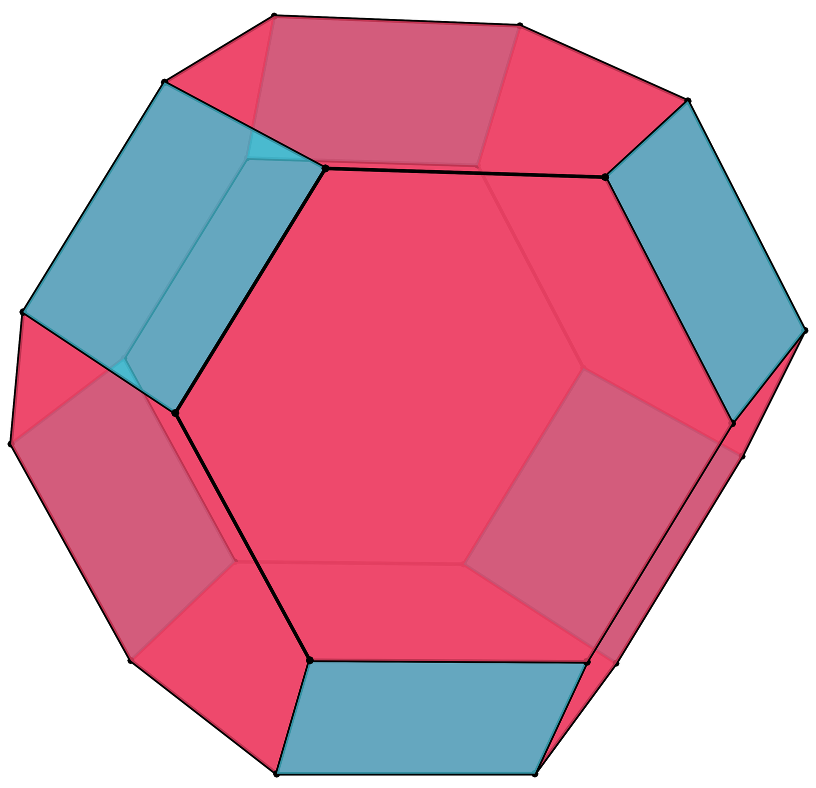
MatchTheNet



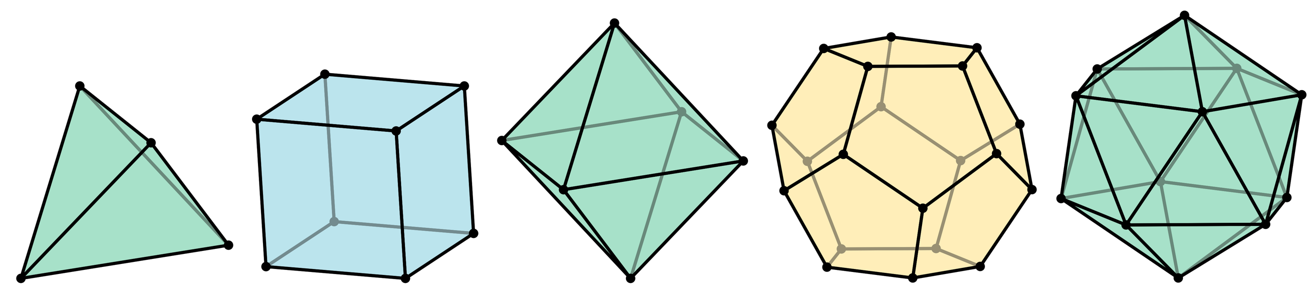
Polytopes

A *polytope* is the convex hull of finitely many points in a Euclidean space. More general are the *polyhedra*, i.e., the sets of solutions to finitely many linear inequalities. A polyhedron is a polytope if and only if it is bounded.

Polytopes and unbounded polyhedra occur in many areas of mathematics and mathematical applications. This includes discrete mathematics and optimization as well as differential and algebraic geometry.

. . . in 3 Dimensions

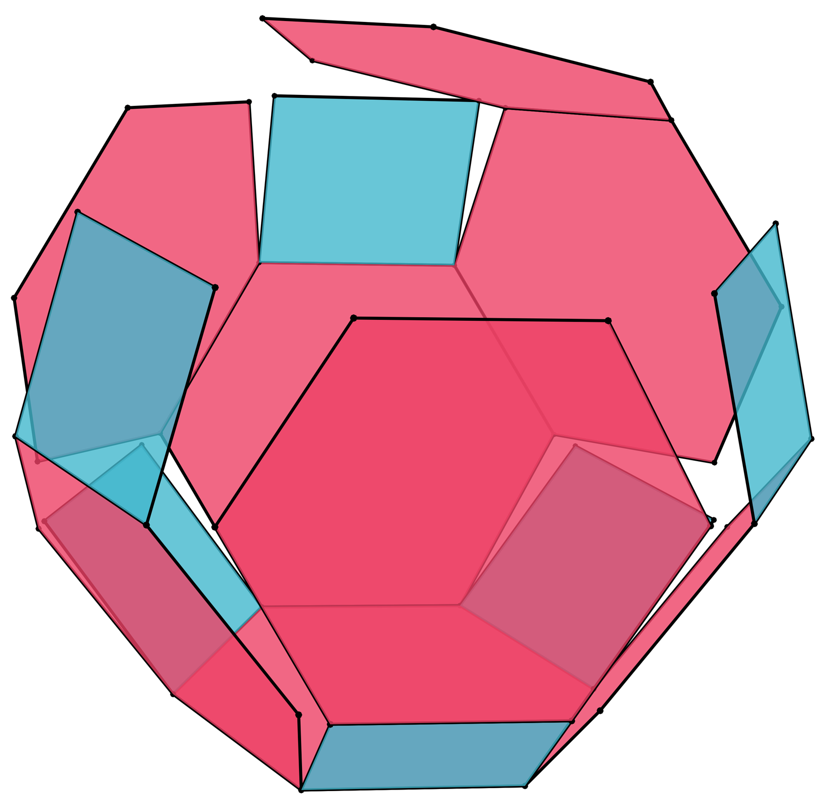
The combinatorics of 3-dimensional polytopes is determined by the vertex-edge graph, which is necessarily planar. Euler's formula reads $v - e + f = 2$, where v = number of vertices, e = number of edges, and f = number of facets.
The most classical examples are the five *Platonic solids*:



They share the property that the group of (linear) automorphisms acts transitively on the set of maximal flags (i.e., incident triplets of vertices, edges and facets). In dimension 3 these are the only polytopes with this property (up to linear isomorphy).

Much more general are the *Johnson solids*. These are those 3-dimensional polytopes whose facets are regular polygons (of various gonalities). As a special case the *Archimedean solids*, by definition, are those Johnson solids which admit a vertex-transitive group.

Again up to linear isomorphy there are 13 Archimedean solids (which are not Platonic), and there are 92 Johnson solids (which are not Archimedean). Notice that some classifications distinguish between chiral copies. This is the reason for other counts, which can occasionally be found, and a common source of errors. The duals of the Archimedean solids are known as the *Catalan solids*. Again there are 13 of those.



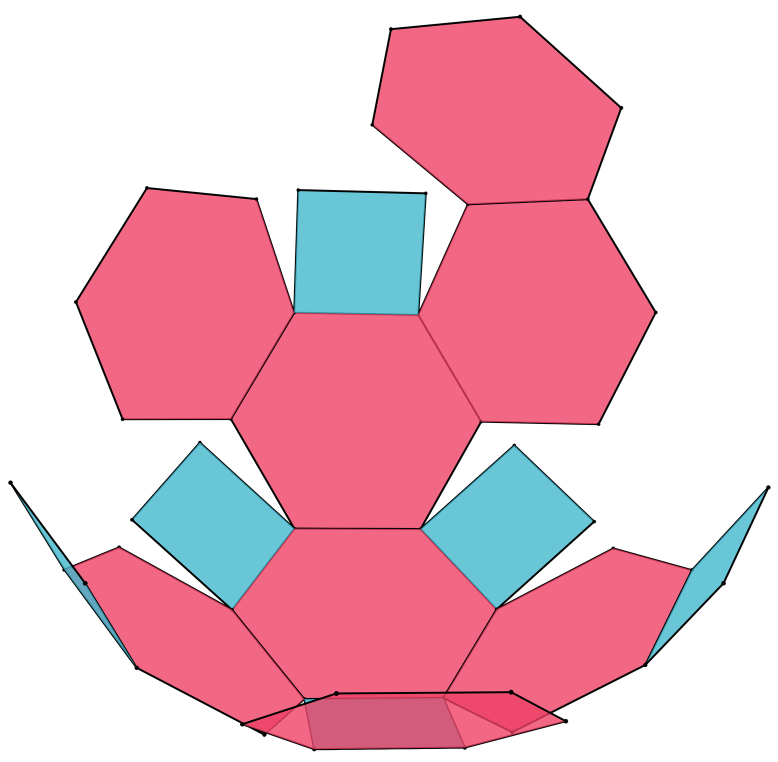
Planar Nets

Let us fix a 3-dimensional polytope P. Choosing a spanning tree T in the dual graph of P defines a *net* of P. This is a set of polygons in the plane which is formed from congruent copies of the facets of P such that the adjacency is dictated by the tree T. The net is *planar* if the set of polygons do not overlap.

It is a very challenging open question whether or not each 3-polytope admits a planar net.

To study planar nets of 3-polytopes is difficult because there are several counter-intuitive effects.
For instance, the following results are known:

- there are 3-polytopes, even tetrahedra, which do have dual spanning trees which lead to non-planar nets;
- there are pairs of non-isomorphic 3-polytopes with the same (unlabeled) planar net.

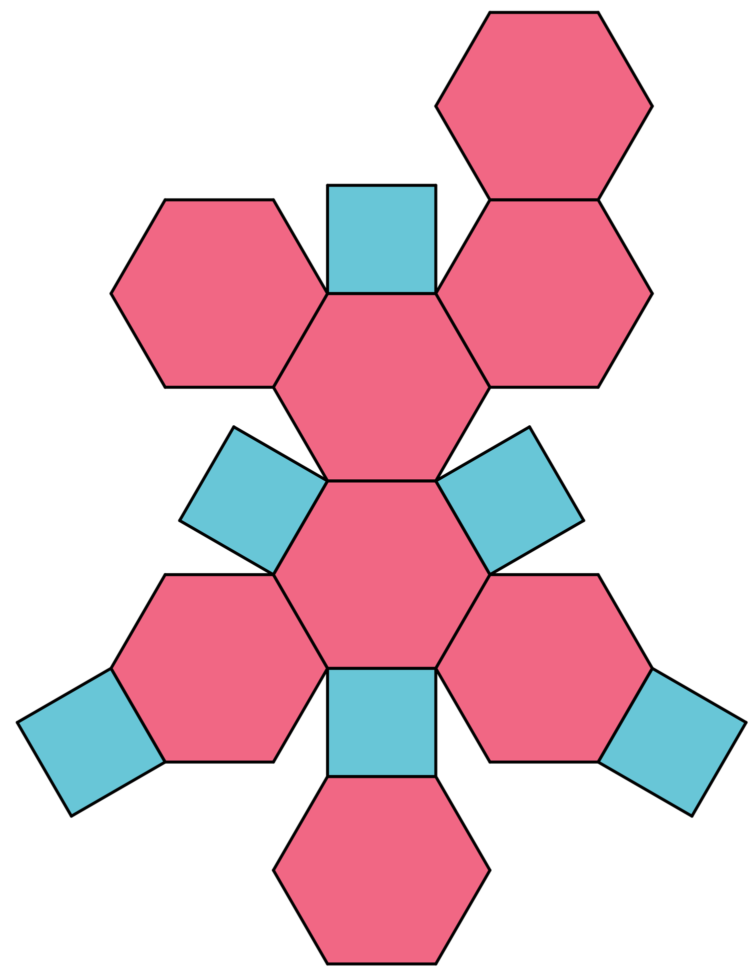


polymake

`polymake` is open source software for research in polyhedral geometry. It deals with polytopes, polyhedra and fans as well as simplicial complexes, matroids, graphs, tropical hypersurfaces, and other objects.
Supported platforms include various flavors of Linux, Free BSD and Mac OS.

An interactive unfolding, as illustrated on this poster, can be constructed with the following `polymake` commands.

```
polytope> $polytope = archimedean_solid('truncated_octahedron');
polytope> $net = fan::planar_net($polytope);
polytope> @colors = ('#0EAD69','#43B8CE','#FFD23F','#EE4266','#540D6E','#888888');
polytope> threejs( $net->VISUAL( VertexLabels => "hidden",
              VertexColor => "black",
              FacetTransparency => 0.8,
              FacetColor => sub {
                $colors[ min($net->MAXIMAL_POLYTOPES->[shift]->size-3, @colors-1) ]
              } ));
```



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