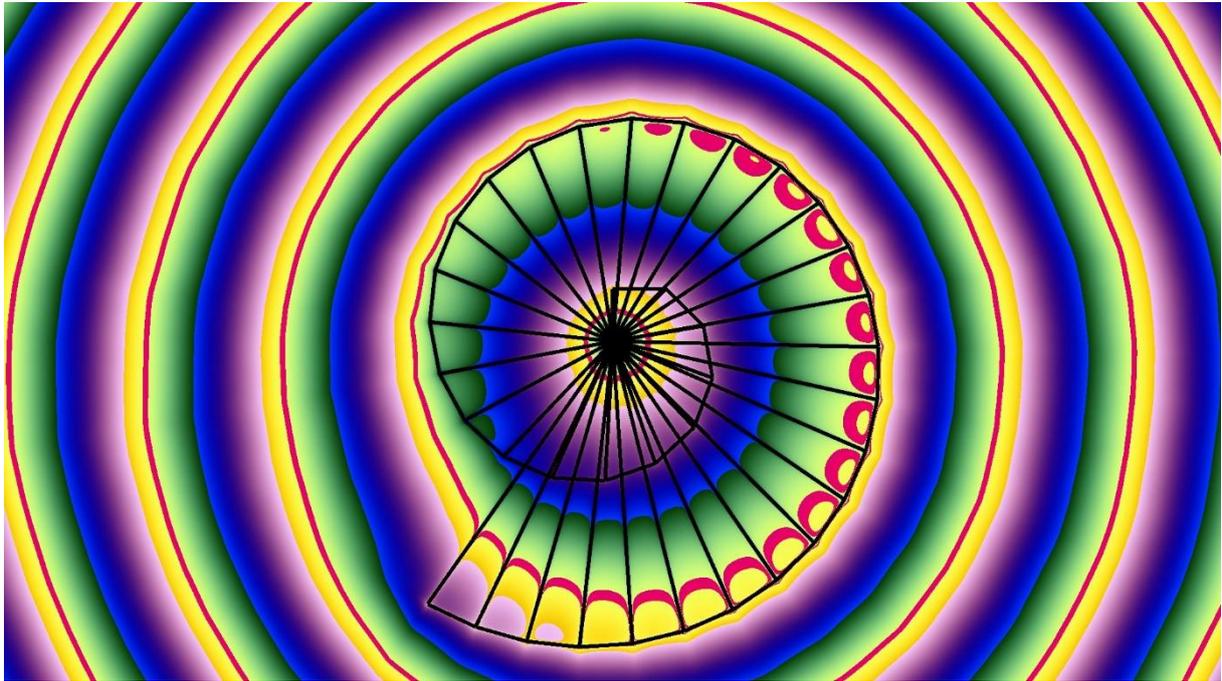


MathLapse - Spiral of Theodorus

Idea and programming by Christian Gaier (Steyr, Austria)



Constructing iteratively square roots of integers with right angle triangles, a spiral is obtained which is named after Theodorus of Cyrene (5th century BC).

The background rhythm is composed and programmed on the drum machine Korg DDD-5. It consists of superimposed triplets and quintuplets yielding a complex rhythmic pattern with mathematical precision.

How to draw “Cybernetic Mandalas”

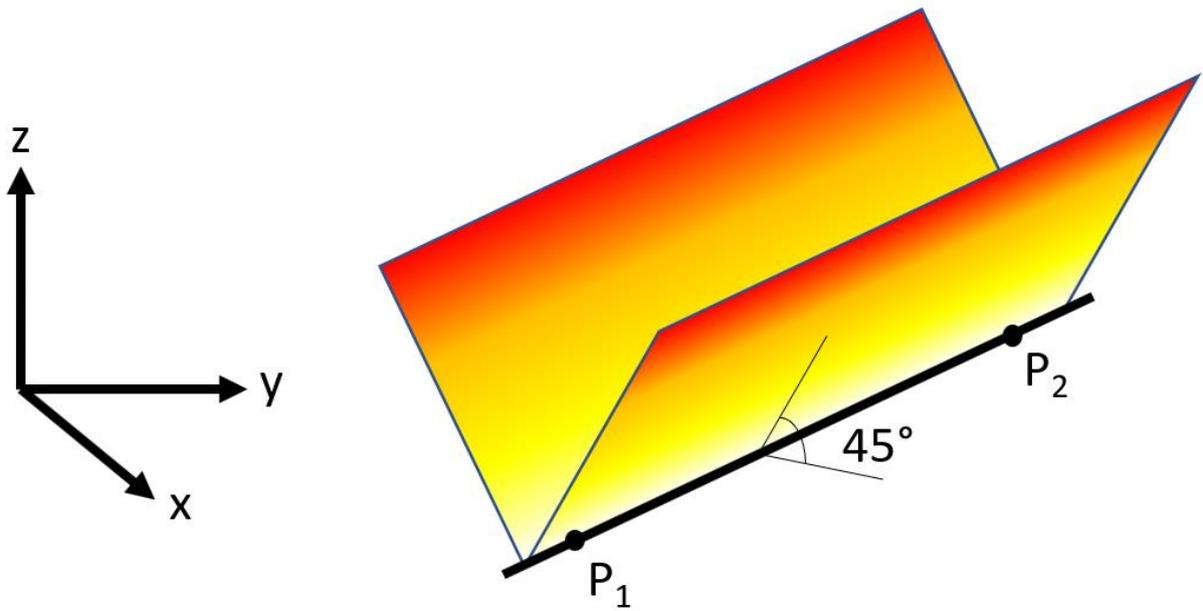
Colorful pictures like the above one with drawings consisting of simple geometric elements (line, circle, arc) can be created by a pinch of mathematics and a few lines of program code. The basic idea is to compose mathematical functions which deliver zero or a positive value for each pixel of the drawing area with the coordinates x and y :

$$f(x, y) \geq 0$$

To each function value a color will be assigned by applying an appropriate mapping rule which generates the byte code for the three primary colors red, green and blue. There are infinite possibilities for creative combinations of colors in dependence of the function value.

For $f(x, y) = 0$ the color black will be assigned to the pixel with coordinates x and y . Therefore, to draw black lines together with a colorful surrounding, a function need to be composed, which is zero at the position of the lines and larger than zero elsewhere.

Let us consider the simplest case of an infinite line with zero thickness and defined by two points. As function a simple plane can be used, whose negative part is folded to the positive domain $z > 0$:



The parameters of the equation for the plane are chosen in such a way to obtain an inclination of one (equivalent to an angle of 45 degree):

$$z = f(x, y) = |a_{12} \cdot x + b_{12} \cdot y + c_{12}|$$

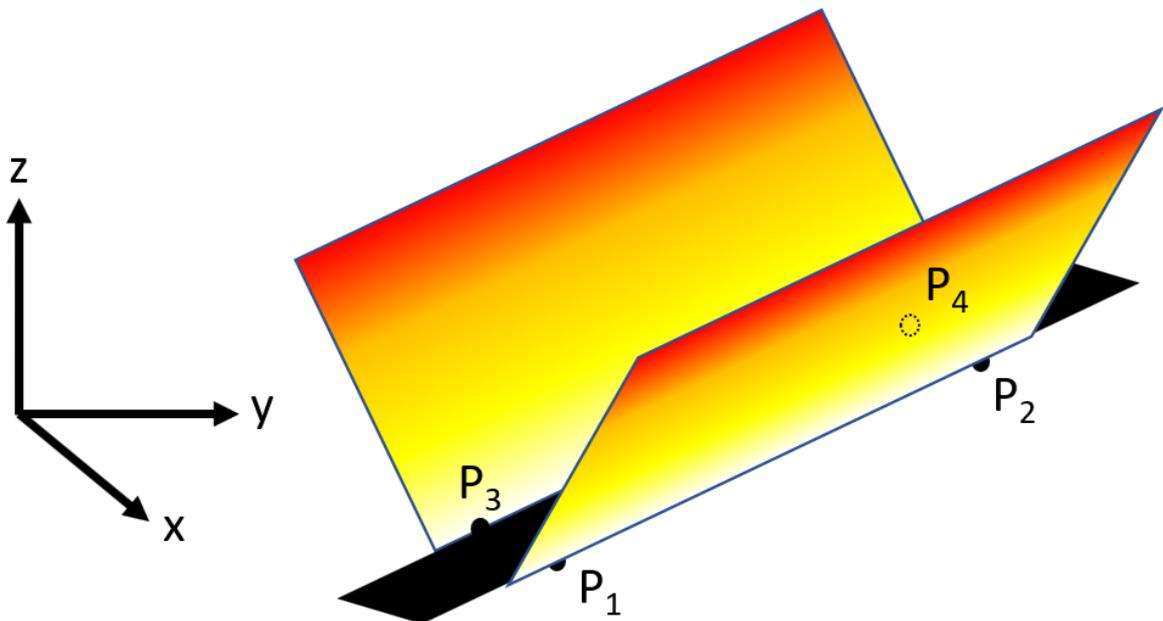
with

$$a_{12} = \frac{y_1 - y_2}{\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}}$$

$$b_{12} = \frac{x_2 - x_1}{\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}}$$

$$c_{12} = -a_{12} \cdot x_1 - b_{12} \cdot y_1$$

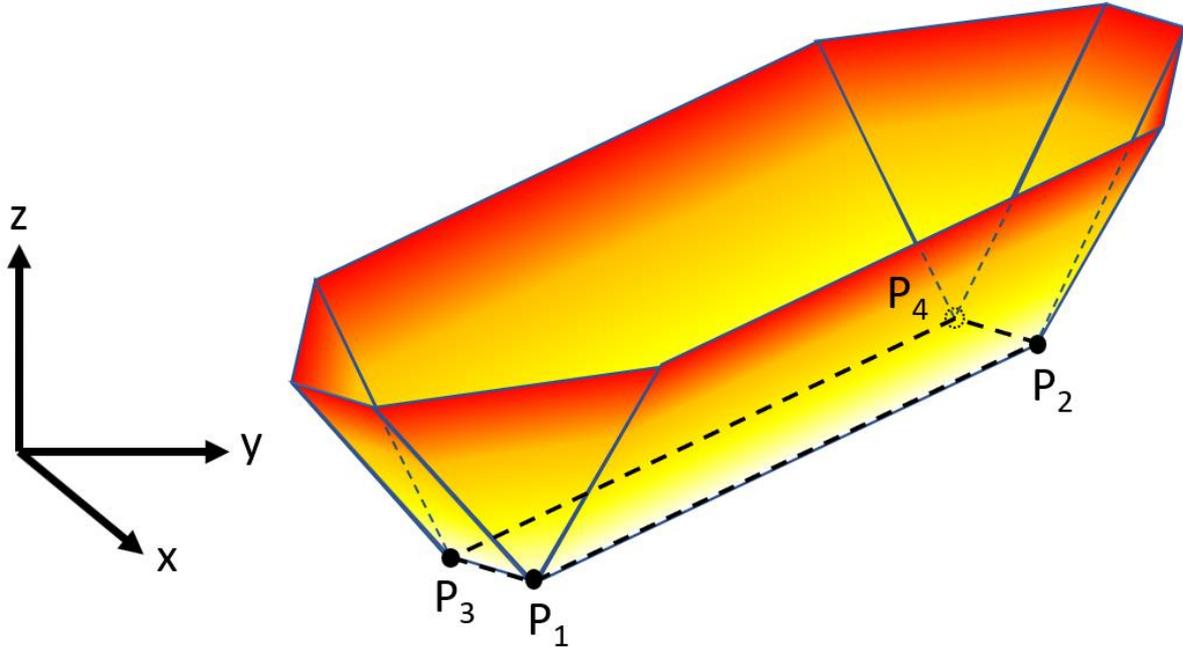
As next step, an infinite line with a thickness larger than zero shall be constructed. This can be easily achieved by adding two of such V-shaped folded planes and ground them to zero:



Placing two points at each side of the line with thickness t , following formula can be composed:

$$z = |a_{12} \cdot x + b_{12} \cdot y + c_{12}| + |a_{34} \cdot x + b_{34} \cdot y + c_{34}| - t$$

Now let us increase the complexity once more. We want to draw a line with a given finite length and a given thickness with a colorful surrounding. Again, this can be easily achieved by adding four of such V-shaped folded planes and ground them to zero:



Placing four points at the corner of the line with length l and thickness t , following formula can be composed:

$$z = |a_{12} \cdot x + b_{12} \cdot y + c_{12}| + |a_{34} \cdot x + b_{34} \cdot y + c_{34}| - t \\ + |a_{13} \cdot x + b_{13} \cdot y + c_{13}| + |a_{24} \cdot x + b_{24} \cdot y + c_{24}| - l$$

Now a function was found to draw a single line with a given length and a given thickness at an arbitrary position of the x-y plane. The areas, where the function delivers zero, will be colored black, whereas to the areas with function values > 0 different colors will be applied. This procedure delivers a black line with a colorful surrounding.

Finally, we want to generate more complex drawings consisting of several black lines with colorful surroundings. For each line numbered with $i = 1, 2, 3, \dots, n$, above formula delivers n values $z_1, z_2, z_3, \dots, z_n$. The final function is the geometric mean value of these partial functions:

$$f(x, y) = \sqrt[n]{\prod_{i=1}^n z_i}$$

This function guarantees, that the black areas of the partial functions z_i with zero value remain black.

For the example presented above the following rules for the assignment of colors have been used:

$$red = 255 \cdot (1.0 - (\sin(0.03 \cdot f(x, y) - 1.6))^2)$$

$$green = 255 \cdot (0.0 - (\sin(0.03 \cdot f(x, y) - 1.3))^2)$$

$$blue = 255 \cdot (0.5 - (\sin(0.03 \cdot f(x, y) - 1.0))^2)$$

These rules are applied for $f(x, y) > 0$.

For $f(x, y) = 0$ the color black will be assigned to the pixel with coordinates x and y :

$$\mathit{red} = 0$$

$$\mathit{green} = 0$$

$$\mathit{blue} = 0$$