands-on maths bag by mmaca

Materials and activities : strategy

2.6) Nim (with 1+3+5+7 multilinks) – version Marienbad



Two player

One can take any number of cubes that have the same colour

The player who takes the last cube is the loser

Have the two players the same chances of winning?

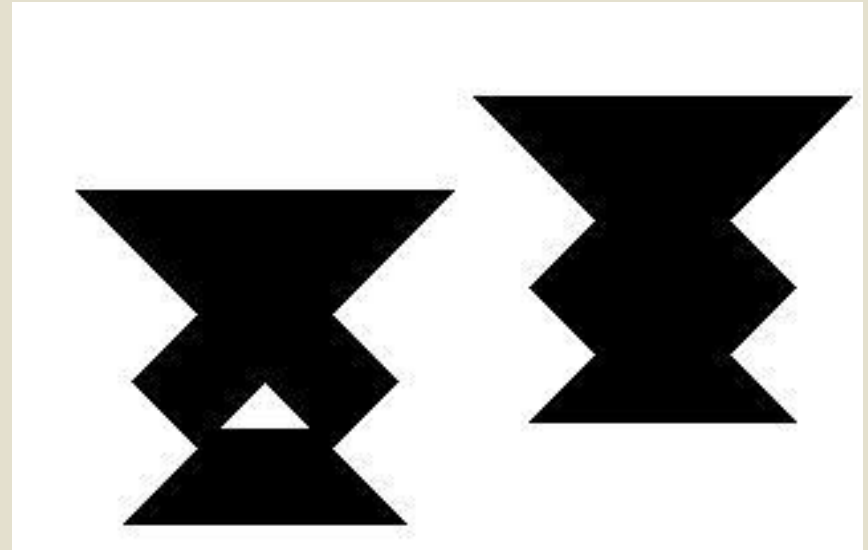
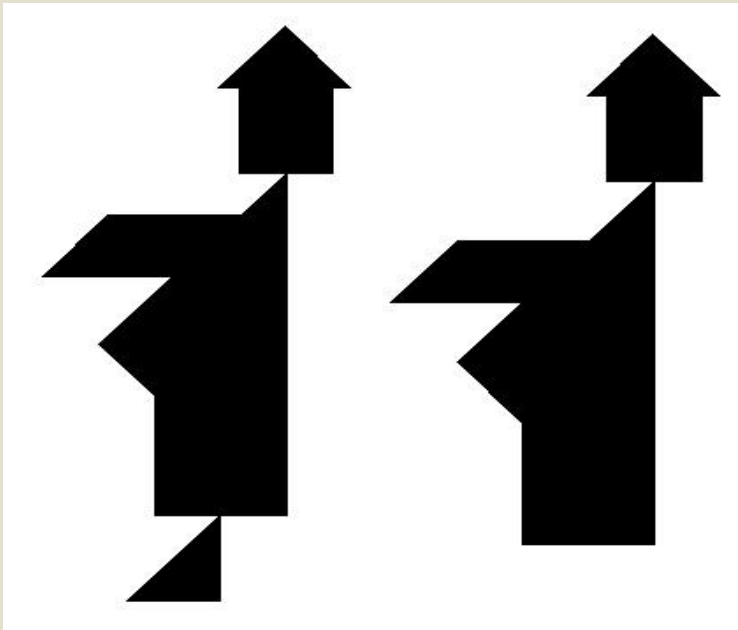
One of the most famous strategy game

Hands-on maths bag by mmaca

Materials and activities : strategy

1.6) Paradoxes (with 1 or 2 tangrams)

All the figures are composed with the 7 tangram shapes

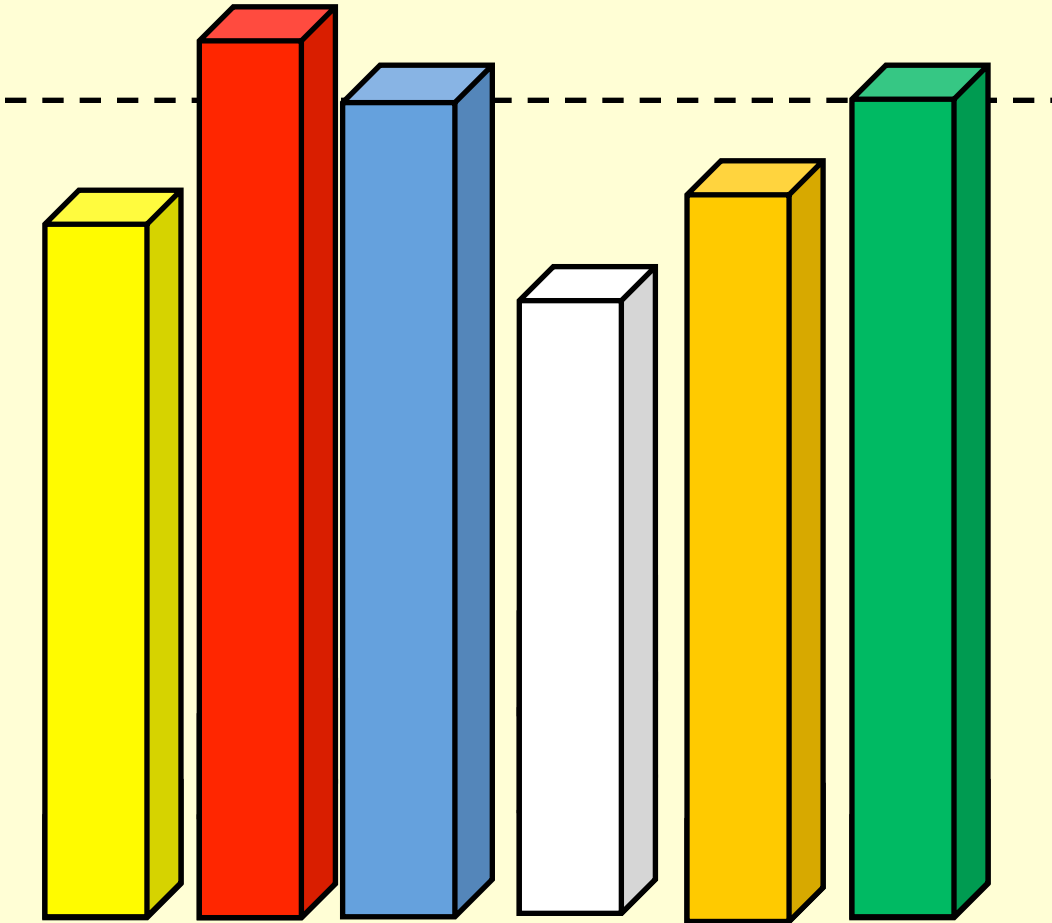


A stimulating challenge of figures composition.
You must change your point of view

Hands-on maths bag by mmaca

Materials and activities: statistics and probability

2.8) Graphs (with 10 multilinks)



Group the 10 cubes for colour and build a 3D graphic.

Combining the cubes of the whole class we can prove the Law of Big Numbers.

An easy but not trivial approach to statistics.

Hands-on maths bag by mmaca

Materials and activities: statistics and probability

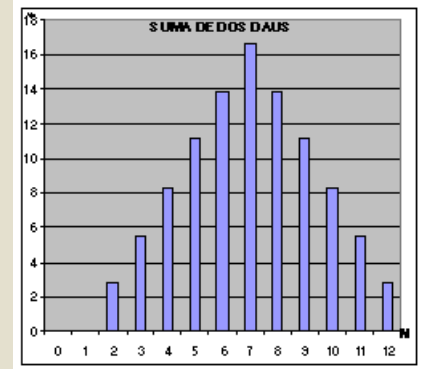
10.1-10.2) Race of numbers (a board, 2 dice)

Two different activities:

1) Adding the values: advance one step when the SUM of dice is the number you chose

2									
3									
4									
5									
6									
7									
8									
9									
10									
11									
12									

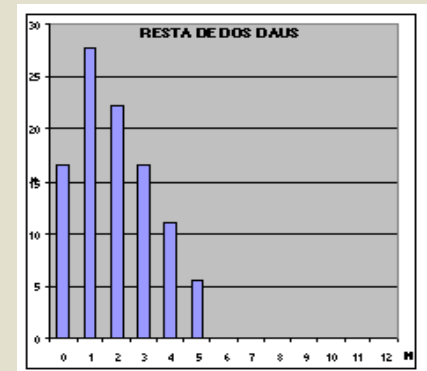
All the “horses” have the same chances for winning?



2) Subtracting the values: advance one step when the DIFFERENCE of dice is the number you chose

0									
1									
2									
3									
4									
5									

A simple, but not trivial, approach to probability



Hands-on maths bag by mmaca

Materials and activities: statistics and probability

10.3) Cross the river (a board, 2 dice, 12 chips)



Where do you will set your 12 chips to transport them faster?

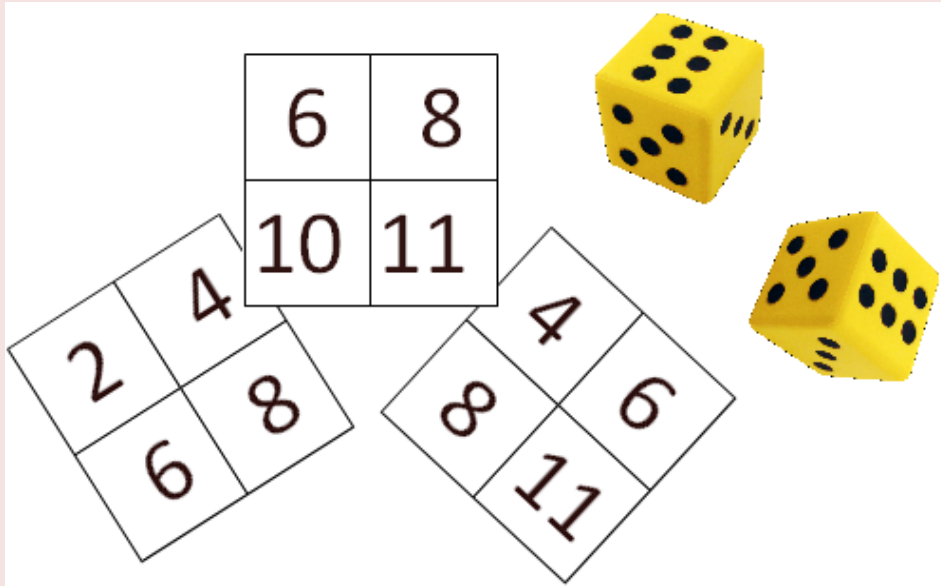


It is a good activity to verify if the concept of probability is well understood.

Hands-on maths bag by mmaca

Materials and activities : statistics and probability

10.4) Probabilistic bingo (32 cards (or more), 2 dice)



All the cards have the same chance of winning?

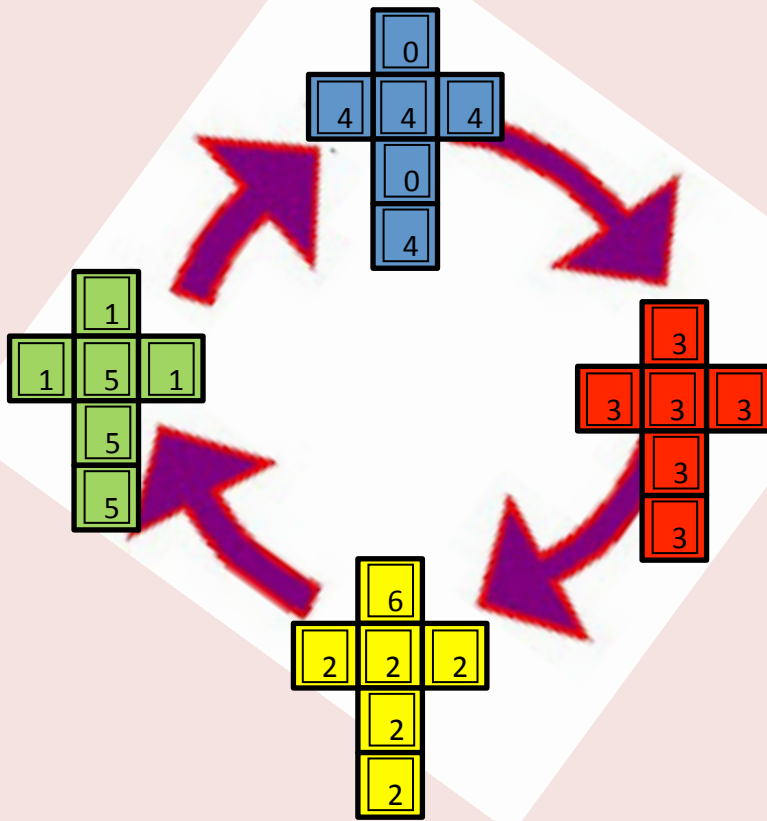
When does fortune play a more important role: choosing the cards or rolling the dice?

Another good activity to verify if the concept of probability is well understood.

Hands-on maths bag by mmaca

Materials and activities: statistics and probability

13) Nontransitive dice (4 special dices)



A simple way to reflect about our perception of probability

With 4 white dice, you can prepare your special dice as in the figure.

Then:

Green wins blue

Blue wins red

Red wins yellow

Yellow wins green

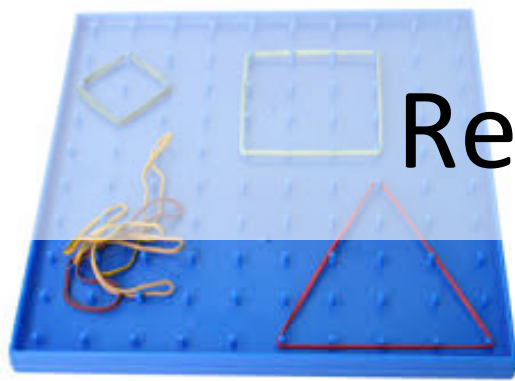
DAU 1						
	1	1	1	5	5	5
DAU 2	0					
	0					
	4					
	4					
	4					
	4					

DAU 2						
	0	0	4	4	4	4
DAU 3	3					
	3					
	3					
	3					
	3					
	3					

DAU 3						
	3	3	3	3	3	3
DAU 4	2					
	2					
	2					
	2					
	6					
	6					

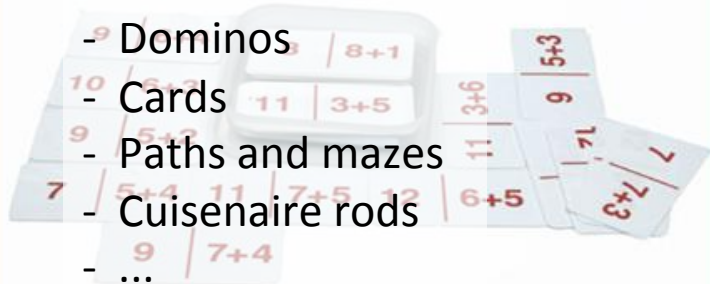
DAU 4						
	2	2	2	2	6	6
DADO 1	1					
	1					
	1					
	5					
	5					
	5					

Reinforce materials



CALCULATION:

- Dominos
- Cards
- Paths and mazes
- Cuisenaire rods
- ...



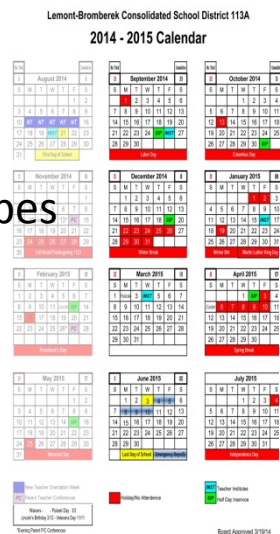
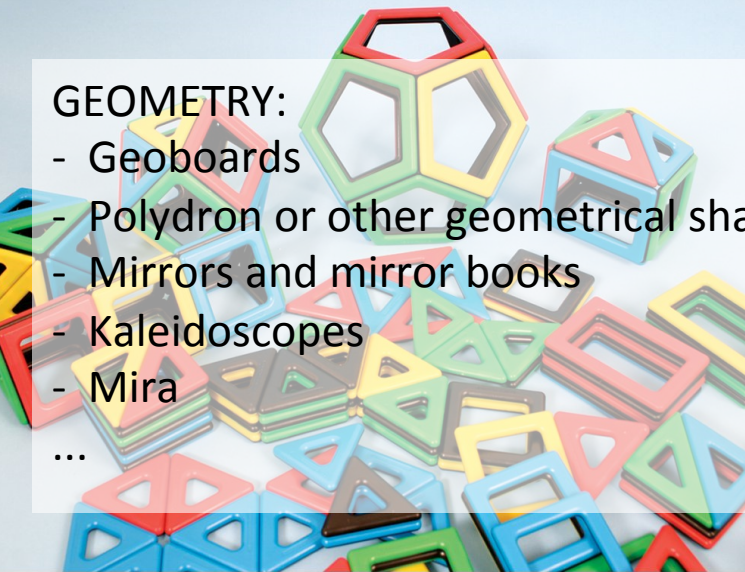
STATISTICS:

- Newspapers and magazines
- Calendars
- School organization
- Fournitures
- ...



GEOMETRY:

- Geoboards
- Polydron or other geometrical shapes
- Mirrors and mirror books
- Kaleidoscopes
- Mira
- ...



Reinforce materials

OBJECTS OF DAILY LIFE:

- Buildings
- Nature
- Structures
- Food
- Strings
- Cans, glasses and bottles
- Boxes and tetrabricks
- Small reeds
- ...

