Snowman

SURFER Experiment
by Andreas Daniel Matt

Motivation
It is winter time and the snow is falling. Time to make a snowman!

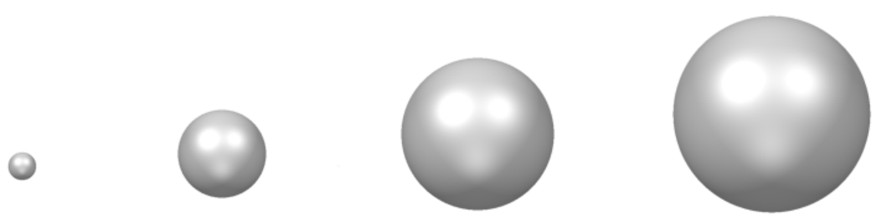
Mathematics behind it
Every snowman starts small: with a tiny snowball that is rolled in the snow to become thicker and thicker. Thus, let's prepare the smallest of all "balls", a point in space.

$$x^2+y^2+z^2=0$$

As you can see in the equation, only the point (0,0,0) solves the equation, only this point is displayed. Or, it should be displayed! However, SURFER does not show it (or at least one cannot see it). This is due to the program's raytracer technology, which makes individual points or single lines difficult or impossible to display.

But the point is there! So now let's roll it and make it bigger. This can be done by increasing the radius *r* of the snowball: $x^2+y^2+z^2=r^2$
Using SURFER, this equation needs to be rewritten to:

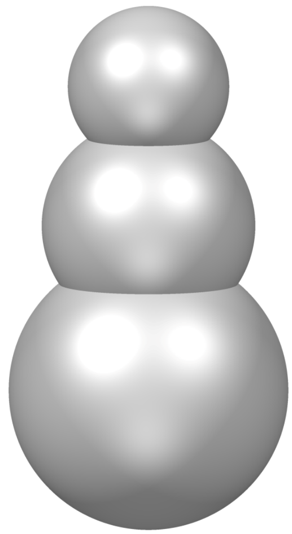
$$x^2+y^2+z^2-a^2=0$$



with *a* increasing from left to right.

All these balls have their center in the origin ($x=y=z=0$). They are displayed in the center of the visualization area of SURFER. To translate the center of the ball and therefore the whole ball itself (or any other object) for example along the y-axis by 1 unit, you subtract 1 from any *y* in the formula.

To add two more snowballs (with different sizes and positions) you simply multiply their formulas. This allows us to build the following snowman:



(x^2+y^2+z^2-1.2)(x^2+(y-1.3)^2+z^2-0.7)*(x^2+(y-2.4)^2+z^2-0.4)=0*
*(first snowball * second snowball * third snowball)*

You can see that the second snowball has been moved 1.3 units up ("*y-1.3*") and the third one 2.4 units up ("*y-2.4*"), while the second and third balls are also smaller than the first one (look at the radii of the three balls).

Now we have to join the three snowballs, i.e. melt them a bit together or add some "extra snow" between the balls. We can call this trick "melting surfaces". It is done via a "noise"-parameter that is added to the equation. Let's say we have two surfaces/equations *f* and *g* and we show both of them via the equation

*f*g=0*

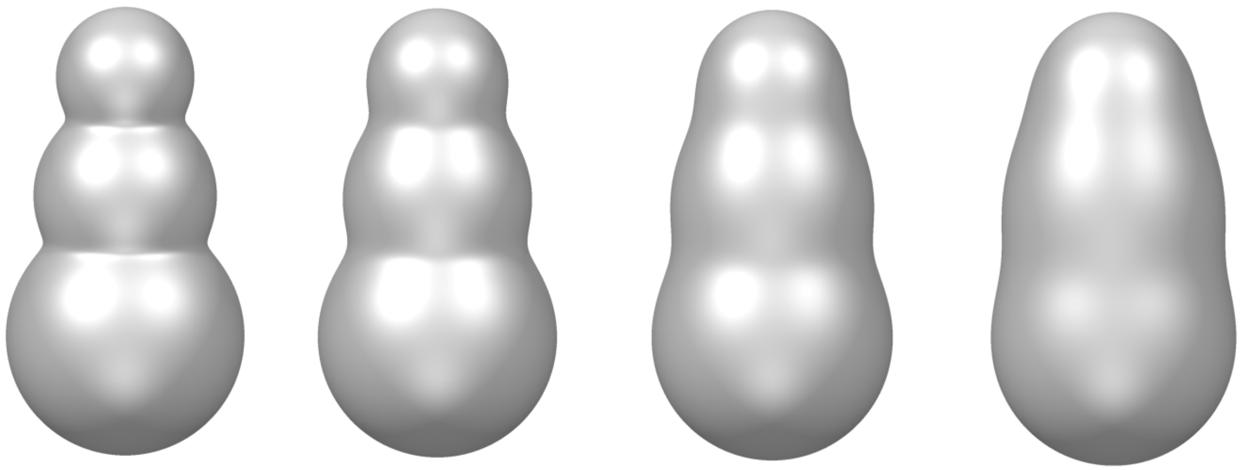
Now instead of having a product that equals zero, we can add a small parameter *a* and observe the result of the following equation

*f*g=a*

If *a* is a small parameter, which is not zero, then the two factors *f* and *g* of the product cannot independently decide the result of the product (if one alone is zero, not everything is zero). Thus they slightly depend on each other to yield *a*.

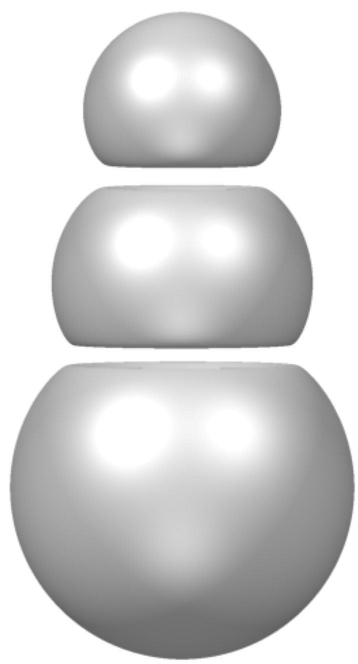
Visually, this results in a connection between the surfaces:

SURFER can be downloaded for free: http://imaginary.org/program/surfer



(x^2+y^2+z^2-1.2)(x^2+(y-1.3)^2+z^2-0.7)*(x^2+(y-2.4)^2+z^2-0.4)-a=0*
For parameters *a=0.01, a=0.1, a=0.4, a=1.0* (from left to right).

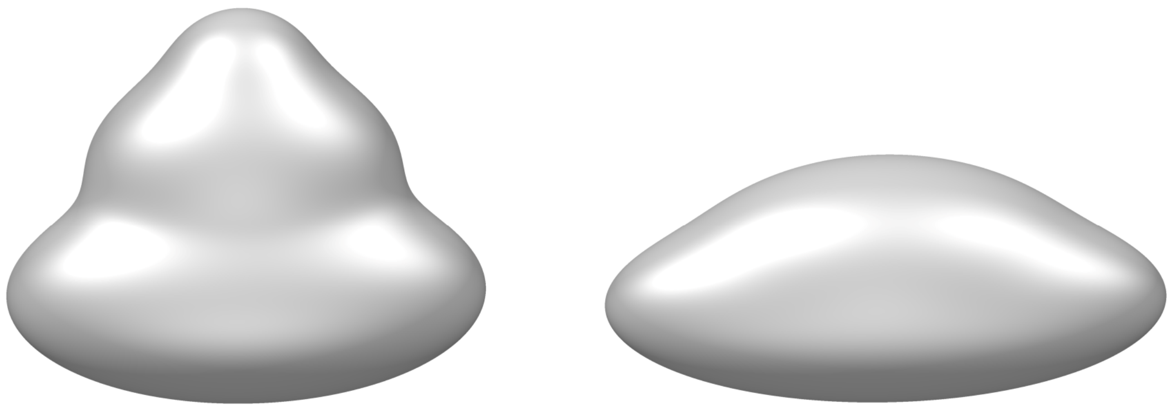
What happens if we use a negative "noise", i.e. $f*g=b$, where $b<0$. As you see instead of three balls fusioned together we obtain a separation of the balls.



(x^2+y^2+z^2-1.2)(x^2+(y-1.3)^2+z^2-0.7)*(x^2+(y-2.4)^2+z^2-0.4)+0.01=0*

Advanced Example

Spring is coming and our snowman is melting. To show the melting even more, we can increase the “noise” parameter from above and also squeeze or stretch the snow balls. To distort them horizontally, we have to add a parameter at the x-coordinates. For example:



*Left: (b*x^2+(y-1.0)^2+z^2-0.3)*(c*x^2+y^2+z^2-0.6)*(d*x^2+(y-1.7)^2+z^2-0.1)-a*
with a=0.1, b=0.33, c=0.21, d=0.7
*Right: (b*x^2+(y-0.3)^2+z^2-0.5)*(c*x^2+y^2+z^2-0.5)*(d*x^2+(y-0.5)^2+z^2-0.1)-a*
with a=0.01, b=0.33, c=0.12, d=0.23

Further Questions (without solutions)

Can you add a nose to the snowman? And a hat? And two arms? And a broom?
Try to find a suitable shape and add it to the snowman by multiplying the formulas. Adjust the shape by translating and distorting as before.

The snowman we prepared is not a real snowman, since it is not filled up with snow - you can easily zoom into the snowman and see that it is empty (yes, it is an algebraic surface). However, the very melted snowman on the right does have a shape inside. Where does it come from? Zoom closely and experiment with even smaller values of *a* to find out more about it (for instance use *0.1*a*).

What would the formula of a solid snowman be, which is completely filled with snow? These are all points inside the spheres (all the snow).