

# Maths behind the game

A consequence of Gauss's Egregium Theorem is that we cannot preserve all of the geometric properties of the Earth (a surface with positive curvature everywhere) when we try to represent it on a flat surface (with zero curvature everywhere): there must be some kind of distortion.

The rough idea behind the game is simple: *to let the players experience this distortion in such a way that they can understand mathematical properties of the map by doing this!*

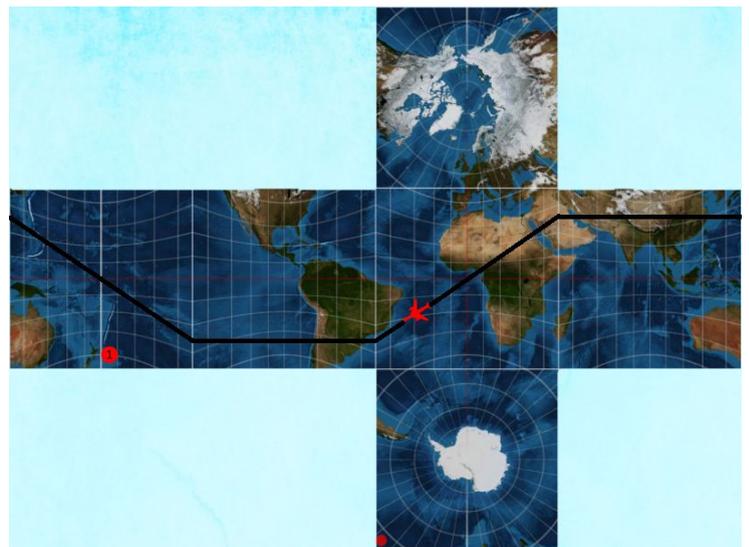
In order to do it we thought about this game: a plane which has constant speed moves above the Earth. The player can only control it by steering to the left or to the right. Hence, if no command is given by the player, the plane will follow a geodesic line at constant speed. The aim is to get 6 checkpoints in the least possible time so the aim is to follow geodesic lines and try to turn as least as possible!

The player, however, sees what is happening not on the Earth, but on a chart (that he can choose among 6 in the game settings).

The effect is that *one can easily visualize geodesic lines on the charts.*

On the right you can see the path that you follow if you fly "straight" on a Fuller projection obtained by starting from a cube.

Simply by playing, you can understand one of the outstanding features of these maps: on each face of the unfold of the polyhedron, geodesic lines are seen as straight lines (a property of Gnomonic projections). However, the speed of the plane (on the chart) is not constant along these geodesic lines and you can observe it by playing.



Not only the speed of the plane is not preserved. Indeed, the shape and the dimension of the plane are also changed as a consequence of the distortion! Consider, for example, a plane that flies "over" the Mercator's chart. From the explicit equations of the Mercator's projection one can prove that the scale factor of this map is proportional to the secant of the latitude. Roughly, this means that one figure that moves towards the pole from the center of the map gets bigger and bigger. What happens to the plane in the game is exactly this as you can see from the following images.



Note that the images are not shrunk or magnified: look at Europe's dimensions in order to convince yourself!

If we press “V” while playing, we modify the appearance of our plane: it becomes a dot with two vectors. These two vectors, have the same length on the Earth and are always orthogonal to each other; one is the direction tangent to the path that the plane is following, whereas the other identifies a normal direction.

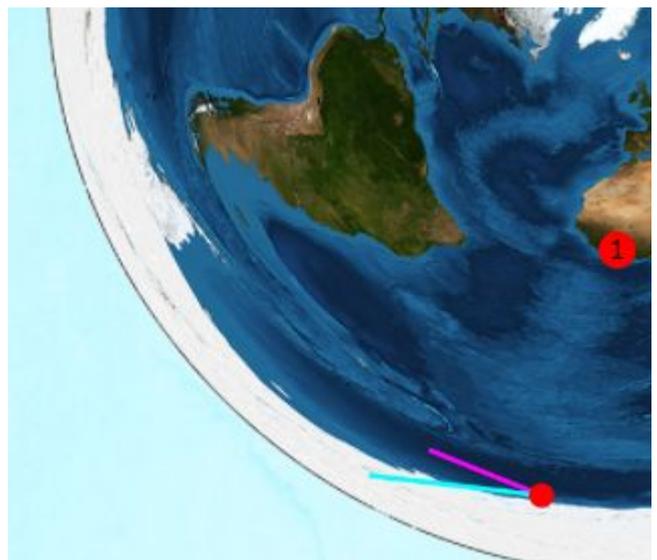
Why is this useful? Because we can understand how the distortion of the map goes along the tangent and normal direction by observing the length of the two vectors. We can also see how a right angle moves on the chart.

Mercator’s map has a peculiarity: it is conformal, i.e. angles on the sphere are correctly measured on the chart. We can see it by flying on our map in the way depicted in the following images: the angle formed by the two vectors is always a right angle.



This may not happen in other charts: for example, these are screenshots of a gameplay respectively on a sinusoidal chart and on an azimuthal equidistant chart.

Do you like these right angles?

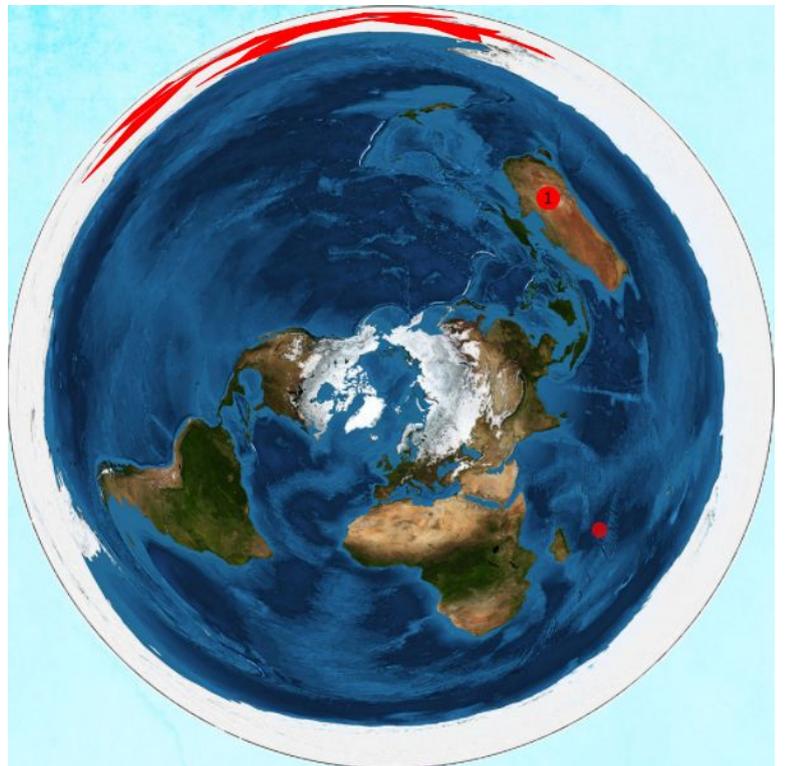


With the “vector mode” you can also convince yourself that some maps are equal-area, i.e. the ratio between two areas measured both on Earth and on the map is always the same. On such a map, the two vectors can appear not orthogonal, but the area of the parallelogram which has them as adjacent sides is constant. There are two maps with this property on which we can play: Gall-Peter’s map and sinusoidal map. If you turn off the “vector mode” you can observe the same fact by looking at how big is your plane: basically, *it behaves like a moving Tissot’s indicatrix!*

Summing up: *you can easily detect angles distortion, directional scale factor, which maps are conformal or equal-area and which are not.*

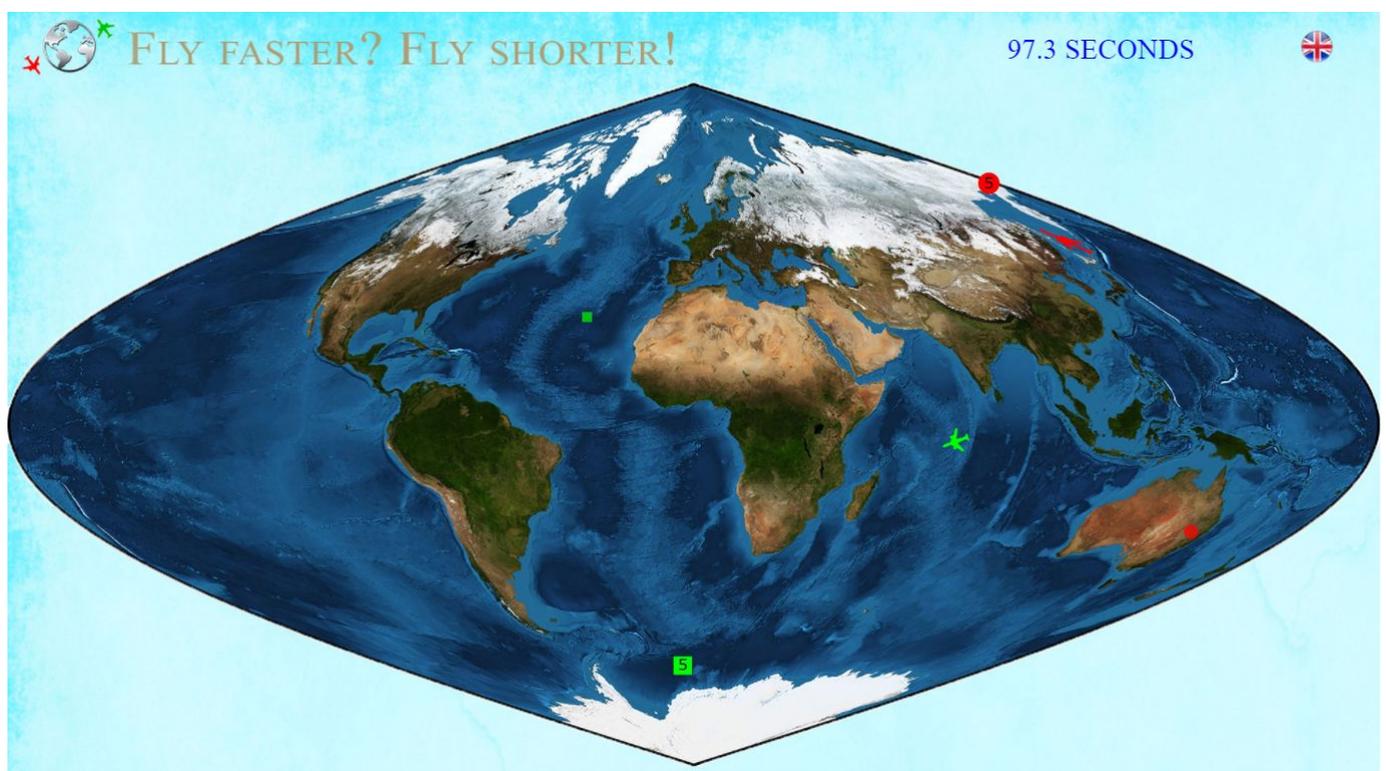
Is this not enough?

- You can see which points of the Earth, if any, is not represented on the chart. For example, in the azimuthal chart, shown here on the right, the South Pole “is” the whole boundary of the circle. And your plane is that “red thing” in the upper part of the picture!
- You can show or hide meridians and parallels in each chart by pressing “G” while playing or while choosing the map in order to help you on your race.
- You can play against a friend!
- You can enable gamepads and play with them.
- If you need it, you can enable colorblind mode.



But most important of all...

*You can learn while having fun!*



*Enjoy the game!*