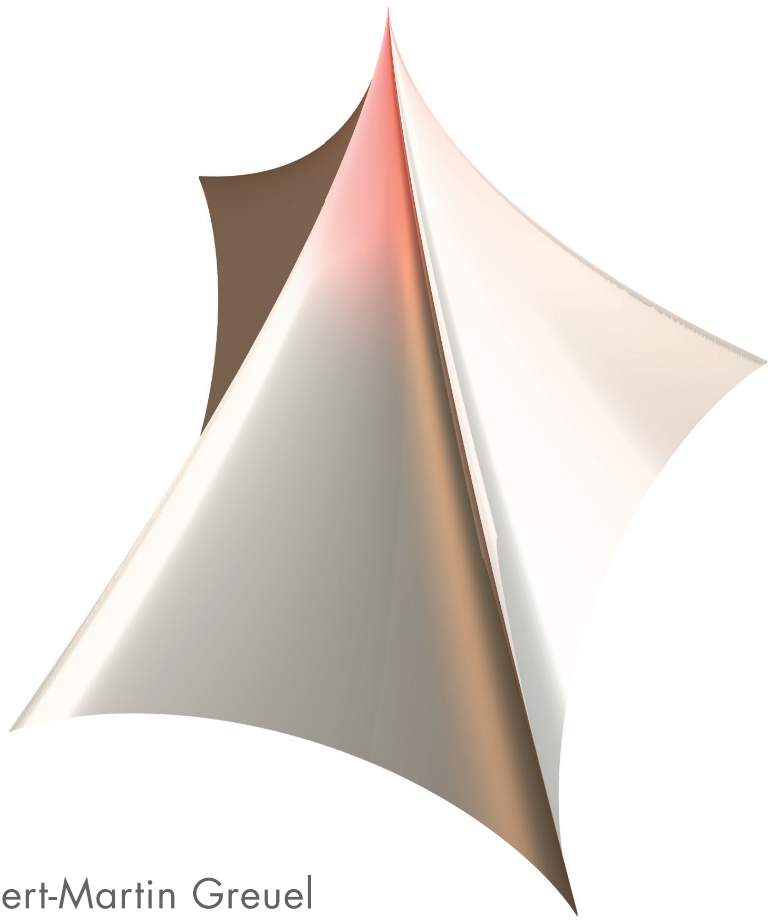
Mathematics Communication for the Future

Mission and Implementation



Gert-Martin Greuel

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Mathematics Communication for the Future – Mission and Implementation

Gert-Martin Greuel

This essay addresses an audience without any special mathematical education. It consists of two parts. The first part is more philosophical in nature, in which I illuminate the tensions between research, application, and communication in mathematics. I shall look at these conflicting tensions and try to reveal some of the causes that lie underneath. I will focus on the question whether it is possible or necessary to transmit an understanding of mathematics to the general public.

The second part deals with the IMAGINARY project, which is a successful attempt to interpret and communicate mathematics to a broad audience. A large part are exhibitions that are mainly about mathematics as an art where mathematical concepts are presented as attractive and interactive visual creations. The aim of IMAGINARY, which is driven by an international collaborating network, is to interest people in mathematics by showing them the beauty of mathematical objects and fill them with inspiration and imagination.

Part I: Mathematics between Research, Application, and Communication

Possibly more than any other science, mathematics of today finds itself between the conflicting demands of research, application, and communication.

A great part of modern mathematics regards itself as searching for inner mathematical structures just for their own sake, only committed to its own axioms and logical conclusions. To do so, neither assumptions nor experience nor applications are needed or desired.

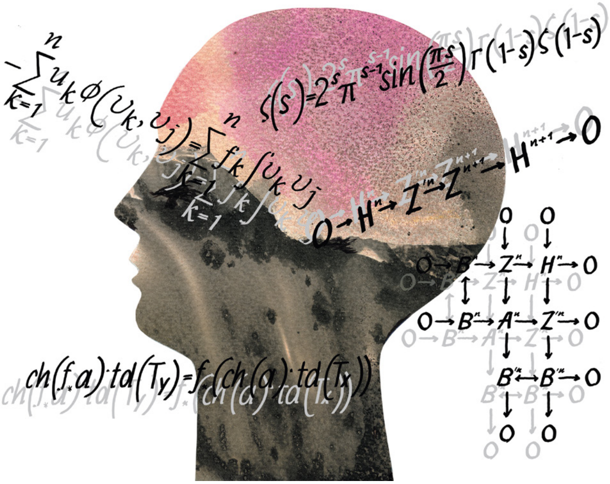
On the other hand, mathematics has become one of the driving forces in scientific progress and moreover, has even become a cornerstone for industrial and economic innovation. However, public opinion stands in strange contrast to this, often displaying a large amount of mathematical ignorance.

Wise Words

Let me introduce my conception regarding research, application, and communication by first quoting some celebrated personalities, and developing my own point of view afterwards.

Research

In this context, I mean by research pure scientific work carried out at universities and research institutes. Trying to explain concrete mathematical research to a non-mathematician is one of the hardest tasks, if at all possible. But it is possible to explain the motivation of a mathematician to do research.



Therefore, my first quote refers to this motivation; it is from Albert Einstein (physicist 1879–1955) from 1932:

> *"The scientist finds his reward in what Henri Poincaré calls the joy of comprehension, and not in the possibilities of application to which any discovery of his may lead."*

Indeed, the "joy of comprehension" is both motivation and reward at the same time and from my own experience I know that most scientists would fully agree with this statement.

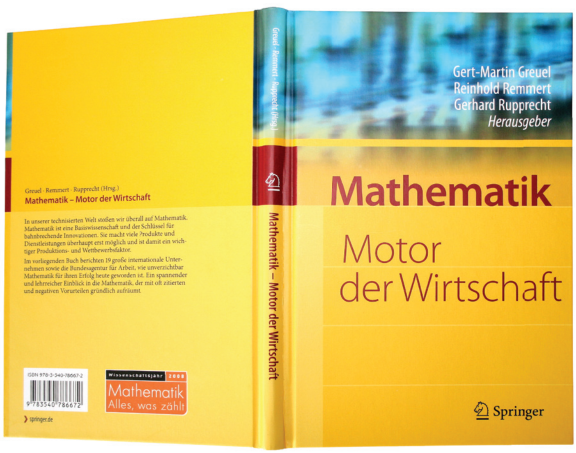
Application

One could hold numerous lectures on the application of mathematics, probably forever. The involvement of mathematics in other sciences, economics, and society is so dynamic that after having demonstrated one application one could immediately continue to lecture on the resulting new applications. The statement concerning the application of mathematics consists of three quotes, in chronological order:

> *"Mathematics is the language in which the universe is written."*
>
> Galileo Galilei (mathematician, physicist, astronomer; 1564–1642).

> *"Mathematical studies are the soul of all industrial progress."*
>
> Alexander von Humboldt (natural scientist, explorer; 1769–1859).

> *"Without mathematics one is left in the dark."*
>
> Werner von Siemens (inventor, industrialist; 1816–1892)



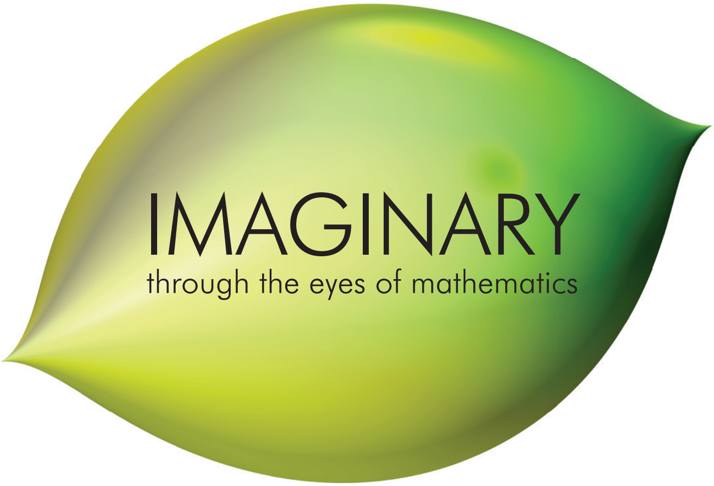
In 2008, the German Year of Mathematics, the book *"Mathematik—Motor der Wirtschaft"*, *(Mathematics— Motor of the Economy)* was published, giving 19 international enterprises and the German Federal Employment Agency a platform to describe how essential mathematics has become for their success. The main point of this publication was not to demonstrate new mathematics, but to show that, in contrast to a great proportion of the general public, the representatives of economy and industry are well aware of the important role of mathematics.

The above statements seem to indicate that there is a direct relation between mathematical research and applications. In fact, recent research directions in geometry are motivated by new developments in theoretical physics, while research in numerical analysis and stochastic is often directed by challenges from various fields of application. On the other hand, the development of an axiomatic foundation of mathematics is guided by trying to formalize mathematical structures in a coherent way and not by the motivation to understand nature or to be useful in the sense of applications. Partly due to this development, it appears that the relationship between research-orientated or pure mathematics on the one hand and application-orientated or applied mathematics on the other hand is not without its strains. Some provocative statements in this article will illustrate this.

Communication

Each of us, whether a mathematician or not, is aware how difficult it is to communicate mathematics. Hans Magnus Enzensberger (German poet and essayist, born 1929) discussed the problem of communicating mathematics on a literary basis in 1999. He writes:

> *"Surely it is an audacious undertaking to attempt to interpret mathematics to a culture distinguished by such profound mathematical ignorance."*



The exhibition *IMAGINARY—through the eyes of mathematics* is one attempt to interpret and communicate mathematics to a broad audience and there exist many other examples of successful communication. Nevertheless, the problem remains and will be discussed later when I shall give some reasons why it is so difficult.

Research and Application – Theory and Practice

Everyday Applications of Mathematics

When we talk about the application of mathematics, we face the fact that mathematics is essential for new and innovative developments in other sciences as well as in the economy and for industry. I do not claim that only mathematics can provide innovation, but it is no exaggeration to claim that mathematics has become a key technology behind almost all common and everyday applications, which includes the design of a car, its electronic components and all security issues, safe data transfer, error correction codes in digital music players and mobile phones, the optimization of logistics in an enterprise and even the design and construction of large production lines. We may say that:

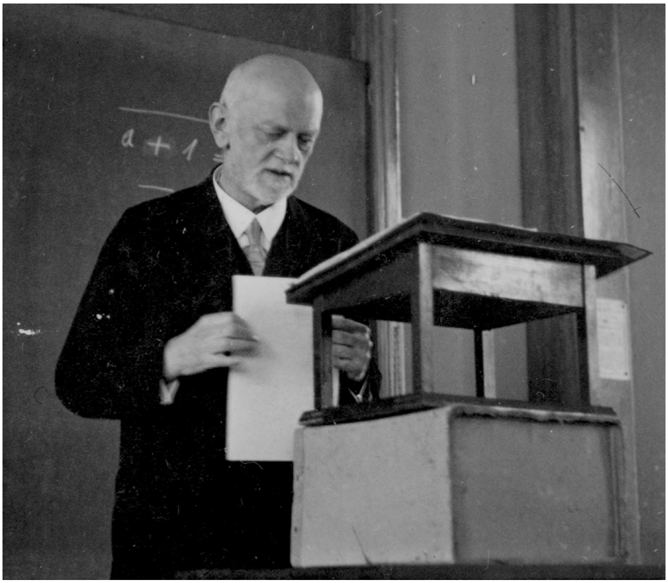
"Mathematics is the technology of technologies."

However, since the mathematical kernel of an innovation is in most cases not visible, the relevance of mathematics is either not acknowledged by the general public or simply attributed to the advances of computers.

Hilbert's Vision

It cannot be denied and is a simple and easily verifiable fact that mathematics is applied in our everyday life and that the application of mathematics in industry and the economy is a part of our utilization of nature. However, according to David Hilbert (mathematician; 1862–1943), mathematics, and only mathematics, is the basis of our understanding of nature in a fundamental sense.

> *"The tool implementing the mediation between theory and practice, between thought and observation is mathematics. Mathematics builds the connecting bridges and is constantly enhancing their capabilities. Therefore it happens that our entire contemporary culture, in so far as it rests on intellectual penetration and utilization of nature, finds its foundation in mathematics."*



David Hilbert, lecture in Göttingen, 1932

Based on his belief, Hilbert tried to lay the foundation of mathematics on pure axiomatic grounds, and he was convinced that it was possible to prove that they were without contradictions. The inscription on his gravestone in Göttingen expresses this vision with the words: *"We must know—we will know"*. Today it is no longer possible to fully adhere to Hilbert's optimism, due to the work of Gödel on mathematical logic showing that the truth of some mathematical theories is not decidable within mathematics. But Hilbert's statement about the utilization of nature is truer than ever.

Gravestone of Hilbert: "We must know – we will know"

On the other hand, this is no reason to glorify mathematics or to consider it superior to other sciences. First of all, the utilization of nature is not possible with mathematics alone. Many other sciences contribute, though differently, in the same substantial way. Secondly, the utilization of nature cannot be considered as an absolute value, as we know today. We are all a part of nature and we know that utilization, as necessary as it is, can destroy nature and therefore part of our life.

Pure Versus Applied Mathematics

Godfrey Harold Hardy

I use the terms "pure" and "applied" mathematics although it might be better to say "science-driven" and "application-driven" mathematics. In any case, here is a very provocative and certainly lordy quotation of Godfrey Harold Hardy (mathematician; 1877–1947) from his much quoted essay A Mathematician's Apology:

> *"It is undeniable that a good deal of elementary mathematics [...] has considerable practical utility. These parts of mathematics are, on the whole, rather dull; they are just the parts which have least aesthetic value. The 'real' mathematicians, the mathematics of Fermat and Euler and Gauss and Abel and Riemann, are almost wholly 'useless'."*

Hardy distinguishes between "elementary" and "real" (in the sense of interesting and deep) mathematics. The essence of his statement has two aspects: elementary mathematics, which can be applied, is unaesthetic and dull, while "real" mathematics is useless.

I think Hardy is wrong in both aspects. Of course there exist interesting and dull mathematics. Mathematics is interesting when new ideas and methods prove to be fruitful in either solving difficult problems or in creating new structures for a deeper understanding. Routine development of known methods almost always turns out to be rather dull, and it is true that many applications of mathematics to, say, engineering problems are routine. However, this is not the whole story. Before applying mathematics, one has to find a good mathematical model for a real world problem, and this is often not at all elementary or trivial but a very creative process. This point is completely missing in Hardy's essay. Maybe, because he did not consider this as mathematics at all.

His other claim must also be refuted. Very deep and interesting results of "real" mathematics have become applicable, as we now know. That is, the border between interesting and dull mathematics is not between pure and applicable mathematics, but goes through any sub-discipline of mathematics, independent of whether it is applied or pure.

Applications Cannot Be Predicted—The Lost Innocence

Nowadays we know better than in Hardy's time that his statement about the uselessness of pure mathematics is wrong. The following quote by Hardy concerns his own research field, number theory.

> *"I have never done anything 'useful'. No discovery of mine has made, or is likely to make, directly or indirectly, for good or ill, the least difference to the amenity of the world."*

Shortly after Hardy's death the methods and results from number theory became the most important elements for public-key cryptography, which today is used millions of times daily for electronic data transfer in mobile phones and electronic banking. For his claim that deep and interesting mathematics is useless, Hardy calls Gauß and Riemann and also Einstein his witnesses:

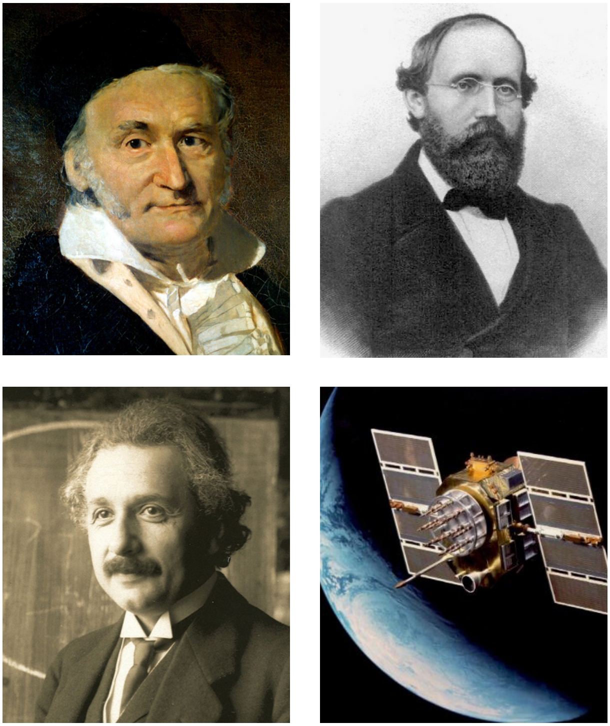
"The great modern achievements of applied mathematics have been in relativity and quantum mechanics, and these subjects are, at present at any rate, almost as 'useless' as the theory of numbers. It is the dull and elementary parts of applied mathematics, as it is the dull and elementary parts of pure mathematics, that work for good or ill. Time may change all this. No one foresaw the applications of matrices and groups and other purely mathematical theories to modern physics, and it may be that some of the 'highbrow' applied mathematics will become 'useful' in as unexpected a way."

The last sentence indicates that Hardy himself was skeptical about his own statements, although he did not really believe in the possibility of "real" mathematics becoming useful. However, the development of GPS, relying on the deep work of Gauss and Riemann on curved spaces and on Einstein's work on relativity, proves the applicability of their "useless" work.

This is not about blaming Hardy because he did not foresee GPS or the use of number theory in cryptography. Nevertheless, his strong statements are somewhat surprising as he was of course aware that, for example, Kepler used the theory of conic sections, a development of Greek mathematics without intended purpose, in order to describe the planetary orbits. So, what was the reason that Hardy insisted on the uselessness of "real" mathematics?

In my opinion we can understand Hardy's strong statements, made in 1940 at the beginning of World War II, only if we know that he was a passionate pacifist. It would have been unbearable for him to see that his own mathematics could be useful for the purpose of war. He was bitterly mistaken.

We all know that nowadays the most sophisticated mathematics, pure and applied, is a decisive factor in the development of modern weapon systems. Without GPS, and hence without the mathematics of Gauss and Riemann, this would have been impossible. Even before that, the atomic bomb marked the first big disillusionment for many scientists regarding the innocence of their work. If there was ever a paradise of innocence with no possibility for mathematics to do 'good or ill' to mankind, it was lost then.



From Gauss through Riemann and Einstein to GPS

Applicability Versus Quality of Research

History shows, and the statements of Hardy prove this, that it is impossible to predict which theoretical developments in mathematics will become "useful" and will have an impact on important applications in the future. The distinction between pure and applied mathematics is more a distinction between fields than between applicability. Quite often we notice that ideas from pure research, only aiming to explore the structure of mathematical objects and their relations, become the basis for innovative ideas creating whole new branches of economic and industrial applications. Besides number theory for cryptography, I would like to mention logic for formal verification in chip design, algebraic geometry for coding theory, computer algebra for robotics, and combinatorics for optimization applied to logistics, to name just a few.

Although the list of applications of pure mathematics could be easily enlarged, it is also clear that some parts of mathematics are closer to applications than others. These are politically preferred and we can see that more and more national and international programs support only research with a strong focus on applications or even on collaboration with industry.

I think the above examples show that it would be a big mistake, if applicability were to become the main or even the only criterion for judging and supporting mathematics. In this connection I like to formulate the following:

> *Thesis. The value of a fundamental science like mathematics cannot be measured by its applicability but only by its quality.*

History has shown that in the long run, quality is the only criterion that matters and that only high-quality research survives. It is worthwhile to emphasize again that any kind of mathematics, either science driven or application driven, can be of high or low quality.

In view of the above and many more examples, one could argue that we would miss unexpected but important applications by restricting mathematical research to a priori applicable mathematics. This is certainly true, it is, however, not the main reason why I consider it a mistake to judge mathematics by its applicability. My main reason is that it would reduce the mathematical sciences to a useful tool, without a right to understand and to further develop the many thousands of years of cultural achievements of the utmost importance. This leads us to reflect on freedom of research.

Freedom of Research and Responsibility

Freedom of research has many facets. One aspect is that of unconditional research, implying in particular that the scientist himself defines the direction of research.

In mathematics there is an even more fundamental aspect. Today's mathematics is often searching for inner mathematical structures, only committed to its own axioms and logical conclusions and thus keeping it free from any external restriction. This was clearly intended by the creators of modern axiomatic mathematics. Georg Cantor (mathematician; 1845–1918), the originator of set theory, proclaims:

> *"The nature of mathematics is its freedom."*

and David Hilbert considers this freedom as a paradise:

> *"Nobody shall expel us from the paradise created for us by Cantor."*

This kind of freedom was emphasized also by Bourbaki who clearly believed that the formal axiomatic method is a better preparation for new interpretations of nature, at least in physics, than any method that tries to derive mathematics from experimental truths. In this connection I like to present a very interesting but little known document from the early days of Oberwolfach after World War II.

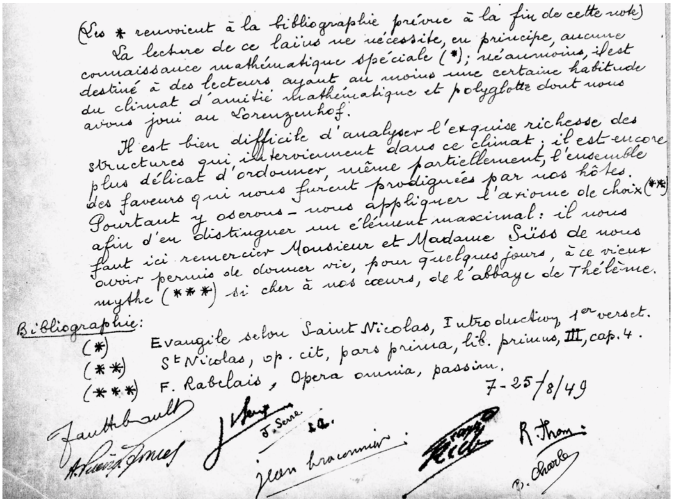
As early as 1946 the first small meetings were held in Oberwolfach at the old hunting lodge "Lorenzenhof". Among the participants were mathematicians like the Frenchman Henri Cartan, whose home country had been an "arch-enemy" for centuries. His family had suffered tremendously under the regime of the National Socialists so that his participation was not at all a matter of course. The first famous guests visiting the Lorenzenhof were Heinz Hopf (a world-famous topologist from Zurich, a German of Jewish descent who had moved from Germany to Switzerland in 1931) and Henri Cartan (the "grand maître" of complex analysis from Paris). It was said that *"Without Hopf and Cartan Oberwolfach would have remained a summer resort for mathematicians, where in a leisurely atmosphere dignified gentlemen would polish classic theories"*.

In August 1949 a group of young "wild" Frenchmen met in Oberwolfach who had taken up the cause of totally rewriting mathematics as a whole, based on the axiomatic method and aiming at a new unification. It was a truly bold venture that only young people would dare to take up. Some of their names have become famous, including Henri Cartan, Jean Dieudonné, Jean Pierre Serre, Georges Reeb, and René Thom. A photo from that time was only discovered a few years ago. It shows part of the group in the autumn of 1949. Cartan himself could not come, due to the consequences of a car accident.

From left to right: René Thom, Jean Arbault, Jean-Pierre Serre and his wife Josiane, Jean Braconnier and Georges Reeb

The Gospel According to St Nicolas and the Freedom of Research

In the first guest book they wrote down the *Evangile selon Saint Nicolas*, a humorous homage on the Lorenzenhof and its famous Oberwolfach atmosphere, endorsed with mathematical hints. The name Evangile selon Saint Nicolas is an allusion to the works of Nicolas Bourbaki, an alias for that group of French mathematicians who wanted to rewrite mathematics entirely from scratch. During that time, almost no one in Germany had heard of Bourbaki. During the Nazi period, the so-called "Deutsche Mathematik" simply missed some important developments in mathematics, for instance in topology, the theory of distributions, and in complex and algebraic geometry. It is one of the most extraordinary achievements of the small Oberwolfach workshops that those mathematicians who stayed in Germany and were not expelled by the Nazis were able to join the world's elite again.



(Les * renvoient à la bibliographie prévue à la fin de cette note)

La lecture de ce laïus ne nécessite, en principe, aucune connaissance mathématique spéciale (*); néanmoins, il est destiné à des lecteurs ayant au moins une certaine habitude du climat d'amitié mathématique et polyglotte dont nous avons joui au Lorenzenhof.

Il est bien difficile d'analyser l'exquise richesse des structures qui interviennent dans ce climat; il est encore plus délicat d'ordonner, même partiellement, l'ensemble des faveurs qui nous furent prodiguées par nos hôtes. Pourtant y oserons-nous appliquer l'axiome de choix (**) afin d'en distinguer un élément maximal: il nous faut ici remercier Monsieur et Madame Süss de nous avoir permis de donner vie, pour quelques jours, à ce vieux mythe (***) si cher à nos cœurs, de l'abbaye de Thélème.

Bibliographie:

(*) Evangile selon Saint Nicolas, Introduction, 1er verset.

(**) St Nicolas, op. cit, pars prima, lib. primus, III, cap. 4.

(***) F. Rabelais, Opera omnia, passim.

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J. Serre

Jean Braconnier

R. Thom

Evangile selon Saint Nicolas from the first guest book of Oberwolfach. The document is signed by Jean Arbault, Jean-Pierre Serre, René Thom, A. Pereira Gomez, Josiane Serre, Georges Reeb, Bernard Charles, and Jean Braconnier.

Evangile selon Saint Nicolas: Reading these lines requires, in principle, no special (*) mathematical knowledge, yet they are intended for readers who have at least developed some feeling for the mathematical and multilingual friendly atmosphere, which we have enjoyed on the Lorenzenhof.

It is extremely difficult to analyze the exquisite diversity of structures that brings this atmosphere into existence; it is even much more difficult to classify the favors that were given to us by our hosts, in their entirety or only in part. Nevertheless, we dare here to apply the axiom of choice (**) in order to reward a maximal element: our thanks to Mr. and Mrs. Suss who allowed us for a few days to give life to this old myth (***) of the Abbaye de Thélème, which is close to our hearts so much.

Literature: (*) Saint Nicholas gospel, introduction, first verse

(**) St. Nicholas, op cit. pars prima, lib. primus, III, Chap. 4

(***) F. Rabelais, Opera omnia, passim

The myth of the Abbaye de Thélème mentioned by the authors of the Evangile selon Saint Nicolas refers to the motto of the Abbey of Thélème, a utopic and idealized "anti-monastery" from Rabelais' Gargantua: the motto was "Fay ce que voudras" (Do as you please). Even today the Oberwolfach Institute is sometimes compared to an isolated monastery where mathematicians live and work together, only devoted to their science.
Bourbaki has become a synonym for the modern development of mathematics being interested only in the development of its internal structures based on a few basic axioms. This restriction of the scientific objective implies a great freedom from external forces and independence from political and social influences.

On the other hand, there are reasons to question this freedom as an absolute value, because it does also imply, implicitly, freedom from responsibility for the consequences of its research. In this connection we must emphasize that this does not excuse the individual scientist as a human being from his responsibilities. The physicist Max Born wrote in 1963:

> *"Although I never participated in the application of scientific knowledge to any destructive purpose, like the construction of the A-bomb or H-bomb, I feel responsible."*

It may be argued that the self-referential character of mathematics, as it appears in the visions of Hilbert and Bourbaki, is, at least partially, responsible for the lack of responsibility. This is emphasized by Egbert Brieskorn (mathematician, 1937–2013), who claims that this attribute implies the possibility of assuming and misusing power:

> *"The restriction on pure perception of nature by combining experiment and theoretical description by means of mathematical structures is the subjective condition to evolve this science as power. The development of mathematics as self-referential science enforces the possibility to seize power for science as a whole. [...] It belongs to the nature of the human being to prepare and to take possession of the reality. We should not feel sorrow about that, however, we should be concerned that the temptations of power is threatening to destroy our humanity."*

The self-referential character appears clearly in Hilbert's and Bourbaki's concept of mathematical structures based on the axiomatic method. This concept was of great influence in the development of mathematics in the twentieth century. It was, however, never without objections and nowadays it is certainly not the driving force anymore. In theoretical mathematics the most influential new ideas arise from a deep interaction with physics, in particular with quantum field theory. Michael Atiyah (mathematician; born 1929) even calls this the *"era of quantum mathematics"*.

Applied mathematics on the other hand, has never been adequately covered by Bourbaki's approach. It is often driven by problems from real world applications. But I do not see that this fact makes it less vulnerable to the temptations of power, maybe even to the contrary.

Not denying the threat of misuse for any kind of mathematics, I like to point out that freedom of research is a precious gift, related to freedom of thought in an even broader sense. Mathematicians for example are educated to use their own brains, to doubt any unsubstantiated claim, and not to believe in authority. A mathematical theorem is true not because any person of high standing or of noble birth claims it, but because we can prove it ourselves. In this sense I like to claim:

> *Thesis.* *Mathematical education can contribute to freedom of thought in a broad sense.*

On the other hand, being aware of the "lost innocence" and the fact that mathematics can be "for good or ill" to mankind, freedom must be accompanied by responsibility. The responsibility for the impact of their work, though not a part of science itself and not easy to recognize, remains the task for each individual mathematician.

> *Thesis.* *Freedom of research must be guaranteed in mathematics and in other sciences. It has to be defended by scientists, but it must be accompanied by responsibility.*

Mathematical Research – Popularization Versus Communication

Having described some of the process of mathematical research, let me now consider the challenge of communicating mathematics and its research.

The Popularization of Mathematics is Impossible

I would like to start with a provocative quotation by Reinhold Remmert (mathematician; born 1930) from 2007:

> *We all know that it is not possible to popularize mathematics. To this day, mathematics does not have the status in the public life of our country it deserves, in view of the significance of mathematical science. Lectures exposing its audience to a Babylonian confusion and that are crowded with formulas making its audience deaf and blind, do in no way serve to the promotion of mathematics. Much less do well-intentioned speeches degrading mathematics to enumeration or even pop art. In Gauss' words mathematics is "regina et ancilla", queen and maidservant in one. The "usefulness of useless thinking" might be propagated with good publicity. Insights into real mathematical research can, in my opinion, not be given.*

Remmert's statement about the popularization of mathematics has a point. However, we have to distinguish between popularization and communication. While his statement may apply to popularization, it does, in my opinion, not apply to the communication of mathematics. Before I explain why, let me start with the difficulties that we face when trying to communicate mathematics.

Structural Difficulties

First of all we may ask why it is not be possible to communicate mathematics. What is different in mathematics compared to other sciences? One could argue that for any research, regardless of whether it's in physics, chemistry, or biology, very specialized knowledge is required so that the popularization of research on the one hand and profoundness and correctness on the other do not go well together. Nevertheless, due to your own experience you will all have the feeling that mathematics might fall into a special category. In my opinion there are two significant structural reasons why it is so difficult to communicate or even popularize mathematics.

The first reason is that objects in modern mathematics are abstract creations of human thought. I do not wish to enter into a discussion of whether we only discover mathematical objects, which exist independently of our thoughts, or whether these objects are abstractions of human experience. Except for very simple ideas, like natural numbers or elementary geometrical figures, mathematical objects are not perceived, even if one can argue that they are not independent of perception. Objects like e.g. groups, vector spaces, or curved spaces in arbitrary dimension cannot be experienced with our five senses. They need a formal definition, which does not rely on our senses. Gaining an understanding of mathematical objects and relations is only possible after a long time of serious theoretical consideration.

Another reason is that mathematics has developed its own language, more than any other science. This is necessarily a result from the previous point that mathematics cannot be experienced directly. Therefore, each mathematical term needs a precise formal definition. This definition includes further terms that must be defined, and so on, so that finally a cascade of terms and definitions is set up that make a simple explanation impossible. But even in ancient times the abstraction from objects of our perception has always been a decisive part of mathematics, which made it difficult to comprehend. In Euclid's words:

"There is no royal road to geometry."

The language of mathematics requires an extremely compact presentation, a symbolism that allows replacing pages of written text by a single symbol. The peak of mathematical precision and compact information is a mathematical formula. But mathematical formulas frighten and deter. Stephen Hawking (physicist; born 1942) wrote in 1988: "… each equation I included in the book would halve the sales".

The importance of "closed" mathematical formulas or equations might change in the future, being at least partly replaced by computer programs. However, this will not make communication easier.

These structural reasons support the thesis of Remmert that the nature of mathematical research cannot be popularized. And all mathematicians engaged in research would agree, that it is nearly impossible to feasibly illustrate to a mathematically untrained person the project one is currently working on. Actually, this experience applies not only to mathematically untrained people but even to mathematicians working in a different field.

The Need to Communicate Mathematics

Nonetheless, the statement that insights into the nature of mathematical research are not possible for a non-mathematician is for me hard to accept. Because this also implies quite a lot of resignation. As much as this statement might be true when limited to genuine mathematical research, it is not true when you take into consideration the fact that mathematical research has become a cultural asset of mankind during its development over 6,000 years.

Furthermore, it is my impression that everyone has a feeling for mathematics even if it is developed to different degrees. Each of you who has been around small children would know that already from an early age they take great pleasure in counting and natural numbers and have a quantitative grasp of their surroundings. They often love to solve little calculations. Regrettably, this interest often gets lost during schooling. I would even go so far as to introduce the following:

> *Thesis. In an overall sense, mathematical thinking is, after speech, the most important human capability. It was this skill especially that helped the human species in the struggle for survival and improved the competitive abilities of societies. I believe that mathematical thinking has a special place in evolution.*

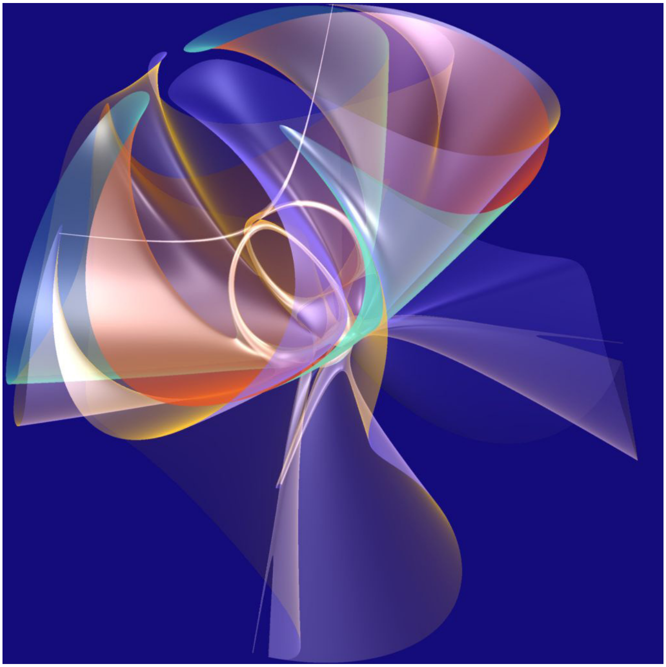
By mathematical thinking I mean analytic and logical thinking in a very broad sense, which is certainly not independent of the ability to speak. Of course, the development of mathematics as a science is a cultural achievement but, in contrast to languages, it developed in a similar way in different societies. We can face the fact that the importance of mathematics for mankind has grown continuously over the centuries, regardless of the cultural and social systems. No modern science is possible without mathematics and societies with highly developed sciences are in general more competitive than others.

Thesis. *Society has the fundamental right to demand an appropriate explanation of mathematics. And it is the duty of mathematicians to face this responsibility.*

However, if mathematicians want to make their science easier to understand it will be at the expense of correctness. And that's a problem for mathematicians. All their professional training is necessarily based on being exact and complete. Mathematicians simply abhor to be inexact or vague. But in order to be understood by society, they will have to be just that. I admit that this remains a continual conflict for every mathematician.

How Can We Raise Public Awareness in Mathematics?

In my experience there are two approaches for raising public interest in mathematics and demonstrating its significance: First, by examples that show the applicability of mathematics, and second, by examples that demonstrate the beauty and elegance of mathematics.



Featherlight, an algebraic surface by Angelika Schwengers

The first approach is certainly the favored one and it is often the only one accepted by politicians. However, we should not underestimate the second approach: it is often much more appealing and even crucial if we wish to get children interested in mathematics.

The elegance of a mathematical proof can really be intellectually fulfilling, e. g. the proof that the square root of 2 is an irrational number, or that there are infinitely many prime numbers. Both proofs can be given in advanced school classes. More accessible and therefore even more suitable for a larger audience is the beauty of geometrical objects. An example of this kind is the mathematical exhibition and communication platform IMAGINARY with its beautiful pictures. For a more detailed description of IMAGINARY see the next article.

General view of the Mathematisches Forschungsinstitut Oberwolfach

Part II: IMAGINARY—Mathematical Creations and Experiences

Let me now describe one example of successful communication of mathematics. As explained in the previous article, this is by no means an easy task, but mathematicians themselves have to make the effort to communicate their science. In fact, many mathematicians do so with remarkable success.

The Beginning of IMAGINARY

IMAGINARY is the name of a collaborative mathematics outreach project that aims to improve the image and understanding of mathematics and in this way awakes an interest and fuels passion for the subject in children and adults. This goal is achieved in different ways: on the one hand by showing the beauty and art in mathematics and on the other hand through surprising applications. To best understand the project we have to go back to its beginning.

IMAGINARY was born at the Mathematische Forschungsinstitut Oberwolfach (MFO) in conjunction with the Year of Mathematics in 2008 in Germany. It started with the travelling exhibition "IMAGINARY – through the eyes of mathematics" shown in 12 German cities and its success has been overwhelming. Its aim was to interest people in mathematics by showing them the beauty of mathematical objects and to fill them with inspiration and stimulate their imaginations.

Due to its tremendous success, soon follow up exhibitions were organised in Austria, Switzerland, Spain, the United Kingdom and the Ukraine. The program SURFER, developed for IMAGINARY, became a centrepiece of the exhibition. It teaches in a playful way the connection between formula and form, between algebra and geometry through beautiful 3-D surfaces. In this way, it bridges the gap between art and mathematics. The visitors of the exhibition get the chance to alter the algebraic equations, see the effects on the displayed surfaces in real time and even get to take a print out back home.

IMAGINARY – Through the Eyes of Mathematics

Exhibitions are the IMAGINARY way to reach out to a broad public in real life. They are shown in galleries, at museums, in schools, banks, universities, parks or train stations. Exhibitions are diverse: they can include images, interactive programs, sculptures, puzzles, games, text boards, etc. You can take all exhibits home and easily stage your own exhibition.

Exhibition at the Leibniz-University of Hannover

Let me give an impression of „IMAGINARY – through the eyes of mathematics“, the original travelling exhibition. Since 2008, this IMAGINARY exhibition has been shown in over 60 cities in Germany alone. But it has also travelled further afield to 4 continents, 29 countries and over 120 cities with more than 1 million visitors in total. In Europe, IMAGINARY has been presented in 17 countries with talks, workshops, media activities and, in most cases, exhibitions.

The first impression of IMAGINARY

SURFER installation at the IMAGINARY exhibition in Belgrade, 2012

What made the exhibition unique from the beginning, is its highly interactive and intuitive nature and its open access and open source philosophy. This is also reflected in the many positive comments left in the guest book by visitors having experienced the unexpected beauty and the "joy of comprehension":

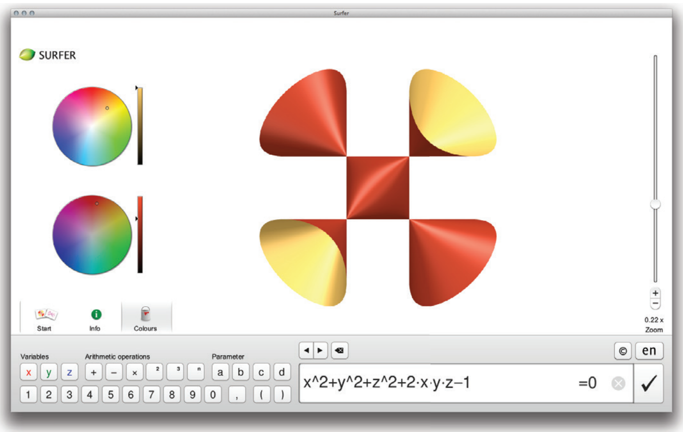
- This already beautiful exhibition is obtaining a special liveliness by excellent leadership.
- Mathematics makes happy.
- Super, especially that you can also use the program in the school.
- A wonderful exhibition. I have spent much time here and met many beautiful things, it had to take place more often and actually as a permanent event!
- Thank you and keep it up!
- It is a fantastically beautiful exhibition.
- The magic world of mathematics is not easy to understand. But you can bring them closer.
- We were again there, because it was so fascinating.
- I should have perhaps studied math ...
- Simply gorgeous, cool programs.

Cedric Villani inaugurating the IMAGINARY exhibition in Paris, 2010

SURFER Creations

The main attraction of an IMAGINARY exhibition is the SURFER, a program that calculates and displays algebraic surfaces in real time. Visitors can enter and change polynomial equations on a large touchscreen with their fingers, shift parameters, determine the colours of the surfaces and turn the figures as they like. The great thing about SURFER is that you don't have to understand the underlying mathematics (algebraic geometry) a priori, you can experiment, try, follow your intuition and creativity and this way learn maths and create unique art work like pictures or animations.

SURFER was developed by the Mathematisches Forschungsinstitut Oberwolfach in collaboration with the Martin Luther University Halle-Wittenberg and the University of Kaiserslautern, mainly by Christian Stussak.

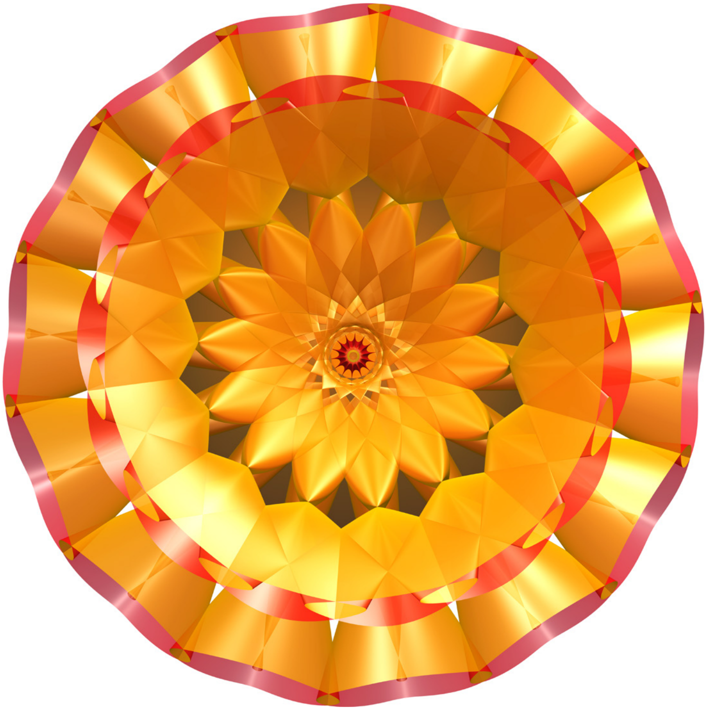


Interface of the program SURFER

Many visitors of an IMAGINARY exhibition downloaded the SURFER and created their own algebraic surfaces, with sometimes really surprising results. For example, Valentina Galata started in 2008 when she was a 17 years old high school student to remodel ‚real world objects‘ based on algebraic surfaces with the SURFER.

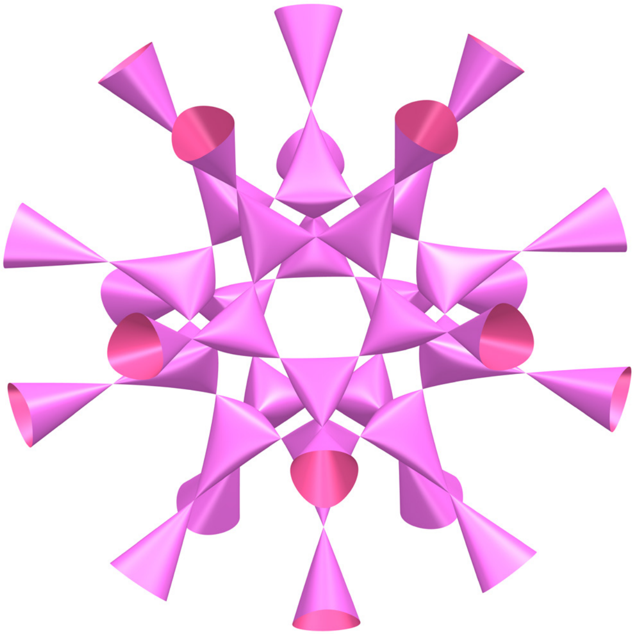
"Cappuccino" by Valentina Galata

Torolf Sauermann, a German surveying technician, once claimed to have found a new world record surface using the SURFER, raising the record for a septic surfaces from 99 to 100 singularities. It then turned out that he had made a counting mistake and found basically the septic of Oliver Labs, a surface with 99 singularities. He thus equalled the record, see the beautiful sunflower image.



"Sunflower" by Torolf Sauermann

The origin of the SURFER is very closely linked with current mathematical research. A first version goes back to Stephan Endrass, a student of the mathematician Wolf Barth, who discoverd the „Barth sextic". The Barth sextic is a beautiful surface of degree 6 with the symmetry of an icosahedron (and with a terrible complicated equation). It holds the world record with 65 simple nodes, the maximum possible number of singularities. From degree d=7 on, the maximum number of singularities on a surface of degree d is unknown.

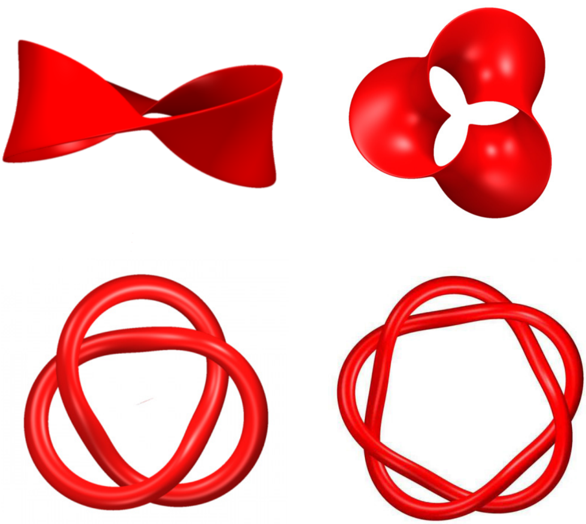


Barth sextic created with the SURFER

The characteristics of the IMAGINARY exhibition „Through the Eyes of Mathematics" can be described as:
× surprising
× interactive
× spreading
× open
× developing

Why the exhibition is surprising?
× it does not look like maths
× art can be found in mathematics
× it has a very low threshold, but with deep mathematics behind
× everybody can do and enjoy pure science with the SURFER
× mathematics becomes acceptable, even fashionable and cool
× mathematics is done in a science 2.0 manner

Besides algebraic geometry, the SURFER is used in other fields of mathematics research. The topologist Stephan Klaus used the SURFER to visualize the Möbius strip and knots. He found a method to construct polynomial equations for every knot type by Fourier decomposition and algebraic variable elimination.



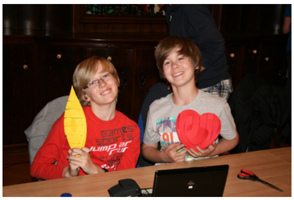
Clockwise from top left: 2-Möbius strip, 3-Möbius strop, trefoil knot, (2,5) – torus knot. Pictures by Stephan Klaus

IMAGINARY in Schools and Classrooms

First schools started to copy the exhibition or parts of it, for example the pictures or the programs by a high school in Saarbrücken. Also first self-organized exhibition were held e.g. in Kiev and IMAGINARY competitions were organized e.g. by a newspapers in Greece. The Girls' Day at the TU Berlin was a one day event to attract school girls to study mathematics by using the SURFER.


Self-organized IMAGINARY exhibitions

Especially the program SURFER is ideal to be used in the "school context". Examples were a 4-days workshop for school students aged 12–14 in Vienna, called "Kinder-Uni-Kunst". The idea was to create mathematical animations with music and while making the films with SURFER learning basic underlying concepts of algebraic geometry.




School workshops with IMAGINARY and explanations in different cities

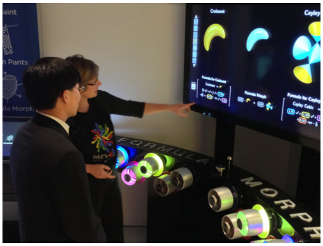
A collection of IMAGINARY worksheets has been developed for school children. The worksheets are of different levels of difficulty, aimed at children and teenagers aged between 5 and 17 years. In particular the SURFER is ideal for giving first insights into algebra and geometry in the classroom.
Different IMAGINARY booklets (for different school levels) with questions and explanations are used during exhibitions for guided school tours or at special workshops.

We are happy to present the schoolbook "Problems for children from 5 to 15" by V. I. Arnold in the Background Material section of IMAGINARY. It has been translated by our users into 6 languages and can be downloaded at imaginary.org/search/node/arnold.

Problems for Children from 5 to 15
V.I. Arnold

International Spreading

The „travelling exhibition" IMAGINARY developed into a "spreading exhibition" through many partners who independently started to stage it and further expand it. An example is the RSME (the Royal Spanish Mathematical Society) who took the exhibition at the occasion of the hundreds anniversary of the RSME, added new texts and translations and staged it in more than 13 cities. The exhibits were also installed and shown in science and mathematics museums.



IMAGINARY exhibits in museums. Left: National Museum of Mathematics, New York. Right: Deutsches Museum in Munich

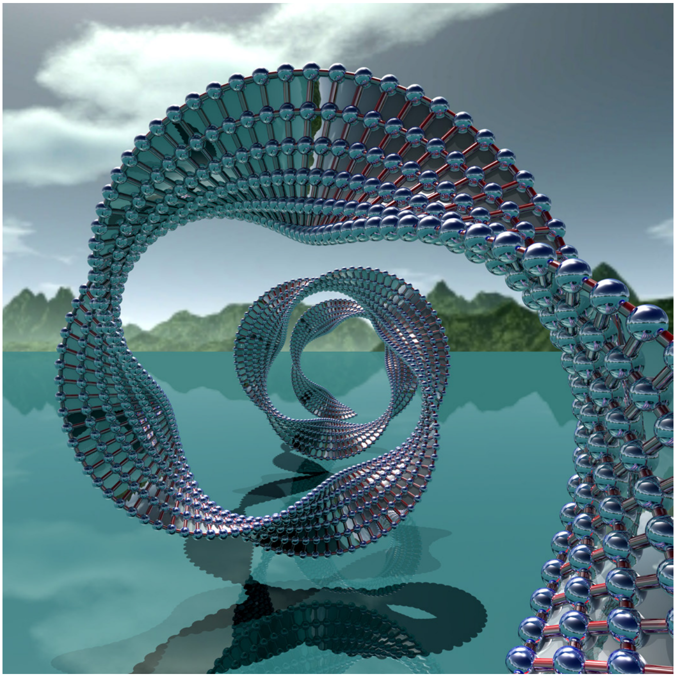
For example the MiMa museum for minerals and mathematics in Oberwolfach, the new MoMath in New York, the „Mathematisches Kabinett" in the Deutsches Museum in Munich, the Formula & Formas exhibition in the National Museum of Natural Sciences and History in Lisbon, and the CosmoCaixa museums in Barcelona and Madrid.

IMAGINARY exhibitions in 4 continents, 30 countries and over 125 cities with more than 1 million visitors

The Mathematics of Planet Earth

However, the original exhibition was not enough; it focused on a very beautiful yet small part of mathematics. The project needed to grow further and the Mathematics of Planet Earth Year 2013 (MPE, mathsofplanetearth.org) presented a good opportunity to do so. A competition for virtual exhibition modules themed around MPE was announced and IMAGINARY offered to provide the required web infrastructure in order to make the modules of the competition available online. At the launch of the MPE year in Europe at the UNESCO in Paris, the web interface to IMAGINARY (imaginary.org) went live, displaying entries for the competition and, of course, the winners.

The web platform provides an online resource for mathematics outreach. One might think of it as a "pick and mix science stand", like a pick and mix sweets stand but instead of sweets one gets to choose between different mathematical tools, all for free. Everything should be easy to digest but also awake an interest to learn more about the non-trivial mathematics behind it. The tools are grouped into different categories, such as exhibitions, galleries, films and hands-on exhibits with instructions on how to recreate them at home, school or university. Currently imaginary.org hosts two full exhibitions: the “IMAGINARY – through the eyes of mathematics” original travelling exhibition is available and free to download.

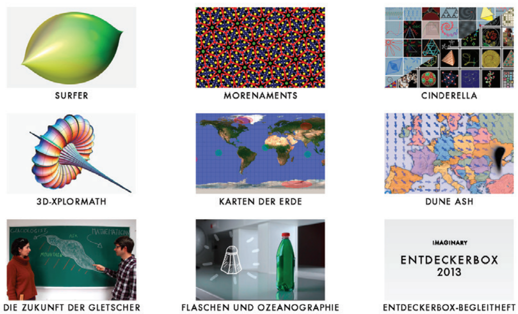


The Björling minimal surface by Ulrich Pinkall

At the same time, a complete MPE exhibition is also available, consisting of a series of modules with a more applied mathematics focus, such as a program that calculates the displacement of volcanic ash clouds (Dune Ash) or a film discussing how mathematical modelling of glacial movement works in order to predict the future behaviour of glaciers.

Interactive station Dune Ash at the launch of Mathematics of Planet Earth, UNESCO in Paris, 2013, simulating volcanic ash dispersion in real time

Of course, exhibits from both exhibitions may be mixed. Furthermore, IMAGINARY also ventured into school education. In December 2013 the so-called ENTDECKERBOX (discovery box) was launched. It is primarily aimed at use in the classroom and provides resources for teachers in order to make mathematics lessons more interactive and interesting for the pupils, see imaginary.org/entdeckerbox. The box contains for example a DVD with the programs running „out of the box", the easiest way to get the progams running on different platforms.



Nine programs and films are included in the IMAGINARY discovery box for schools

Bottles and oceanography, a film submitted at the MPE competition, part of the IMAGINARY discovery box

Who stands behind IMAGINARY?

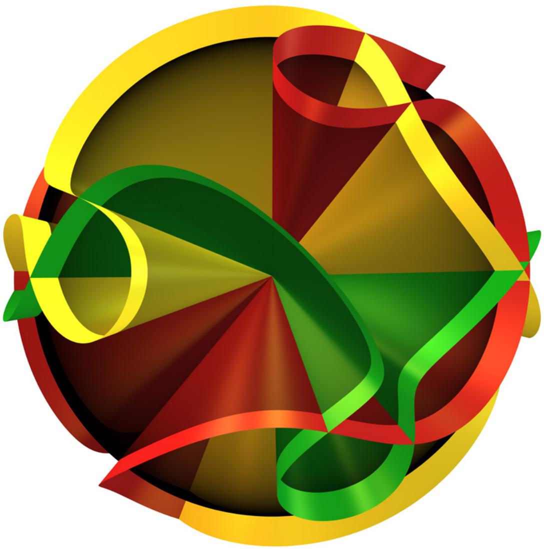
IMAGINARY is a project by the MFO, accounted by its director Gerhard Huisken, with funding from the Klaus Tschira Stiftung. It is maintained by a committed core team (mathematicians, software engineers, graphic designers, etc.), who run the project, develop the Internet platform and give advice on how to coordinate exhibitions, but also dream up new ventures of where IMAGINARY will go in the future.

At the same time, and most importantly, it is community driven. This means that anyone who has an interesting piece of software, film or other type of interactive material can upload this to the website and make it available to the rest of the community. In this way, the community becomes an integral part in the communication process by not only experiencing but also creating content and thus advancing mathematics communication to the 21st century. Of course, anyone can just use the material and create a mathematics event, exhibition or workshop. Due to this community driven aspect of the website, the German news outlet “Spiegel Online” called it the “YouTube of Mathematics”.

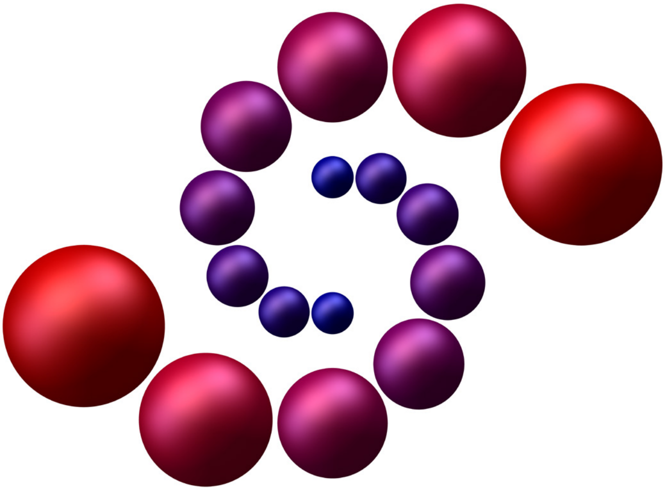
New Horizons

As a result of its success in general, and particularly in the last two years, IMAGINARY has seen a lot of media coverage across Europe for its numerous successful exhibitions. Gert-Martin Greuel, the former director of MFO and scientific advisor of IMAGINARY, and Andreas Daniel Matt, the curator and project manager of IMAGINARY, both also initiators of the project, were awarded the German Media Prize for Mathematics in November 2013.

In 2014, many new exhibitions have been or will be launched around the world. In particular, IMAGINARY has started a collaboration with the African Institute for Mathematical Sciences (AIMS) and, in association with AIMS, an interactive IMAGINARY event was organised for the first time in Africa at the 10th anniversary pi-day celebrations in Dar es Salaam, Tanzania. In November 2014, a workshop and exhibition will be organised in Cape Town to plan future mathematics communication activities with partners on the African continent. IMAGINARY's travelling exhibitions are currently on tour or planned for the coming months in Germany, Russia, Spain, Norway, Portugal, and Hungary, see imaginary.org/events. New project in France and in Turkey are on the way.



But most certainly, the highlight is the IMAGINARY exhibition organised by the NIMS institute at the ICM in Seoul, South Korea, in August 2014. It will be the largest IMAGINARY exhibition so far, featuring all modules of previous years, as well as new software, images, films and sculptures.



Logo of the NIMS IMAGINARY exhibition at the ICM 2014

The IMAGINARY pick and mix science stand has become a franchise (non-commercial and open), where you can taste new maths and bring your own ideas. It has grown from a single exhibition into a collaborative framework and community driven movement with IMAGINARY teams in several countries. Following the IMAGINARY philosophy, we invite all mathematicians to continue to send us their latest images, films or software programs to be included in the IMAGINARY platform.
So far the IMAGINARY projects are designed to be comprehensible for a general public without any special mathematical education, which is one of the reasons for its big success. It is specially attractive for young people and school children and we hope that some of them get interested in the mathematical science.

Young IMAGINARY fans

The question now is how does IMAGINARY continue? A new project is to connect modern mathematics and current research to outreach. Mathematicians visiting the MFO are asked to write about their current work but for a general public. These so-called "snapshots of modern mathematics" are then reviewed and edited and distributed through the project. Another new idea is to increase networking between mathematics communicators. To kick start this, an open map of all mathematics museums on our planet was prepared – yes, there are many more than one might think!

IMAGINARY is constantly evolving and always welcomes exciting new contributions and ideas (for example the game http://2048.imaginary.org). It is a project by and for the community. We hope that many institutions make use of the content and infrastructure of IMAGINARY, and take an active part in shaping its future.

Text sources:

Part 1 is based on the following article (containing full references to the quotations): Gert-Martin Greuel: Mathematics between Research, Application and Communication. In: E. Behrends et al. (eds.), Raising Public Awareness of Mathematics , Springer-Verlag Berlin Heidelberg (2012), 359-386.

For Part 2, see also the article: Gert-Martin Greuel, Andreas D. Matt & Antonia S. J. S. Mey: IMAGINARY – Mathematics Communication for the 21st Century. In EMS Newsletter 92, June 2014.

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Algebraic knots by Stephan Klaus

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"Björling" by Ulrich Pinkall

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From film "Bottles and Oceanography", presented by Inria and Interstices

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