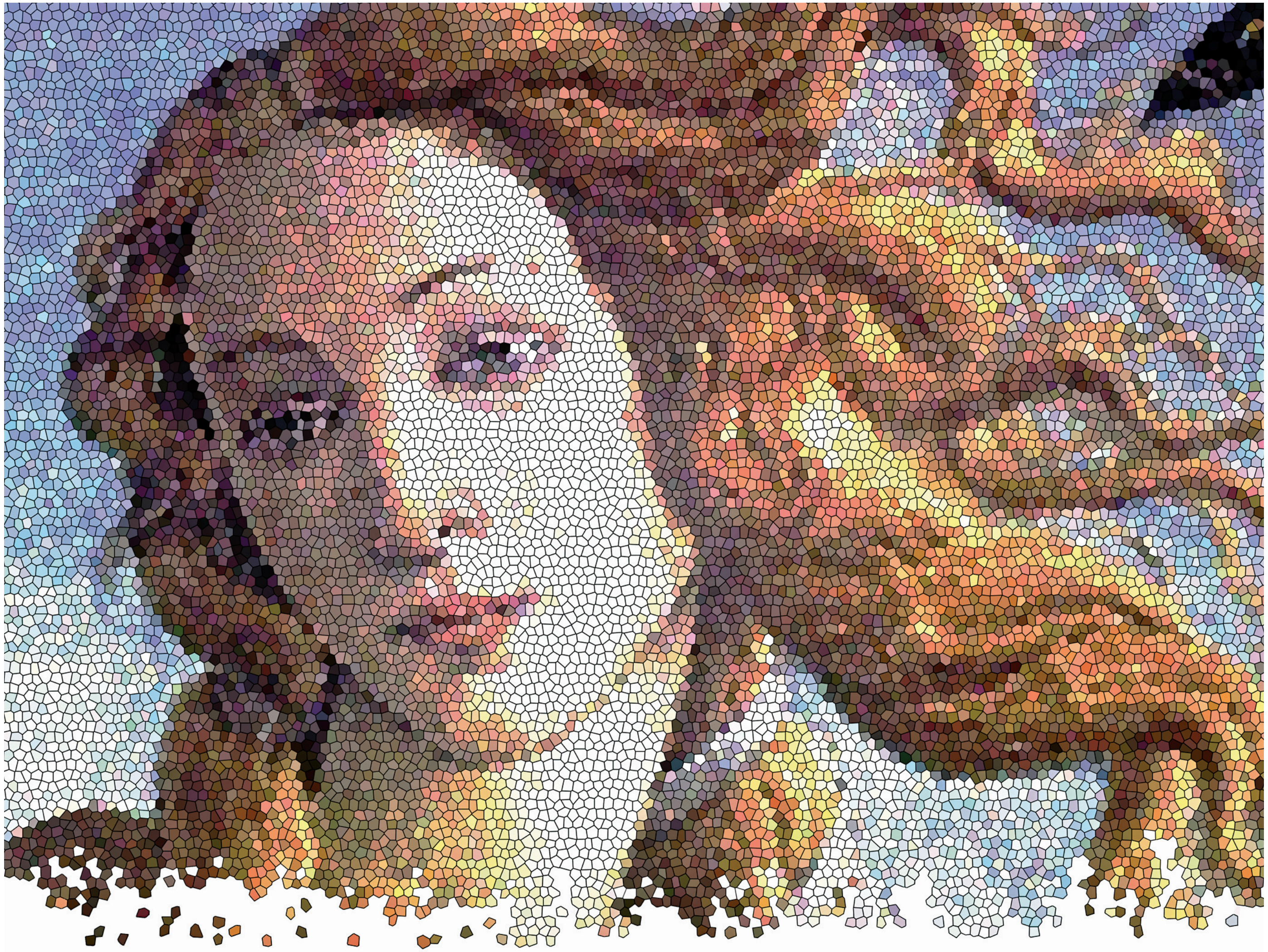
# Martmatics



*A presentation by Fernando Corbalán*

MATHS LAB

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15 JAEM
JORNADAS SOBRE EL APRENDIZAJE Y LA ENSEÑANZA DE LAS MATEMÁTICAS

cprgijón

UNIVERSIDAD DE OVIEDO
Cátedra de Inteligencia Analítica Avanzada

laboral
Centro de Arte y Creación Industrial

# Preface

MATHSLAB is an open space where one can explore the relationship between art, technology and creativity. In the attempt to provide the solution for this venue is MartMATICS presented to focus on the bond between art and mathematics.

Mathematics is a substantial part of culture. It has developed side by side with the rest of human knowledge, having an influence on and being influenced by the other disciplines, in particular, by art. A demonstration of the recognition of its social importance is the fact that Mathematics is the only subject which is studied in every educational system worldwide and in all compulsory educational levels.

But this situation thought to be a privilege is not when we analyze the study of Mathematics in real daily life at school. Unfortunately, nowadays Mathematics is thought only to be part of a course syllabus and not considered a major aspect of our daily lives outside of the education center. The practice of calculus algorithms is jailed up to the classroom atmosphere and considered unimportant in real life. Mathematics has been stereotyped to not having importance after graduation.

Mathematics is an essential part of Humanities and is studied throughout the first world countries, where it reaches the entire population. However, in the last few decades it has received the unfortunate reputation of being a subject outside the realm of Culture. Furthermore, it seldomly appears in the media, only on rare occasions featured exclusively with a negative connotation (wrong election pools, unfair draws or incorrect social models) in contrast to what happens with other sciences. We are not conditioned to recognize them in everyday life, therefore, they may end up being just a school subject.

For some time an effort has been made all around the world, our country included, to improve the teaching of mathematics. Some of the effort has been focused on trying to integrate the knowledge and interdisciplinary approach along with the dissemination of mathematical results applied to diverse social fields. This resulted in significant advances in the general culture, and particularly in the scientific one. It not only helped break that erroneous and accommodating situation of thinking that there were two cultures, a literary/humanistic one and a scientific/technological one, but it also showed the coexistence of the two without conflict.

In the attempt to find a connection, MartMATHICS is developed to show how Mathematics and Art have been related to each other throughout history and how they are still linked today. There are many bonds, but we have chosen the most attractive and significant ones. We do not want to block other ideas or possibilities, but wish to open the door to new thoughts that can lure people into becoming more interested in mathematics.

We would like to encourage «hands-on» visits, not only in MATHSLAB, but also in class or at home, so there are leaflets with suggestions for possible follow-up activities connected to each topic in the presentation.

In conclusion, I would like to offer MartMATHICS to all the visitors of MATHSLAB, but especially to all pupils who are going to spend some time with us, and also to Mathematics, Art or Visual Arts teachers. We hope the participants will enjoy it and that it will be the beginning or continuation of a productive and pleasant intellectual journey in the territory of Art and Mathematics: Beauty.

**Fernando Corbalán**

**Note.-** We would like this presentation to give Mathematics an important role in the cultural universe, so would welcome any comment, advice or suggestion for further improvement.

# 1 In Search for Beauty: *The Common Territory of Art and Mathematics*

«Mathematics is the archetype of the beauty of the world».

**Johannes Kepler**, astronomer (1571-1630)

«A mathematician, like a painter or poet, is a maker of patterns. If his patterns are more permanent than theirs, it is because they are made with ideas. [...] The only material a mathematician works with is ideas. As a consequence, it is very likely that the patterns a mathematician creates will last for a long time, as ideas are slower to get old than words. The mathematician's patterns, like the painter's or the poet's must be beautiful; the ideas, like the colours or the words must fit together in a harmonious way. Beauty is the first test: there is no permanent place in this world for nasty mathematics from an esthetical point of view».

**Godfrey H. Hardy,** mathematician (1877-1947)

«Equations are important to me, because politics is for the present, but an equation is something for eternity».

**Albert Einstein,** physicist (1879-1955)

«Numbers provoke my imagination they strengthen, activate and stimulate it. I feel their activity in my body, in my senses, they provoke new images […]. I can say that all of my works since 1953 are consciously related to numbers, they stem from them».

**Pablo Palazuelo,** artist (1915-2007)

«How close is the process of scientific creation to the art! Artists and scientists not only share the process but also many ambitions: the ambition of universality, beauty, coherence, rigour, reaching the elegance and conciseness of a mathematical formula […] we feel very close, but while scientists generate great certainties that will perish, artists try to communicate […] eternal doubts».

**Oscar Tusquets,** architect and designer (1885-1955)

«Symmetry, either defined in a wide or restricted sense, is an idea by means of which the man through the ages has tried to comprehend and create order, beauty and perfection».

**Hermann Weyl,** mathematician (1885-1955)

«The forms that best express beauty are: order, symmetry and precision».

**Aristoteles,** philosopher (384-322 B.C.)

«The laws of symmetry are some of the richest sources of artistic creation».

**Maurits Cornelius Escher,** artist (1898-1972)

«Mathematicians may generate eternal certainties but like to believe that their doubts will perish».

**José Luis Fernández Pérez,** mathematician

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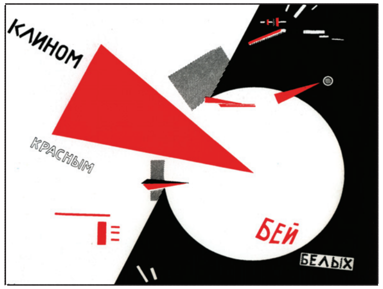
# 2 Elementary Shapes

«Geometry is like a keyboard of graphic language: curves, either regular or irregular, lines, angles, circles, arcs are the few universal elements that can express everything».

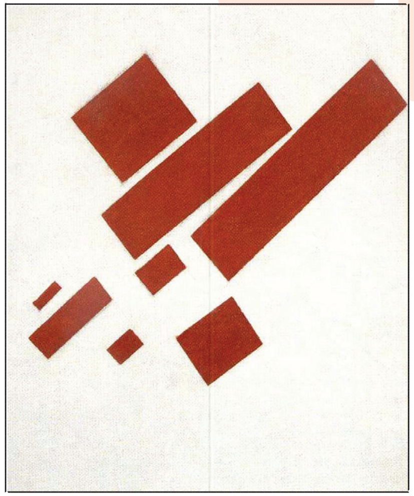
**J. Torres-García**

The elementary shapes (lines, triangles, quadrilaterals, circles…) sometimes appear directly in paintings, drawings, engravings, etc. They constitute the fundamental core of some paintings and pictorial movements.

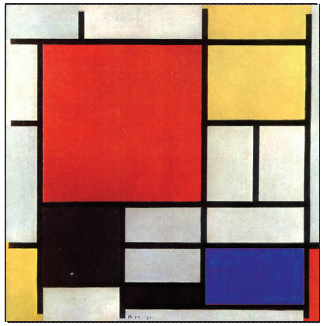
Here are some representative examples:



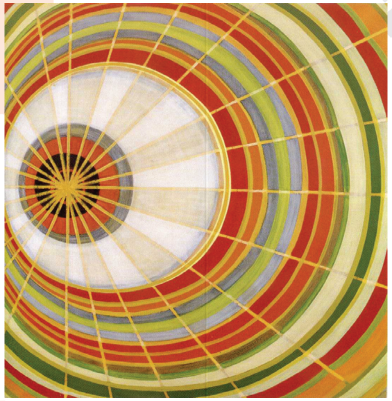

*Beat the Whites with the Red Wedge.*
**Lissitzky,** 1919.



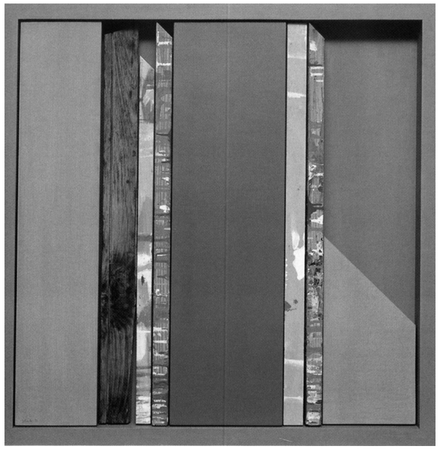
*Supremacist Painting. Eight Red Rectangles*
**Malevich,** 1915.



*Composition*
**Piet Mondrian,** 1921.



*If then III*
**Charo Pradas,** 1994.



*Alistado y diagonal*
**Gerardo Rueda,** 1990.

*Paris Review Poster.*
**Roy Lichtenstein,** 1966.

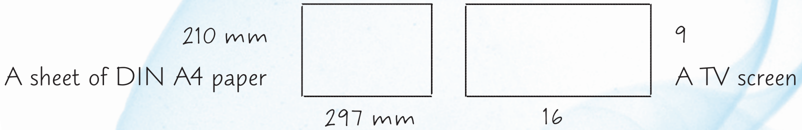
# 3 The Golden Ratio

*To you, happy prison of the retina,*
*golden section, heavenly quadrature,*
*mysterious fountain of measure*
*that the harmonic universe originates.*

**Rafael Alberti**

Are all rectangles alike? That depends on how you look at them…

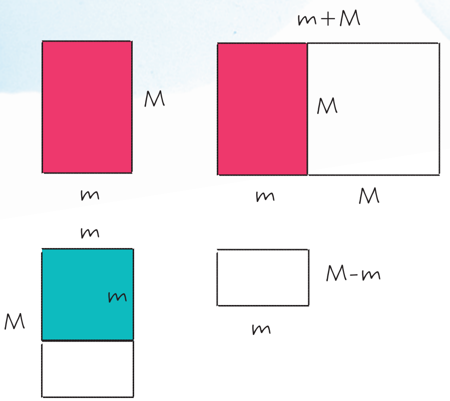
It has generally been accepted that some rectangles are alike only if the ratio of their sides is the same. For example, the rectangles of the following figure are NOT alike:




We'll now see a rectangle that was and still is very important in art: the Golden Rectangle (GR).

A rectangle is GOLDEN ONLY if by adjoining a square section whose side is the same as the longest side of the rectangle, we obtain another rectangle of the same shape. If from a golden rectangle you cut off a square whose side length is equal to the shortest sides, the piece that remains is also a Golden Rectangle.

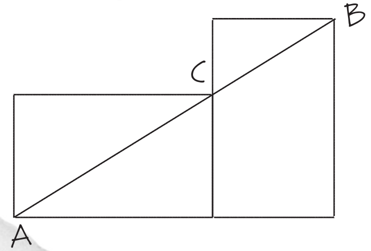
The quotient between the longest and shortest side of a GR is called the golden number $\phi$.




$$\rightarrow \frac{M}{m} = \frac{m+M}{M} = \phi$$

$$\rightarrow \frac{M}{m} = \frac{m}{M-m} = \phi$$

A rectangle is golden if when we place it as in the figure and draw the line through AB, we pass through C.




As you can see, the same thing happens with credit cards: they are golden rectangles.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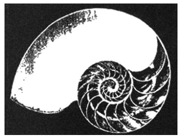
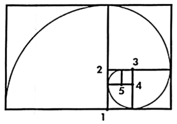
# 4 Where to find the Golden Ratio

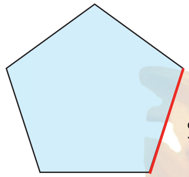
Passing from one golden rectangle to the next by adding squares allows growth preserving form.





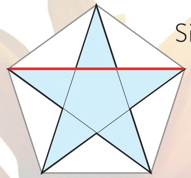

We can find the golden ratio in unexpected places:

Regular pentagon



Side l

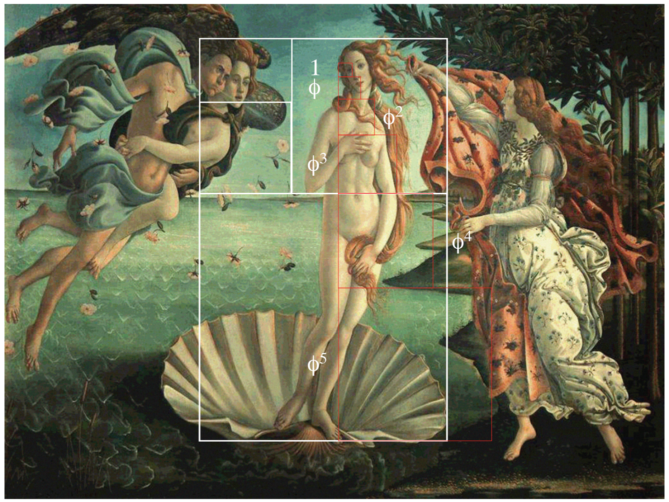
Pentagonal star



Side m

$$\text{Ratio} = \frac{\text{side star}}{\text{side pentagon}} = \frac{m}{l} = \phi$$

The starred pentagon has been used many times in the composition of paintings.




*Birth of Venus* by
**Sandro Botticelli.**
The mathematician T. Cook described some golden ratios like the ones in the picture.

*Leda Atómica.*
**Salvador Dalí,** 1949.

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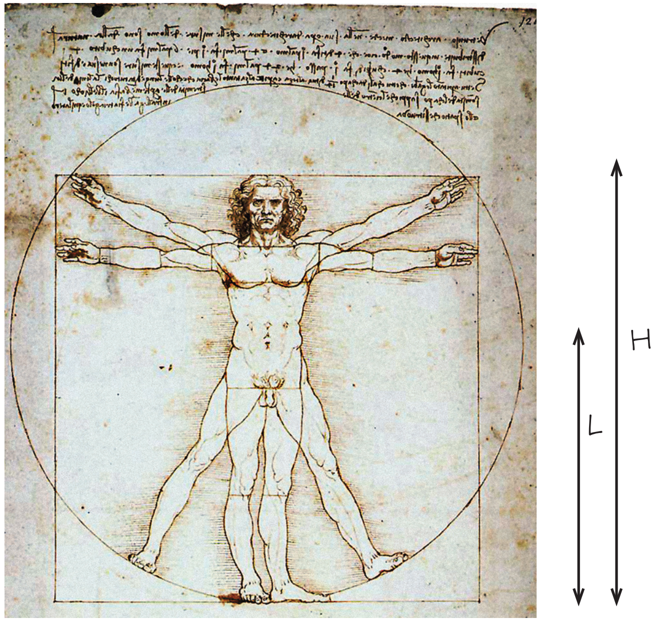
# 5 The idealized human figure… according to its height

*«Let no one who is not a mathematician read my works.»*
**Leonardo da Vinci**
(opening of the *Trattato della Pintura*)

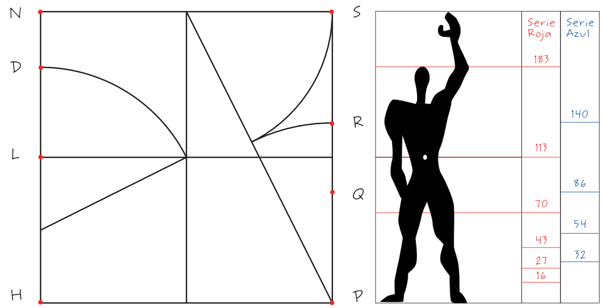



The so-called "Vitruvius man"

Many artists of all times have studied the body proportions of the «ideal human figure». One of the best known treatises is the one by **Leonardo da Vinci** drawn in one of his diaries around 1492. There we find many relationships between the measurements of the different parts of the human body, such as the following: *«the length of the extended arms of a man is the same as his height»*.

And there is another relation, this time difficult to see and is not what it appears to be at first glance: the ratio between the **total height** and the **height from the ground to the navel** is exactly ϕ.

$$H/L = \phi$$



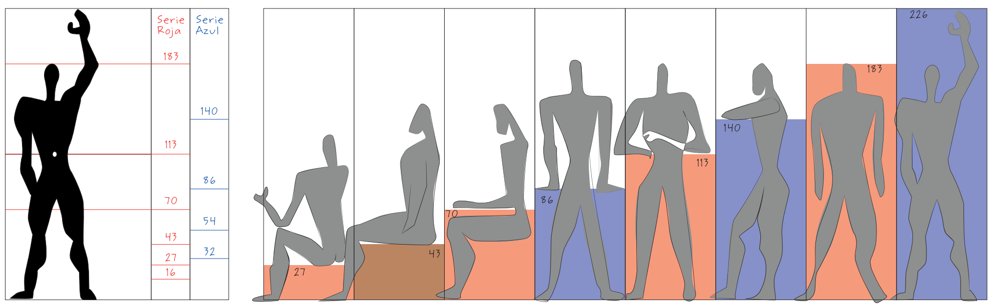

Modulor for a height of 1.83 m (measure in mm).

*«Behind the walls do the gods play: they play with numbers, which the universe is made of.»*

**Le Corbusier**

## Le Corbusier's Modulor

**Le Corbusier** (1887-1965) was one of the most influential contemporary architects. In order to design houses and furniture he used the **Modulor,** which he defined as *a measuring device based on the human height and Mathematics. A stylised human figure with one arm raised determines three intervals to the points of the body that determine the occupation of space - the foot, the solar plexus, the head, and the fingertips of the raised arm. These three intervals define a series of Fibonacci's golden sections.*

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# 6 Perspective (1)

*«Perspective is the rein and rudder of painting»*
**Leonardo da Vinci**

Perspective is a method of representing three-dimensional images on a planar surface, such as, canvas. This method is not intuitive, but instead depends on conventions between the artist and the observer.

Other civilizations used different conventions. The detail scene of the Egyptian papyrus «*The Book of the Dead*» below shows a heart being weighed on the scales by a divine judge to see if it can pass to the afterlife. The figures are represented in different sizes to depict social distinctions. But it is still a planar representation!

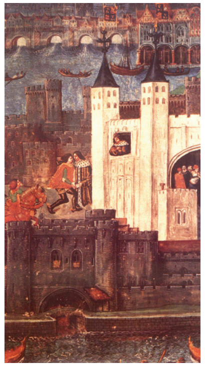
Papyrus from the *Book of the Dead*, compiled during 1570 BC and 600 BC.

Fragment of a Roman mosaic, El Bardo Museum (Tunis)

In the Roman mosaic you can see the intention of «leaving the plane», showing a third dimension by representing what is seen from different viewpoints.

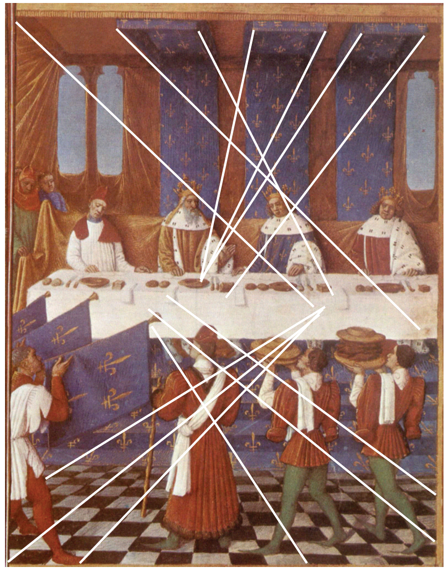
You can observe that they are similar to children's drawings in the way they try to express volume. They may also recall certain *cubist* drawings, like some of Picasso's that you will find on another poster.

During the Middle Ages, pictorial representations lacked depth, that coherent proportion of the distance between the viewer and the figures. In the paintings from that time, the most important person or the one carrying the action is emphasized by his or her size and is the centre of attraction, even among the other figures in the painting.



Illuminated manuscript from the 15th century showing London Bridge and the Tower of London.

During the 14th and 15th century, several painters approached to representations on canvas much nearer to what the eye actually sees. But their paintings still lacked precision, being drawn by rule of thumb. Observe in this painting by Fouquet that the different «intuitive vanishing points» don't converge even in one point. Perspective hadn't been «mathematized» yet.



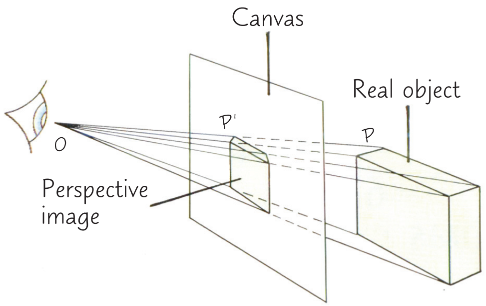
*The Royal Banquet.* **Jean Fouquet,** (1420-1480).

# 7 Perspective (2)

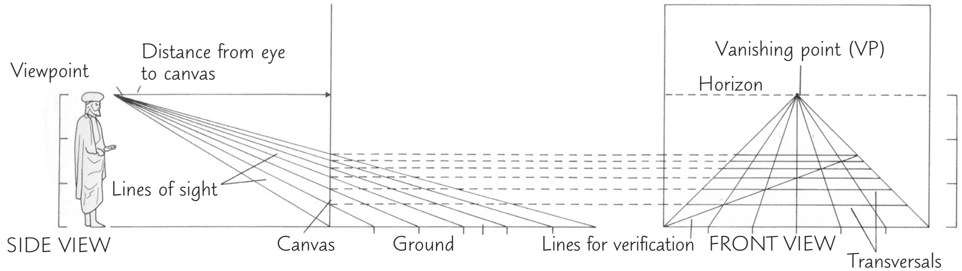
*«The first requirement for a painter is to know geometry»*
**León Battista Alberti**

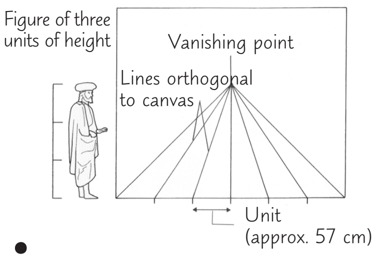
Perspective as it is known exists since the first half of the 15th century. Italian and French artists tried to paint showing an accurate reflection of reality.

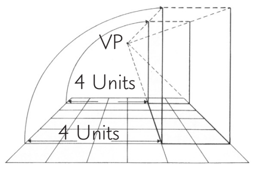
The mathematical model is explained in the figure: the point O is the observer's eye, in front of which we imagine a vertical plane, the canvas. We join any point P of the object to O with a line; the point P' of intersection of the line with the canvas is its representation.




In his *Treatise on Painting* (1435), **León Battista Alberti** explains his method of representing reality. Observe how useful the floor tiles are to check the correctness of the drawing and the perspective.

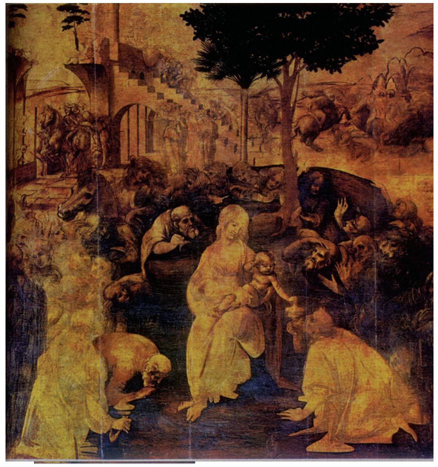


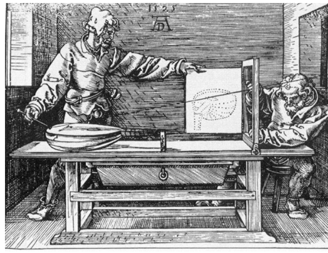




Controlling the heights

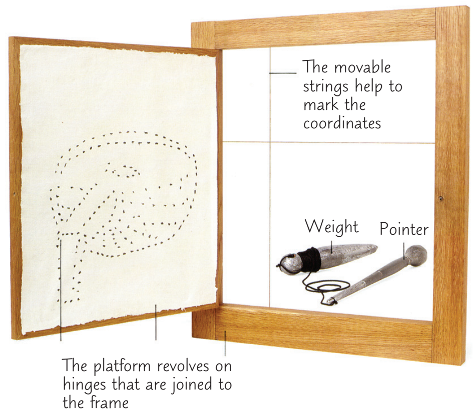
**Leonardo da Vinci** (1452-1519) exhibits linear perspective with a single vanishing point in this unfinished painting *The adoration of the Magi*. The rectangular tiling of the floor «flees» toward the vanishing point behind the head of the rider of the reared-up horse.



**Albrecht Dürer** (1471-1528), moved by his great interest for the application of Mathematics in Art, illustrated all these techniques. In the engravings below, he describes some of the apparatus that helped the artist to get precise perspective drawings.



Etching by A. Dürer showing instruments used at his time to help trace perspective drawings



# 8 Perspective (3)

*«The canvas is an open window through which we can see the painted object»*
**León Battista Alberti**

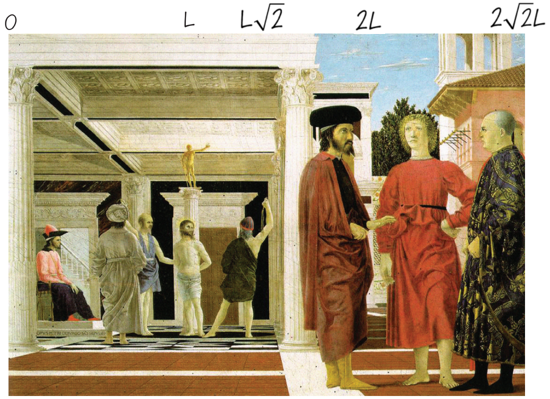
As geometric techniques were generalized to get perspective drawings, artists began to risk depicting more and more complex phenomena.

Here we will see some examples of the early «virtuosity» in the use of perspective.

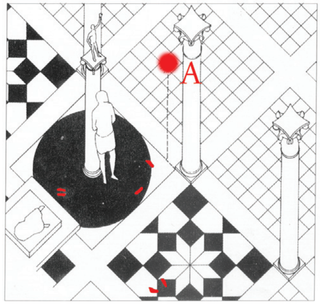
**Piero della Francesca** painted *The Flagellation* of Christ around 1460. A few years later, another artist **Kemp** said, *«No other image can exude such a detailed air of geometric control. Never has any painting been so scrupulously planned.»*

This painting has indeed been studied very closely.

Della Francesa was very interested in geometry. He wrote the treatise on *The Five Regular Solids,* in which he fit all forms into regular polyhedra. This technique was widely used in artistic drawings.




Some researches have pointed out its accurate composition (in this case, using the « ratio $\sqrt{2}$ » , like in the sheets of paper in DIN format, instead of the golden ratio).




In Mathematics the inverse of an operation or transformation is the one that leaves the objects as they were. The «mathematization» of perspective allows us to reconstruct with precision the space represented by the painting: on the left, you can see the reconstruction of the pavement, the disposition of the figures and even the height attributed to Christ at that time: 178 cm. Thus, we have «inverted» the laws of perspective. Another curiosity: the point A marks where Christ is looking at in the painting.

Painting is no longer a handicraft and becomes one of the noblest professions. Painters show their accuracy in the representation of ingenious perspectives, exhibiting their command of Mathematics.

Vault of the church of St Ignatius of Rome.
**Andrea del Pozzo**
(1642-1709)
This painting represents a culmination of the trompe l'oeil technique, a way of creating perspective illusions. The cupola is actually a barrel vault, and the architecture over the windows is also only painted.

# 9 Anamorphosis

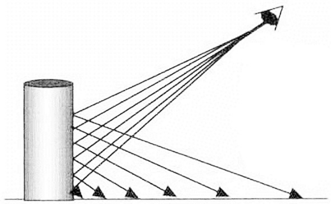
*«At a fountain in Granada, visitors are told the story of a Caliph that selected his new Great Vizier. For this, he invited the candidates to identify an object that rested or floated in shallow water. All except one immediately said that the object was an orange. The last one picked it up and identified it as half an orange. The post was his.»*
**Lawrence Wright**

An anamorphosis is a painting or drawing showing either a distorted unclear projection or perspective or a regular 'formed again' image, depending on where it is seen from or where it is reflected.

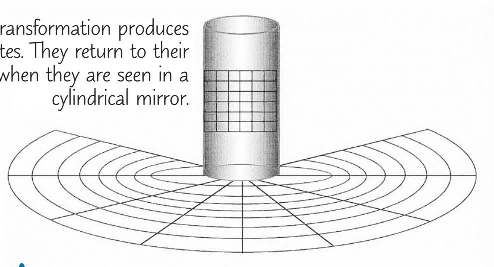
There are two types of anamorphosis. A first type is the **oblique** (perspective) anamorphosis, an example of which we find in the famous painting *The Ambassadors* by **Hans Holbein.**

The skull becomes recognizable when we look at it from a point very close to the painting from the lower right (you'd better close one eye).

**Hans Holbein's** *The Ambassadors*, full of symbolism.

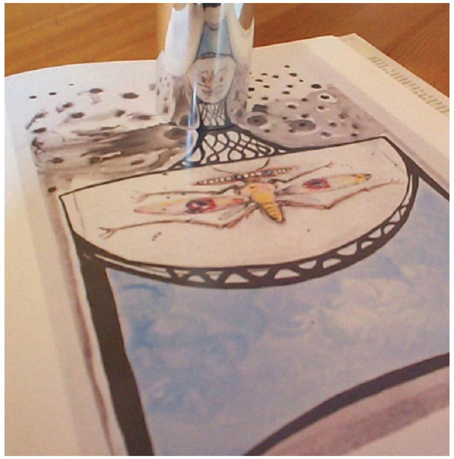


Another type is catophric anamorphosis, where one must look at the image reflected in a **cylindrical, conical** or **pyramidal** mirror in order to reconstruct the original image.



The anamorphic transformation produces polar coordinates. They return to their cartesian origins when they are seen in a cylindrical mirror.

Here is an example of anamorphosis in a **conical** mirror. The effect is even more surprising, because the boundary is reflected in the centre and viceversa.



The game of anamorphosis by **Salvador Dalí.** In *Insect and Clown*, both the anamorphic image and the reflected one have a meaning for the observer, but they are quite different objects.

Is anamorphosis only a game? Does it have any application? These techniques are used in the broadcast of sports events, when you can see images of advertisements projected onto the track, circuit or field.



You can also see it in the traffic signs painted on streets and highways. These messages on road paving, which must be seen from an unusual perspective, would be so shortened as to be illegible if they didn't have techniques similar to oblique anamorphosis.



From a certain distance, the height of the sign seems similar to that of the word CAR, but from a lateral view we can see that it is actually twice as high. In fact, this transformation is a simplification of anamorphosis.

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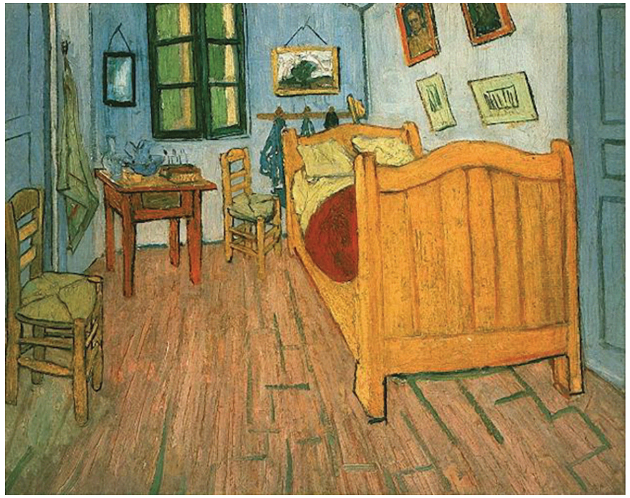
# 10 The path towards abstraction (1)

*«The history of imitation is not the same as the history of Art.»*
**W. Worringer**

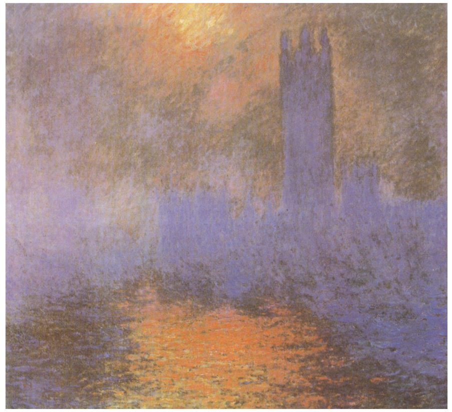
Space knowledge and the mastery of perspective was developed even more accurately, giving rise for example to the «optical illusions» you can find on poster 12. In **Velazquez'** painting *Las Meninas* (1656), the viewpoint - the eye that describes the scene - is multiple: it could be the painter, the lady-in-waiting or the visitor. And the mirror in the background reflects the upper bodies of the King and the Queen introducing the viewer into the painting.

*Van Gogh's Bedroom in Arles*
**Vincent Van Gogh.**



Some centuries later, towards the end of the 19th century, the impressionist painters suggested a new kind of representation, «studying the relationships between colours, brush strokes or lights» **(C. Corrales).** Painters begin to give up imitating reality.

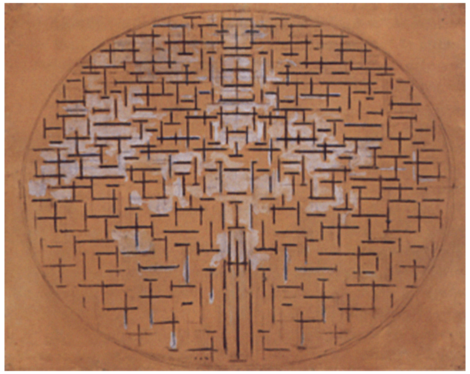
**Claude Monet:**
Left,
*Cathedral of Rouen*
Right,
*The London Parliament*



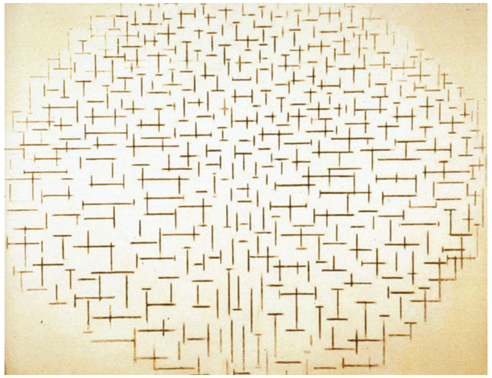
# 11 The path towards abstraction (2)

*«Cubism reduces the figures of the physical world to the maximum basic underlying forms.»*
**D. H. Kahnweiler**

In the way to abstraction we will look just at two outstanding examples: the one **Mondrian** represents which focuses on the search of the skeleton which is under the objects he paints, which he doesn't change.

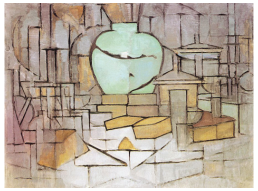


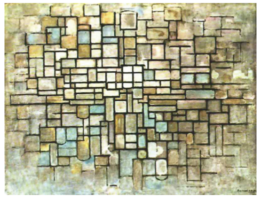
Painting from the series *Church facades.*

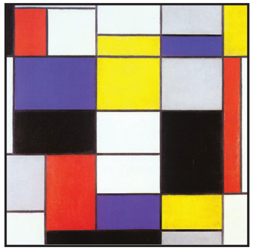


Painting from the series *Ports and the Sea.*

In this way the paintings of coloured rectangles (like those of the poster «Elementary Figures») are skeletons of buildings.







Progressive levels of abstraction in paintings by **Mondrian.**

The **cubist** movement paints three-dimensional volumes using two-dimensional planes. In each one, the painter captures what he sees. The fixed viewpoint is abandoned; instead the artist moves around the object and represents what he sees while in motion, all in different levels of abstraction.

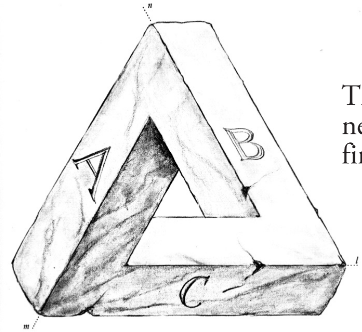
Here you can see some of the more than 54 different studies that Picasso did for his recreation of Velázquez' *Las Meninas.*

In this way, the cubists achieved a global description given the local descriptions and the relationships between them.

Martmatics

# 12 Impossible objects

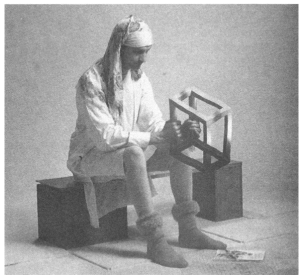
*«The Tribar, impossibility in its purest form.»*
**Roger Penrose**

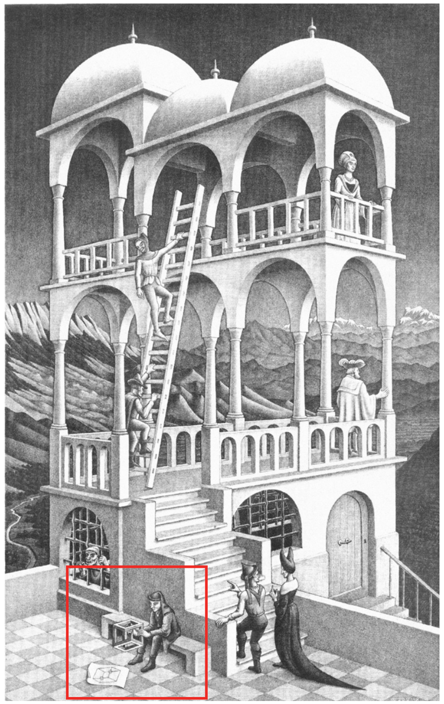


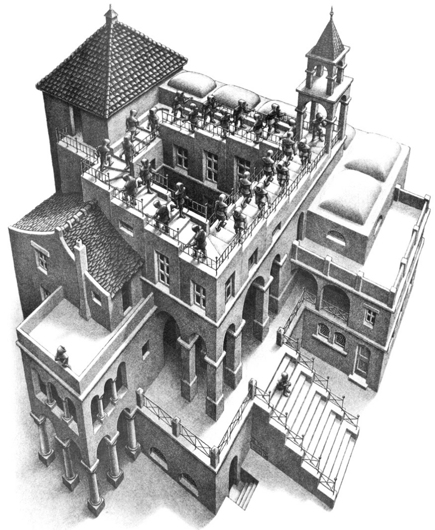

The conscious construction of impossible objects is a relatively new practice. In 1934 the Swede **Oscar Reutersvärd** drew the first *Tribar*.

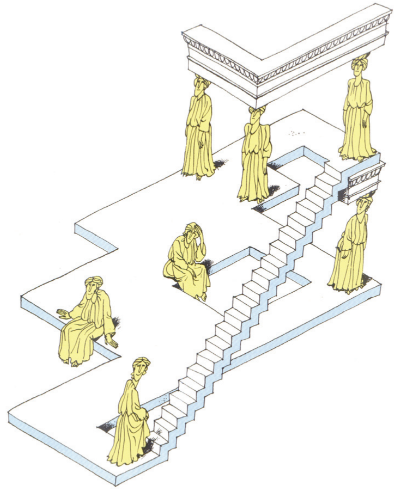
In 1954 **Penrose** visited an exhibition of woodcuts by **M. C. Escher.** Intrigued by what he saw, he published an article about impossible objects, especially the *Tribar* and continuously ascending stairs. Later, Escher in turn used his contributions in various woodcuts.

In *Belvédère* (1958) **Escher** rejoices in the creation of impossible objects. Some artists like **M. Hamaeckers** (right) constructed models for «photographing» impossible objects. After a slight change of viewpoint, the resulting image would be quite different.

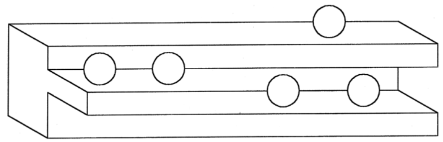


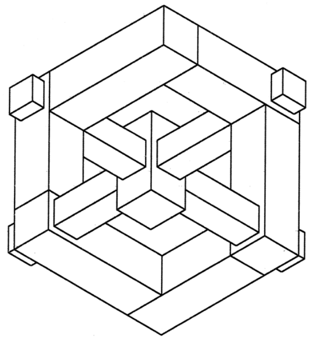






Based on **Penrose's** article on continuously ascending (or descending) stairs, **M. C. Escher** composed his woodcut *Ascending and descending* in 1960.





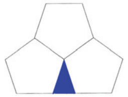
These are other examples of impossible objects.

Martmatics

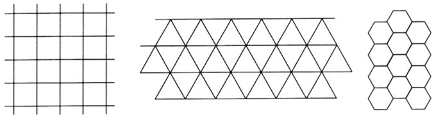
# 13 Tessellations (1)

Put your feet on the ground and look at the forms of the tiles on the floor. We want to cover the entire floor (or a wall or some other planar surface) with equal tiles and, moreover, these tiles should be regular polygons. With which tiles can we succeed? It cannot be done if all are regular polygons: for example, it is impossible if we use regular pentagons. Why? Because if we want to cover the entire floor with tiles, the sum of the angles incident to any vertex in the tiling must be 360º. Therefore, 360º must be an integer multiple of the angle of the regular polygon, and that angle must be a divisor of 360º. The angles of the regular polygons are:

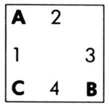
| Number of sides | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|
| Angle (degrees) | 60 | 90 | 108 | 120 | 128.6 | 135 |



The divisors of 360º (namely 120º, 90º and 60º) correspond to the regular hexagon, square and equilateral triangle. In consequence, these are the only regular polygons with which the plane can be tiled. As a regular hexagon can be divided into six equilateral triangles, there are only two ways to fill the plane with regular polygons: with squares or regular triangles.



Let's explain simply how to make artistic tessellations starting from a square net. We take a square and mark its sides and angles in the way shown below on the left. After rotating the square four times around the vertex B, we return to the original square obtaining a figure of four squares, the upper left part of Figure A. Displacing this horizontally and vertically by twice the original side length, it allows us to tile the entire plane. Observe that the sides labelled 1 and 2 (and those labelled 3 and 4) will always coincide.



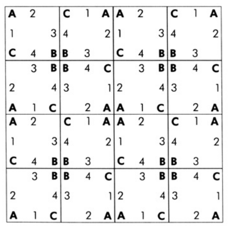
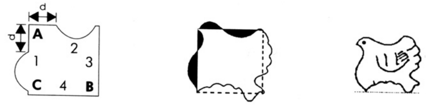
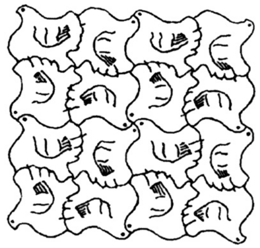



Figure A

Martmatics

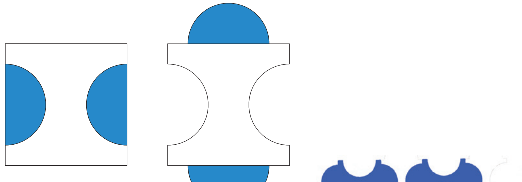
We can vary the original squares by adding a shape along side 1, and removing it along side 2. We can modify sides 3 and 4 in a similar way, until we end up with a tile that we like. In order to obtain a pigeon, we can add and subtract the pieces in the following figure, and draw eyes and wings: this tile will fill the plane.



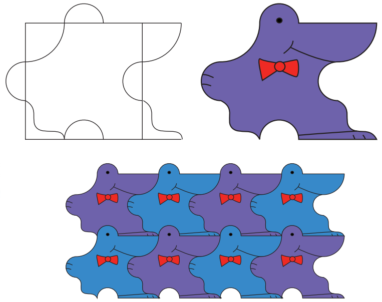


In order to do this, we need to reproduce the same motions as in Figure A (translation, rotation and reflection). In this way we get a tessellation made of pigeons, like the one on the right (designed by **S. Haak**). The only limitation is our own imagination and our skill, the technique is simple.

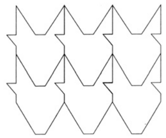
Here we present some examples designed by around 14-15 year-old students, where the way of filling the plane is not always the same as in the pigeon example.



"Gothic" tessellation



"Penguin" tessellation



"Bull" tessellation

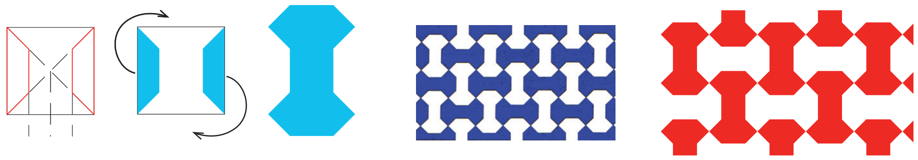
"Tile" tessellation

# 14 Tessellations (2)

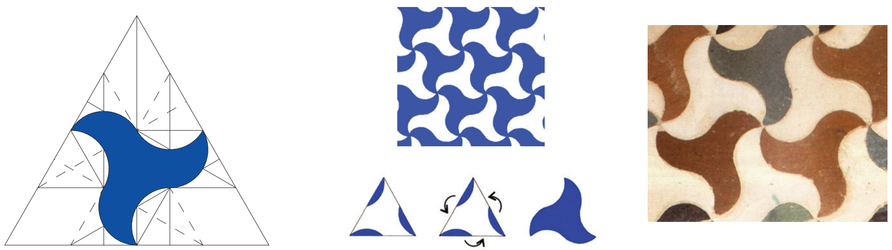
*«For muslims, all power comes from Allah, and he protects everything. That is why he must be present in all Nazarene palaces. How should he be manifested if his religion 'prohibits' the use of his image? The answer is mathematics. A fundamental characteristic of geometric decoration is the use of one single design that multiplies to cover the entire plane. In this way, one represents unity - Allah is one - and multiplicity - he is everywhere.»*

**Rafael Pérez Gómez**

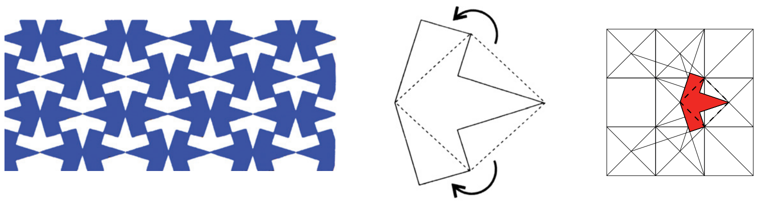
We are now presenting three of the tessellations that appear in the Alhambra, built during the Nazarene dynasty (the Kings of Granada, Spain before the conquest of the Catholic Kings in 1492). The first one is called the «bone»; we are going to show below how it is made and how to construct a tessellation based on it:



Next is the «bird», which is constructed from a triangular base and yields tessellations like the following:



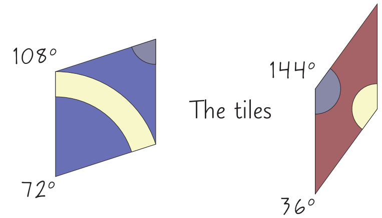
And in the third example from the Alhambra, we are going to start with the tessellation itself, made of «nails». On the right, you can see how it was made:

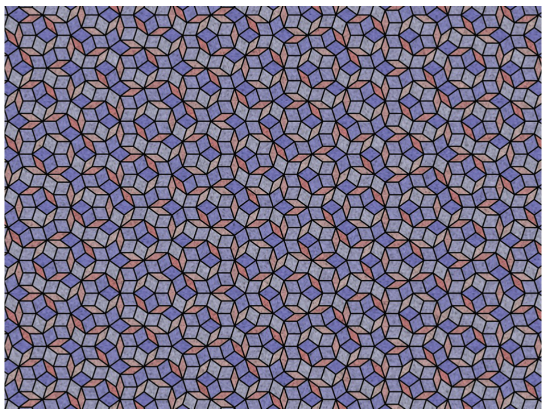


You can observe that one can get surprising results from a very simple base figure. Moreover, in the same way as it is done in the Alhambra, one can paint the tiles in different colours and enhance the effect they create.

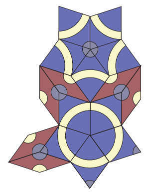
There is a huge number of other possibilities to construct tessellations, like those found by the Dutch **M. C. Escher** (see poster 22), or those proposed by the mathematician **Roger Penrose**. He found a way to tile the plane in an **aperiodic** way, which means that it is impossible to find a set of motives that would constantly repeat by way of translation, rotation or reflection. One way to achieve these «Penrose tessellations» is using the tiles below and the following a set of rules:

1. The colours must coincide at the boundary.
2. There must be no gaps between tiles.



On the left you can observe a portion of the plane tiled by a well-formed **Penrose** tessellation.
On the right the beginning of another tessellation.

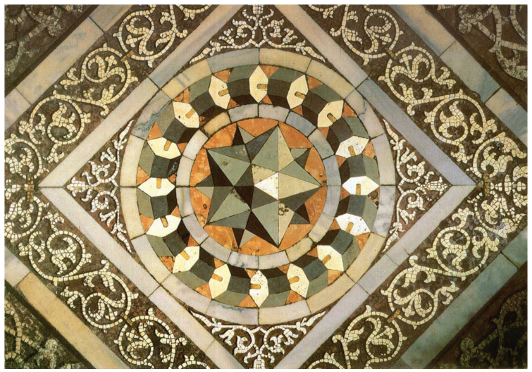


# 15 Decoration and symmetry

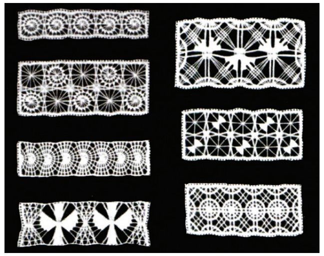
There is a lot of mathematics in decorative elements, especially figures, elementary forms and the use of symmetry. This has been true through all the times. Here you have some samples. Enjoy them!

Pre-Columbian designs

Photograph from the series *The Earth seen from the Sky* by **Y. Arthus-Bertrand.**



Tiling of St Mark's Cathedral, Venice, by **Uccello.**

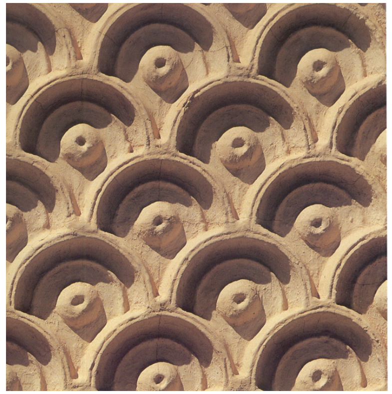


Geometric lace

Desk-lamp by **K. Jucker** and **W. Wagenfeld,** in which you can appreciate the Bauhaus programme (poster 21): the use of industrial materials (metal and glass), the transparency of the function of each component (for example, you can see the rising cable) and an aesthetic form based in the harmony of simple geometric bodies (spheres, cylinders, cubes...)

Hanging lamp with traction mechanism, by **Marianne Brandt** and **Hans Przyrembel,** Bauhaus.

**Marianne Brandt.** Nickeled brass ashtray.



Symmetric decoration element in the pavilions of Finca Güell, by **Gaudí.**

# 16 Advertisement (1)

*«The air we breathe is composed of oxygen, nitrogen and advertisements.»*
**R. Guerin**

Advertisement is a part of the art of our daily life and surely one of the manifestations of art that is most «seen», even though we don't often realize it.

We will concentrate on a mathematical interest and taste aspect: The pictograms of commercial entities. It is worth pointing out that companies put a lot of time and effort into designing and marketing their logotypes and these incorporate very often clear geometric elements. This indicates that geometry is important outside school.

Here you can see some pictograms: Look at them to see what they might contain and think about how you would make them. You can read the accompanying text, but try to do so after having thought about it for a while.

If from an equilateral triangle you cut off a smaller equilateral triangle from each side, you obtain the logotype on the left. But you can also build it from a rhombus that turns by 60° and 120° around one of its vertices. The final sensation is very dynamic, like a turning propeller. Moreover, it is also the translation of the name 'diamond'.

Mitsubishi

In the pictogram of Zaragoza council, there are several triangles because this is the shape of the region. Besides we also «see» something else, which is only suggested: the initial of Zaragoza.

DIPUTACION Ð ZARAGOZA

TOYOTA

This could be considered a plane pictogram or a planar representation of a body in space: it is similar to a sphere, but in this case it is actually the body generated by an ellipse turning about its major axis (called an ellipsoid).

Deutsche Bank

DIA

There is a subliminal publicity in both logotypes: the percent sign %. In the first case, it is referred to the credits percentage whereas in the second case, it means what the prices are cheap.

RENAULT

Caixanova

There are many logotypes that use a Möbius band (obtained by joining the ends of a strip of paper after twisting it through 180°, see poster 19), or false Möbius bands (that come about by sticking after different twists). Here is two of them, but if you search for them in the street you will come across a lot more. How would you obtain these logotypes from a strip of paper?

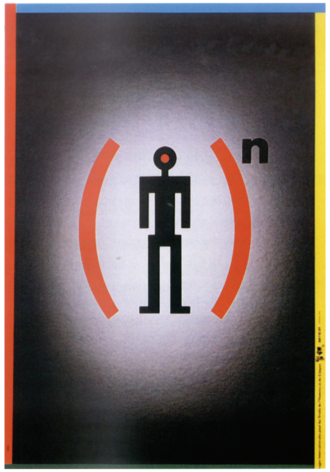
Adia

The pictogram of the company Adia is an impossible figure: there is no way to arrange the three bars so that they form an impossible figure (see panel 13). The logo of Helvetia Seguros may seem the same impossible figure but it is not.

Helvetia Seguros

# 17 Advertisement (2)

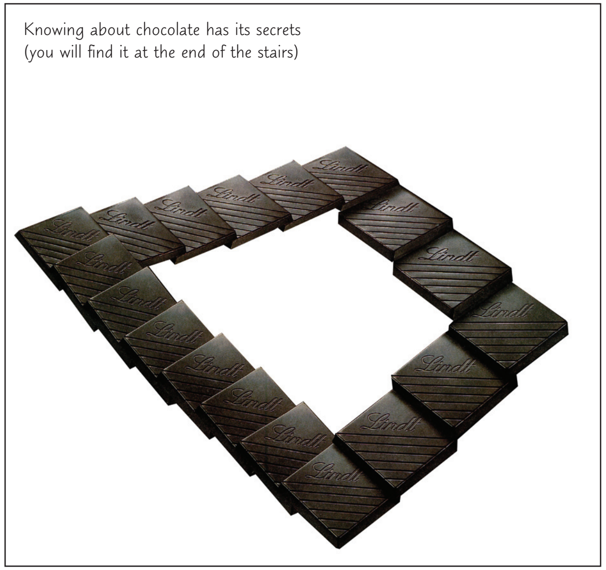
*«Advertisement is like grass growing. You never see it happen, but every week you have to mow the lawn.»*
**Andy Tharsis**



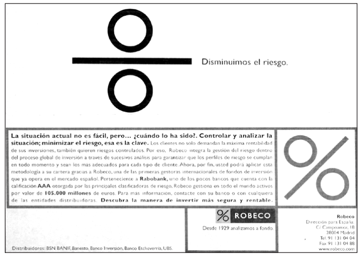
*Poster of the Bicentennial Exposition of Human and Civil Rights*, by **Pere Torrent**, known as *Peret.*

This poster, surprising in its simplicity, makes a playful use of a mathematical equation: a human pictogram in parenthesis **raised to the *n*-th power** suggests that what is important is humanity.

This advert smells not only of chocolate but it also «smells of Escher». In fact, on the poster **Impossible Objects** you will find the woodcut «Ascending and Descending stairs», on which this image is based on to emphasize the hermetic secret of the master chocolate makers: where do the stairs end? The secret of the master chocolate makers lies at the end of the stairs.








Advertisements sometimes use mathematical terminology, in this case, in a somewhat confused way: in order to symbolize decreasing risk, the percent sign has been converted to the **indeterminate** fraction $\frac{0}{0}$ -it sounds rather risky, doesn't it?

NEWMAN

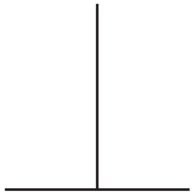
A spelling that lets you read the trademark either upwards or downwards, that's why the clothes of both trademarks clearly exhibit their logos even when they are in the hangers.

oysho

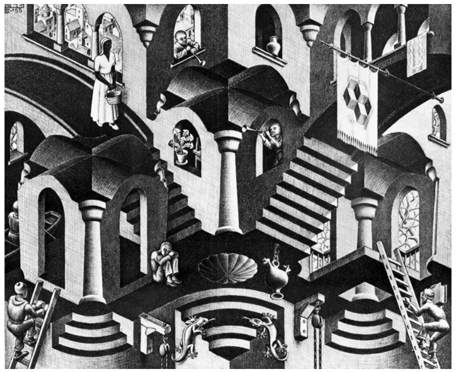
# 18 Optical illusions

Perspective is a convention that sometimes gives raise to problems. Observe, for example, the woodcut *Absurd Perspectives* (1754) by **William Hogarth.**

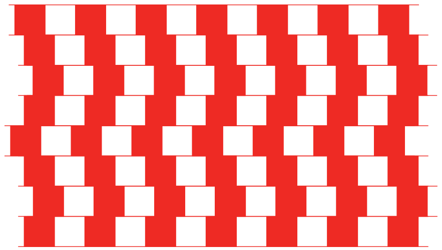
Horizontal objects tend to seem shorter because our retinas are curved.



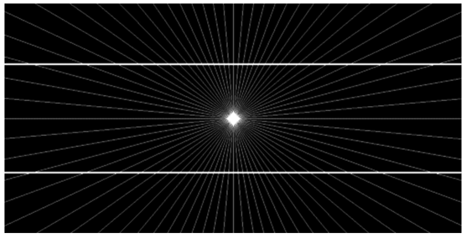
If an image can be interpreted in more than one way, our mind will go back and forth between different interpretations.



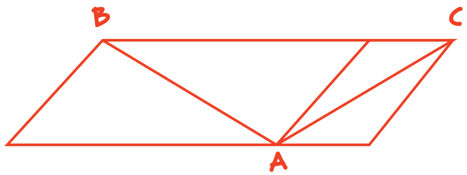
In this woodcut, **Hogarth** played with the laws of perspective and optical illusions. You are bound to find some paradoxes here!



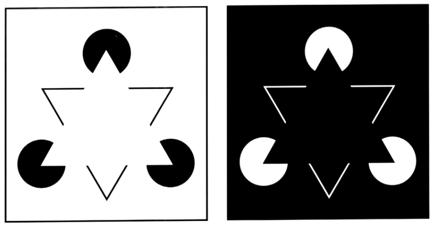
Look at the horizontal lines from the right or left hand-side and you will find that they are actually parallel.



Two parallel lines appear to curve outward from the centre when drawn over a bundle of lines that converge in a point.

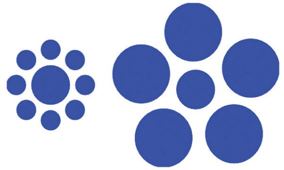


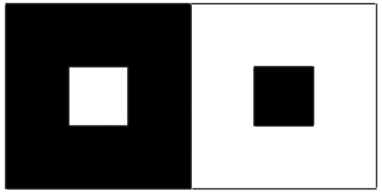

The segments AB and AC have the same length, even if they may seem different to us.



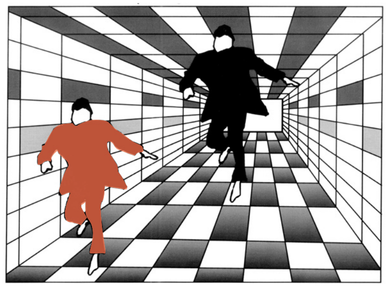
Your eye tends to complete the suggested figures. Here you can see triangles that are not really drawn.

The inner circles are the same size ...aren't they?





On our retina, the image of a bright region invades that of a dark one, making the dark one appears smaller.

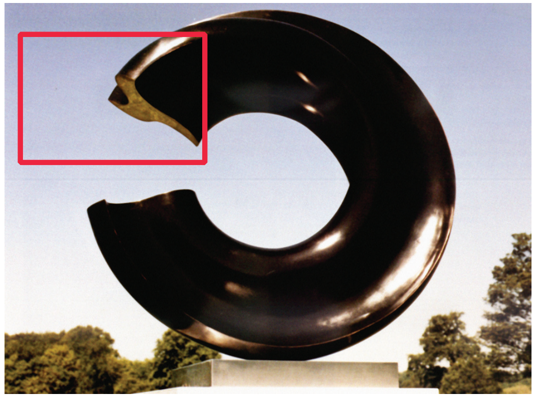


On a perspective grid, figures that are «farther away» appear larger than the «closer» ones. You can check that both figures are the same height.

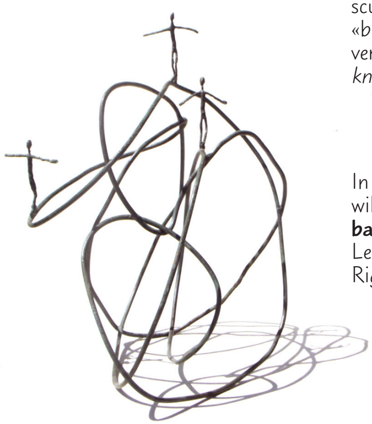
# 19 Sculpture (1)

*«With the time I discovered that forms and space are exactly the same thing. You cannot understand space without understanding form.»*
**Henry Moore (English sculptor)**

This sculpture with strong geometric content is on Europe Square (Zaragoza, Spain).



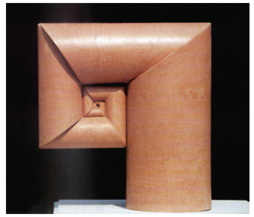
*Music of the Spheres*
**John Robinson** frequently alludes to Mathematics in his sculptures. In this case, the cross-section has the form of the «bird»-tile from the Alhambra (poster 14). If you follow the vertices of the bird, you will find that the whole solid is a *trefoil knot,* a variant of a Möbius band.

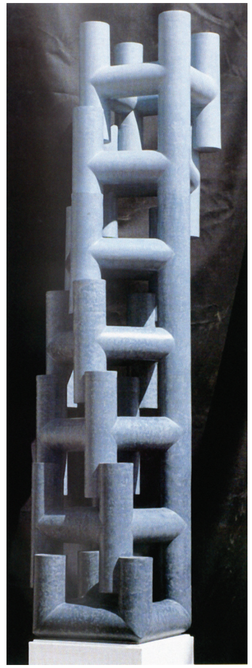


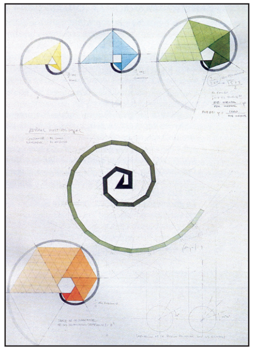
In the work of **Alonso Márquez** you will often find references to **Möbius bands.**
Left: *Encuentros y Desencuentros;*
Right: *Eterno Retorno.*


*Fibonacci sequence 1-55*
On this chimney in Turku, Finland, we can find the first terms of the Fibonacci sequence in 2 metre high neon lights. For its author, the Italian **Mario Merz,** it is «a metaphor of the human search for order and harmony in chaos».

*Spiral* $2\sqrt{2}$.
The rectangle « $\sqrt{2}$ » (that appears in the DIN series of paper) yields this spiral when it is successively divided in two. **Javier Carvajal** calculated the (complementary) angles at the joints and found out that they are 54° 44' and 35° 16'.







**Javier Carvajal** has investigated the relationship between *Number and Form* in many sculptures since the 1990s.
Left and centre: *Multipolygonal spiral* and preliminary drawing. The angles by which successive segments are joined correspond to those of the sequence of regular polygons: triangle (60°), square (90°), pentagon (108°)...
Right: *Tower of turning cubes.*

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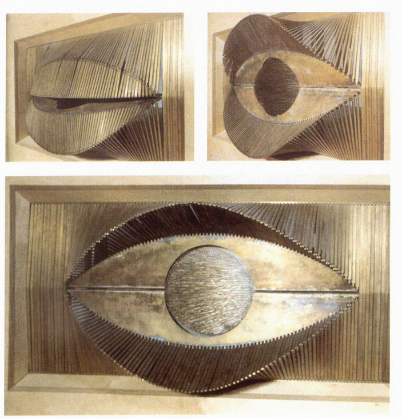
# 20 Sculpture (2)

*«There is no better backdrop for a sculpture than the sky, because there you have a contrast between a solid form and its opposite: space.»*
**Henry Moore (English sculptor)**



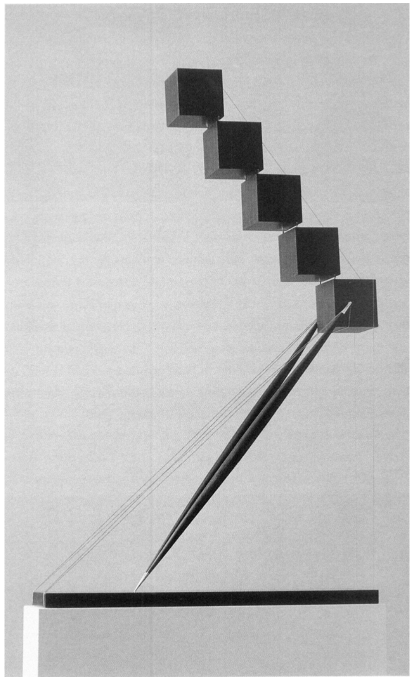
*S9 Winking Eye*, 1985. Brass. **Santiago Calatrava.**

*Monument to UNESCO*, 1986. **J. Jobin and J. Vallières.**

This sculpture in Québec commemorates the declaration of that city as World Heritage by the UNESCO. It reproduces in 3D the logotype of «World Heritage»: a circle (a sphere that symbolizes the world) that contains a square (cube) representing the establishment of humanity.

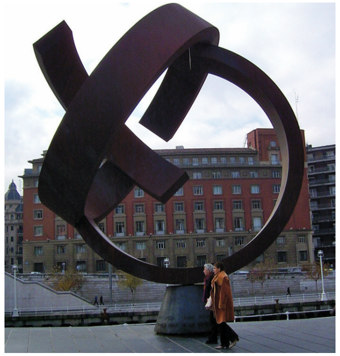
In the centre of the cube there is a glass prism that represents Québec in the heart of Mankind.

*S55 S.T.*, 1995. Ebony and chrome steel. **Santiago Calatrava.**



*Door of illustration*, Madrid 1990. It is a stainless steel monument designed by **Andreu Alfaro** that through semicircles reinvents the traditional city entrance doors.

**Oteiza.** *Ovoid variant of the sphere clearing*, Bilbao.



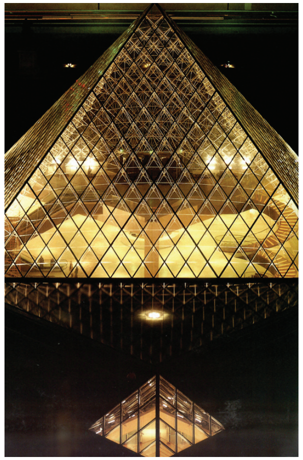
*Like a star (Estrella varada)*, Alicante, 1978. **Sempere** and the theme of star made from a dodecahedron where steel bars come out of each face, being the upper ones longer.

# 21 Architecture

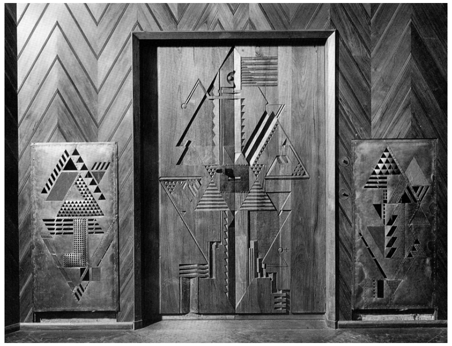
*«Human beings come into world with two eyes, but they learn to see only with patient learning.»*
**Walter Gropius (Bauhaus architect)**

*Cathedral in Toronto*, by **Santiago Calatrava.**

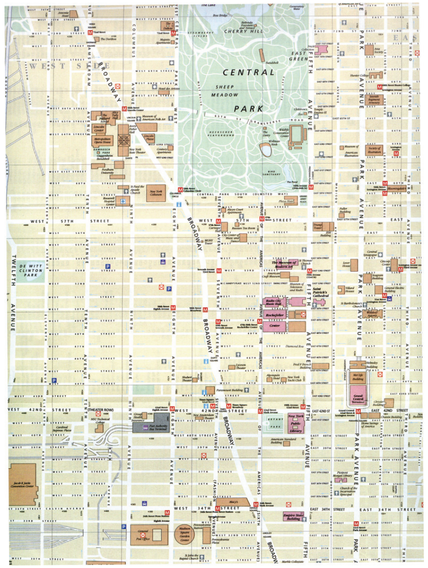
We can frequently observe elementary figures in architectural design. Above, the cubical volume of the *Grande Arche* de la Défense, Paris, by the Dane **J. O. Van Spreckelsen.** Below, the pyramids accessing the Louvre Museum, by **I. M. Pei.**



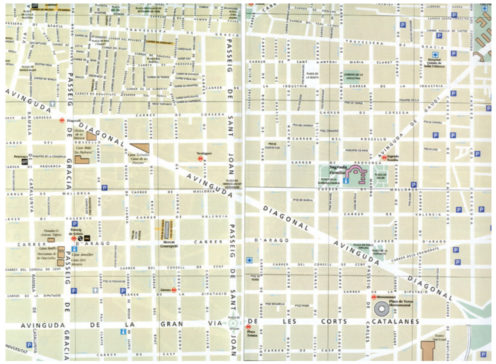
Hyperboloid on a building in the harbour of Kobe (Japan).



During the 1920s and 1930s, the German **Bauhaus** school brought his idea of «unified works of art» with numerous mathematical motives. In their constructions as well as in their lectures, technique, design, art and craftsmanship came together. Here is the interior and exterior details of *Casa Sommerfeld*, by **Gropius** and **Meyer,** the first big common project of Bauhaus.




Mathematical rationalism is sometimes reflected in urban planning. Below you can see two good examples, the **Ensanche de Barcelona** and the avenues in **Manhattan.**

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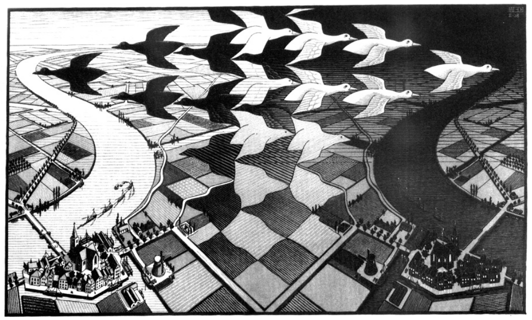
# 22 M.C. Escher

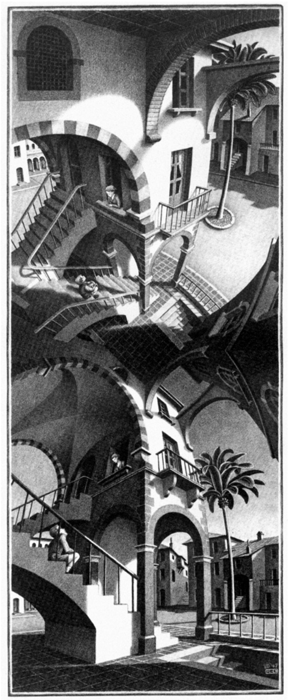
*«Escher's prints have a strange and even abnormal fascinating touch.»*
**Bruno Ernst**

Maurits Cornelius **Escher** (1898-1972), a Dutch designer and engraver, was for a long time an artist difficult to classify. While art critics generally ignored him, some mathematicians were surprised by how Escher managed to materialize the essence of some rather abstract concepts. The general public has kept on buying his prints, a real world-wide bestseller.

Nothing captured Escher's imagination more than the **regular partition of surfaces** -remember what you saw in the poster on **Tessellations.** In 1936 he visited the Alhambra in Granada, and studied its ornaments very closely. After experimenting with translation, rotation and reflection of motives, he composed pictures like *Day and Night* (1939).

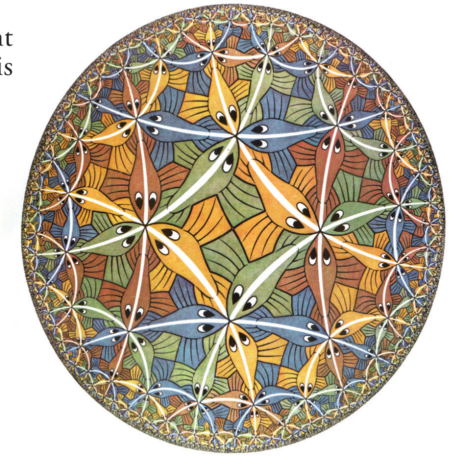


*Day and Night* (1939) is possibly the most popular of Escher's prints. Some of the favourite themes of the artist are present in this picture: the regular partition of the plane (the black and white birds complement each other), the symmetry between the town in daylight and the one in the night, the cultivated fields, surfaces that transform themselves into birds, solids, and the ideas of change and cycle (metamorphosis).



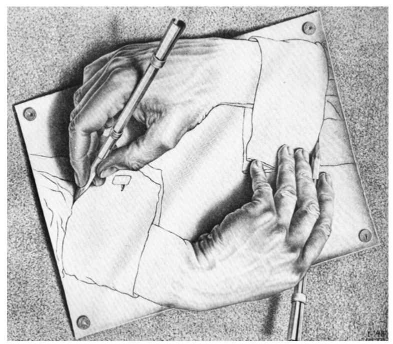
In *Above and Below* (1947) the object of study appears to be the *relativity* of perspective. The concept of perspective, elaborated in the Renaissance, is pushed to its limit (the vertical vanishing point in the zenith or the parallel converging curves), immersing the viewer in perplexity. Escher is «playing» with the very structure of space.

The approximation to infinity is another recurrent theme in his work. For example, here you see his coloured woodcut *Circular Limit III*.



The fish become infinitesimally small towards the boundary of the disk. Along each curve, fish of the same colour appear to grow and shrink as they approach and leave the centre. Observe that also here the plane is also covered.

Escher also explores drawing as a way of cheating: the artists delights in the manoeuvre, and the spectator lets himself be fooled consciously, just as in the demonstration of a magic sleight of hand. Some tricks are shown here, in *Drawing hands* (1948) and *Art Gallery* (1956).



In *Drawing hands,* Escher - who wrote with the right hand, but drew and engraved with the left - rejoices in the concept of symmetry. The paper, flat, contains the two hands, which have volume, and each in turn draws a hand... on paper! The ideas of cycles and metamorphoses are also very common in Escher's work. All this to fool and surprise the spectator.

If we describe the woodcut *Art Gallery* in a clockwise direction from the top left corner, we feel the vertigo of being inside and then outside, of seeing a flat painting turn into three-dimensional space.

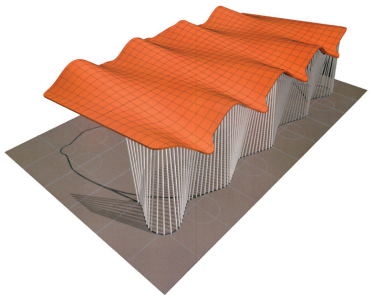
# 23 Gaudí (1)

*«When making surfaces, geometry doesn't complicate the construction but rather simplifies it. The most difficult part is expressing geometric things in the language of algebra, because in that language not everything can be expressed, and there is room for misunderstandings. These disappear when they are solidified into spatial objects.»*

**Antonio Gaudí**

**Gaudí** (1852-1926) is one of the world's most well-known architects, and one of the major tourist attractions of Spain, and in particular of the city of Barcelona, even more after 2002, which had been celebrated as «Gaudí's Year».

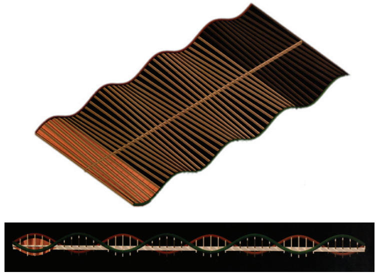
Some of **Gaudí's** hallmarks are the care in the adaptation of natural forms to the terrain he constructed in, the use of light, and the spiritual or transcendental character of much of his work. But underneath all that there is an elaborate and careful mathematical work. He even referred to himself as a «geometer»: «I am a geometer, which means a synthesizer».

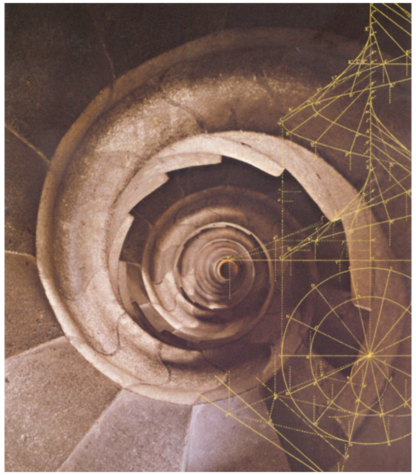


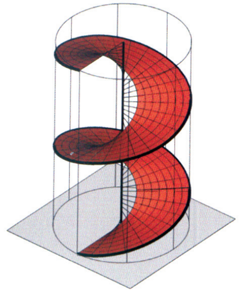
*The Sagrada Familia* as seen from *Casa Milá* - La Pedrera - Barcelona, Spain

The use of lines and Gaudí's sense of infinity was frequent in **ruled surfaces** *(surfaces generated by a line that moves along one or more curves).* He was one of the first architects that realized their architectural interest.

The roof and facade of the "Escoles" of Sagrada Familia made of ruled surfaces, lines sustained by sinusoidal curves.







Helicoidal staircase of Sagrada Familia, another ruled surface.

One-sheeted hyperboloid obtained by the revolution of a hyperbola around its axe. Column in *Parc Güell*, Barcelona, Spain.

# 24 Gaudí (2)

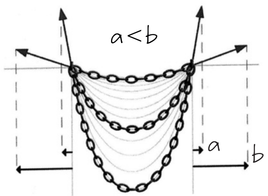
*«The closed curve is the meaning of limitation, just like the straight line is the expression of infinity.»*
**Antonio Gaudí**

Catenary arches on the ground floor of *Las Teresianas College* (Barcelona)

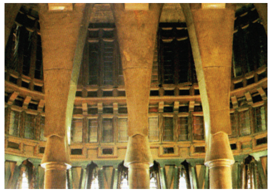
The catenary arch -or **catenary**- is the form acquired by a chain when it is suspended from two points and only supports its own weight.

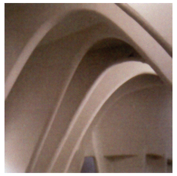
If the weight supported is distributed uniformly with respect to the horizontal distance, when suspending it from two points it yields a **parabola.** If it supports different weights at some of its points, it adopts the shape of a **funicular arch.**

By turning these archs upside down, we obtain the profiles of arches with the same length supporting the same weight.




The entrance and hall of parabolic arches in *Palau Güell* (Barcelona, Spain).





Catenary arches in *Casa Batlló* (Barcelona, Spain).

The thinner these arches are, the less lateral pressure they discharge on the walls and fewer buttresses are necessary.

In spite of the optimality of the catenary arch with respect to resistance, for aesthetic reasons it was little employed before Gaudí used it profusely in all of his work.

Catenary arches in *La Pedrera.*

Spirals in the roof of *La Pedrera.*

Spinning a thread from a cylinder of cone yields beautiful spirals.

The Archimedean and Logarithmic **spirals**, so frequent in Nature (horns of animals, snails, sunflowers…) are also used in Gaudí's work for decorative purposes: ironwork grids, balconies, mosaics…

And of course, all conic sections - circles, ellipses, parabolas and hyperbolas - are found as sections or slices of ruled surfaces, as seen on the poster Gaudí (1).

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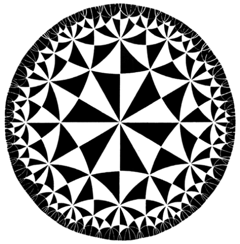
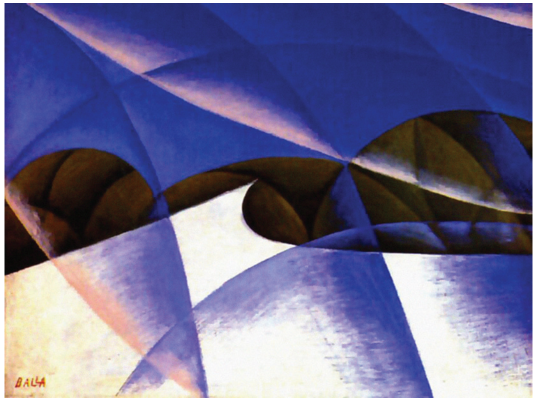
# 25 Mathematics as an artistic object



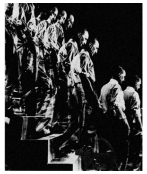
Mathematics has not only influenced in Art, but has also directly inspired artists. Here you can see some samples to close this exhibition.

Escher created spectacular renditions of spaces constructed by mathematicians, like in this circular model of the Universe by **Henri Poincaré.**

Different artistic movements from the beginning of the 20th century, like *futurism* or *surrealism,* have used the new geometries of their time. It is worth pointing out the «temporal dimension»: Motion was one of their main motivations.




*Abstract Velocity*, by **Giacomo Balla.**



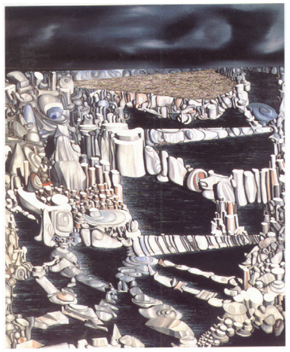
*Nude descending stairs*, by **Marcel Duchamp.**

*«Geometry, the science of space, its dimensions and relations, have always determined the norms and rules of painting.»*
**H. B. Chipp**

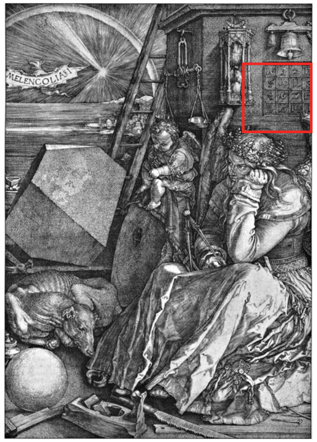
You can find here other examples of mathematical inspiration at different times in history.

*Portrait of Lucca Pacioli*, by **Jacopo de Barbari.**

*Zero*, by **Jasper Johns.**



*The imaginaries*, by **Yves Tanguy.**




*Melancolia*, by **Alberto Durero**

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