Fibonacci Chain

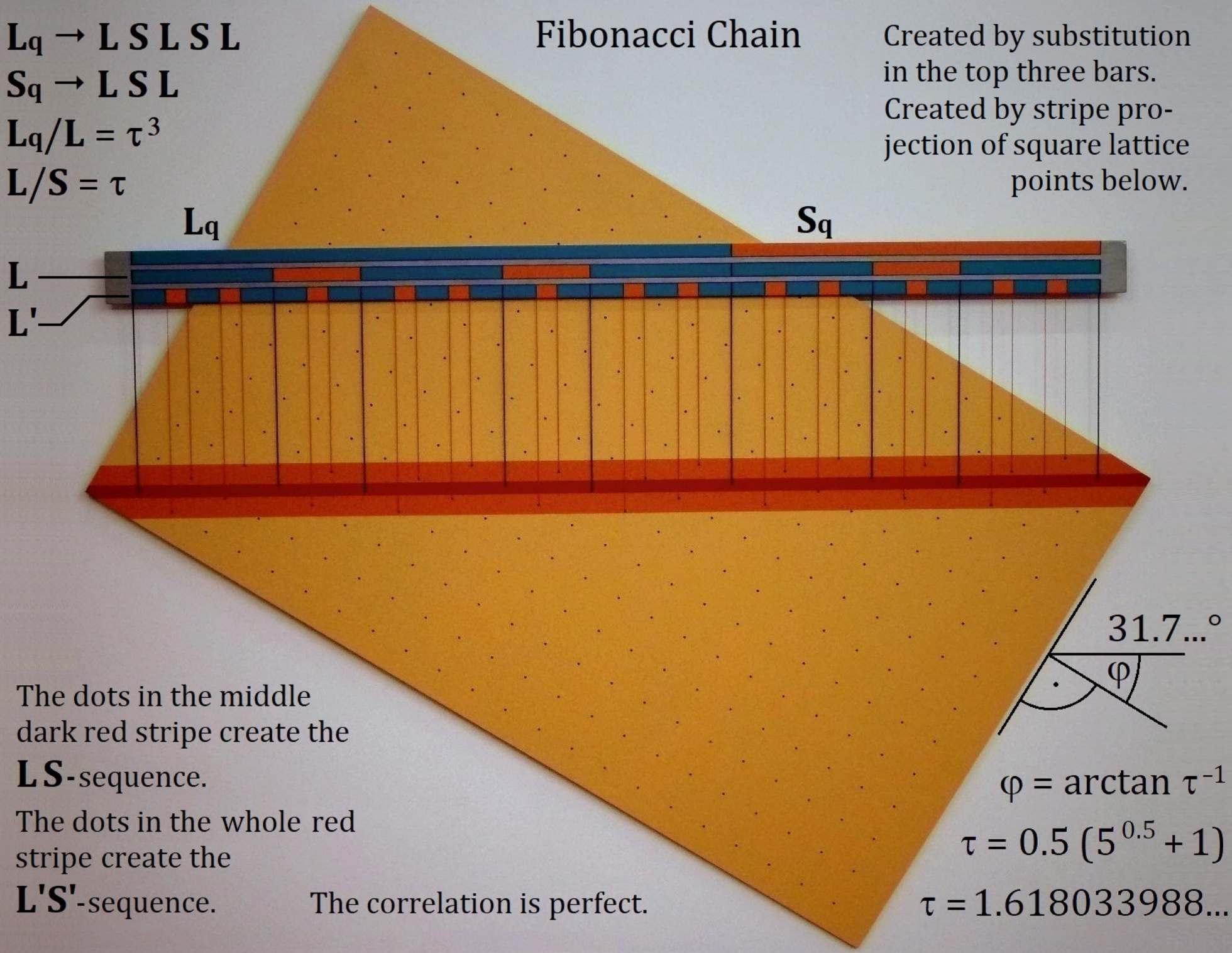
$L_q \rightarrow L\,S\,L\,S\,L$

$S_q \rightarrow L\,S\,L$

$L_q/L = \tau^3$

$L/S = \tau$

Created by substitution in the top three bars. Created by stripe projection of square lattice points below.



The dots in the middle dark red stripe create the **L S**-sequence.

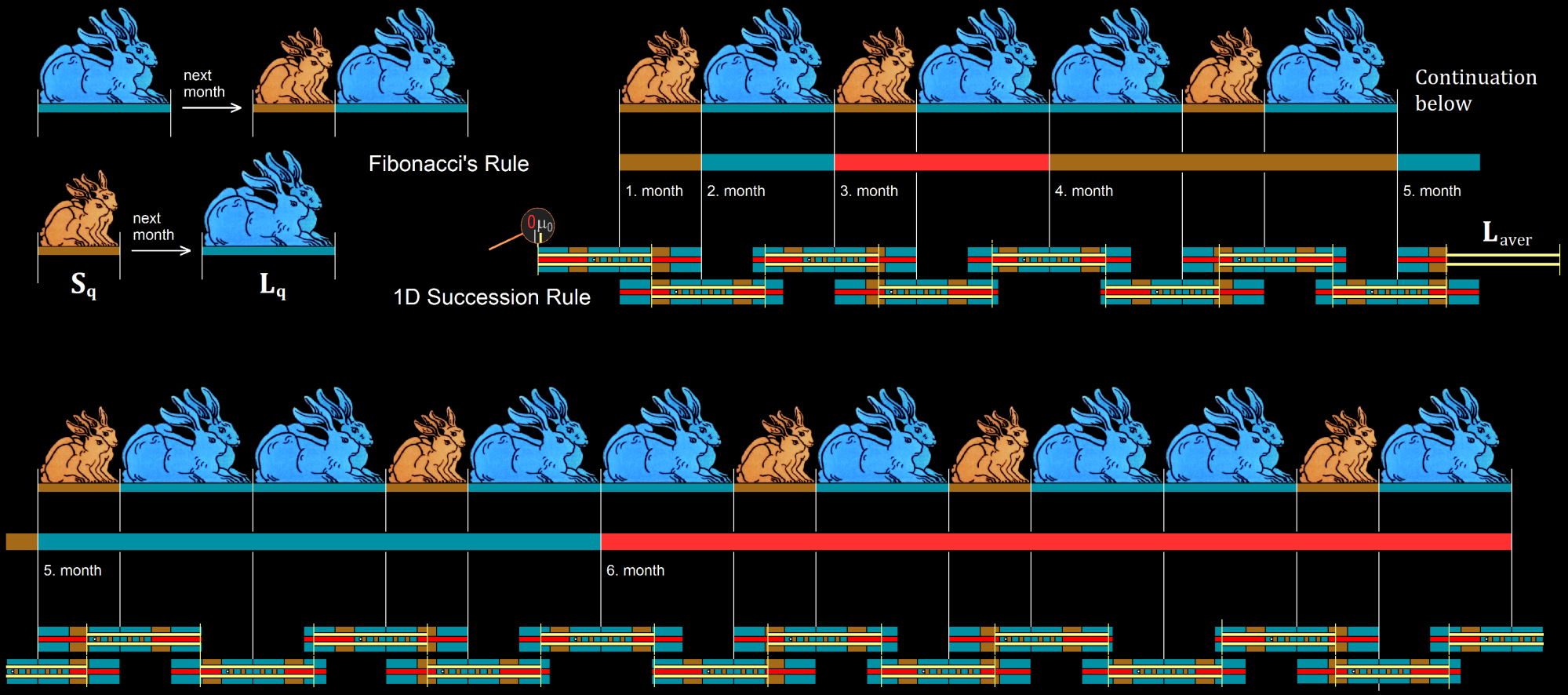
The dots in the whole red stripe create the **L'S'**-sequence.

The correlation is perfect.

$\varphi = \arctan \tau^{-1}$

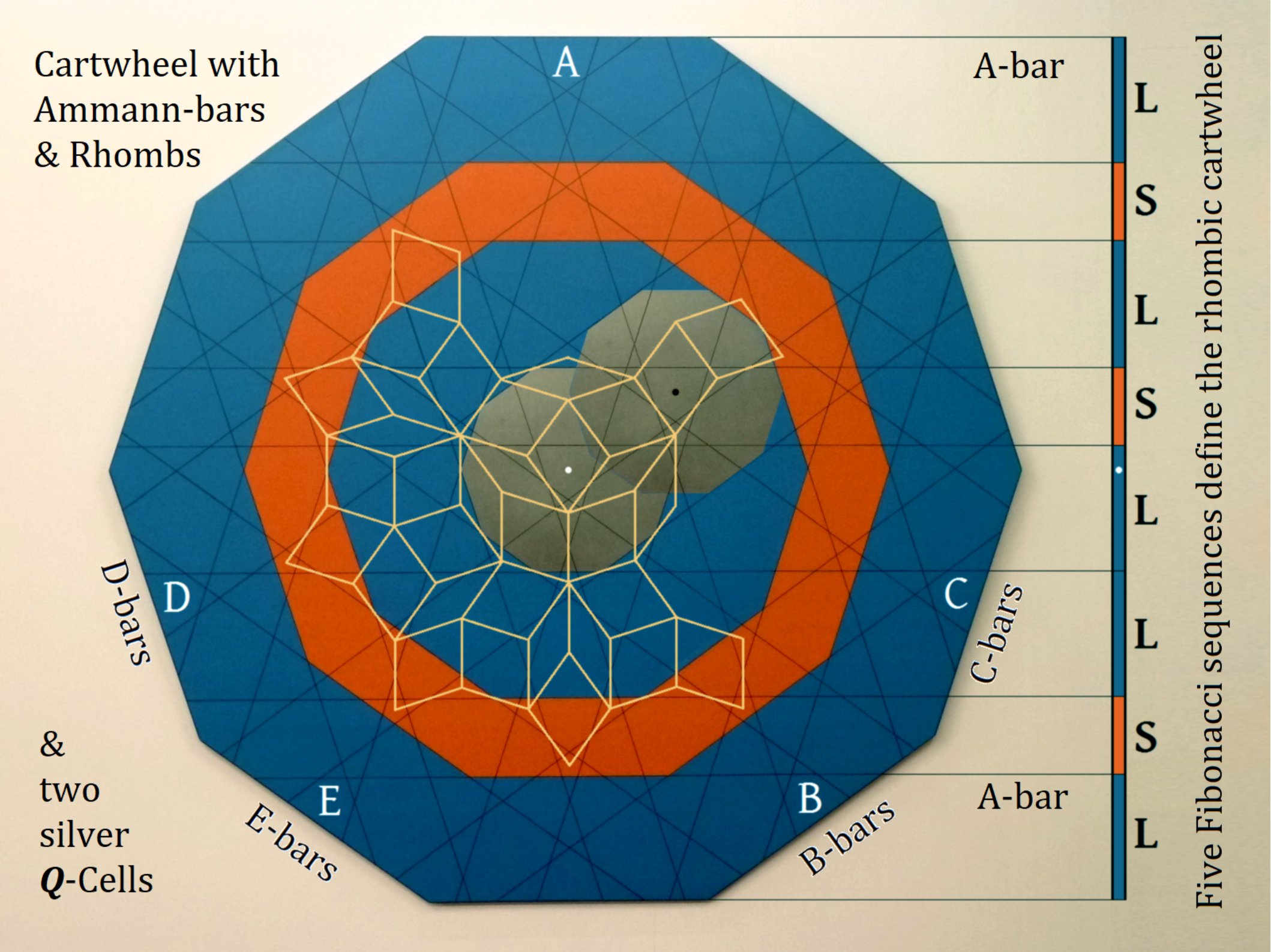
$\tau = 0.5\,(5^{0.5} + 1)$

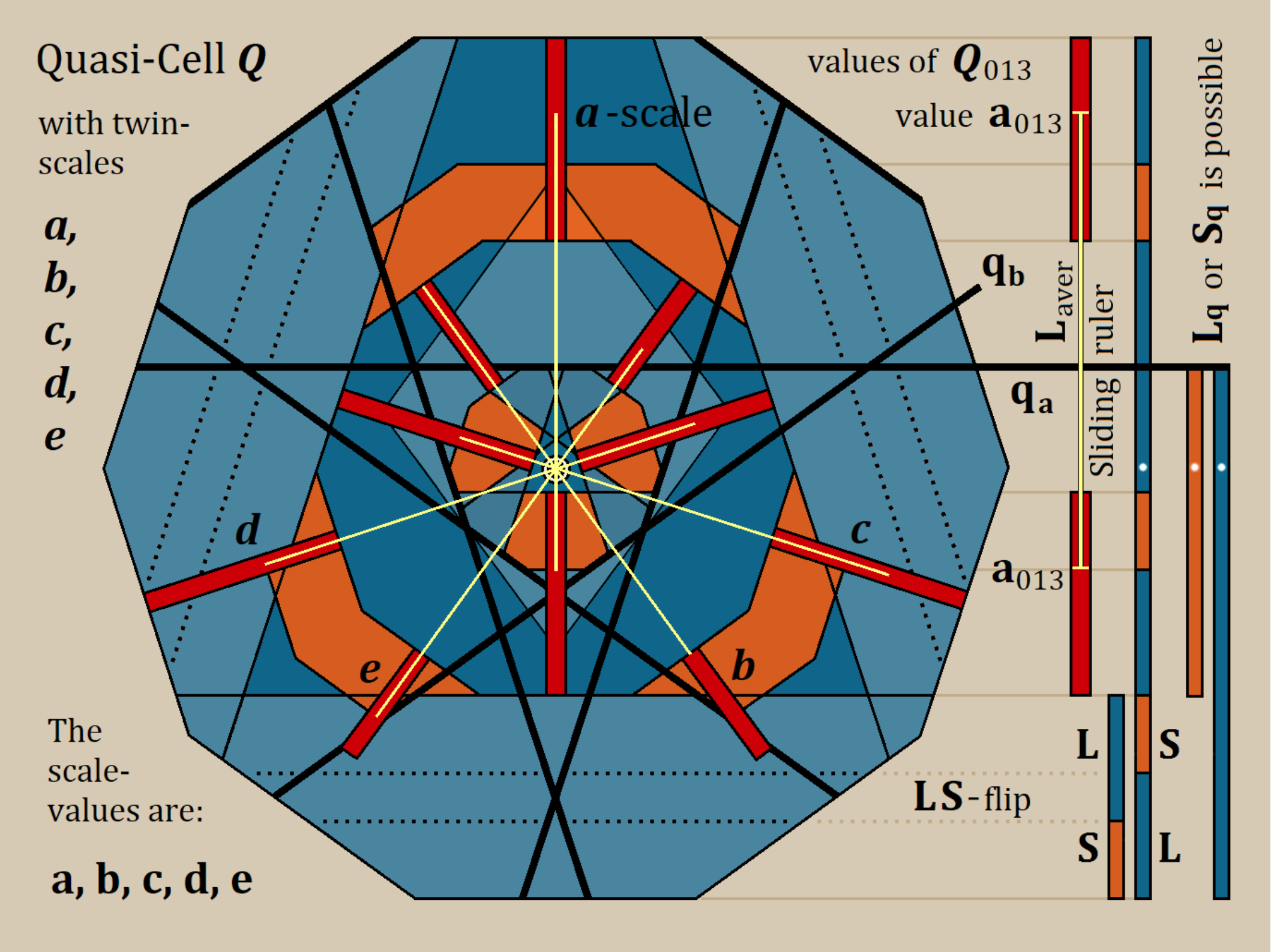
$\tau = 1.618033988...$

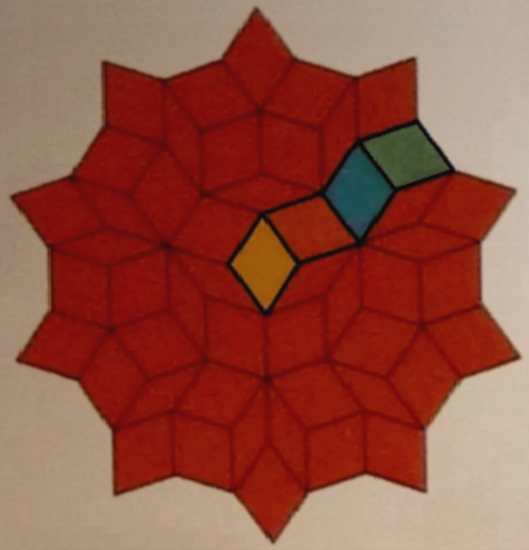


The first aperiodic sequence was found by Leonardo Fibonacci (c. 1170 - c. 1240-50). He described the growth of a population of rabbit couples under idealized conditions: A newborn couple is sexually mature after one month and then gives birth to another couple every month.
May the rabbits be given eternal life.

Since in a Fibonacci sequence the number of adult pairs to the young pairs is approaching the golden ratio τ (= 1.618...), the sequence is suitable for modelling quasicrystals with tenfold rotational symmetrie. But for this, a successively generating rule is needed. If the old and young pairs assigned lengths $\mathbf{L_q}$ and $\mathbf{S_q}$ that are in the golden ratio τ to each other, an average length $\mathbf{L}_{aver}$ can be calculated.
With this average length, it is possible to construct quasicells **Q** that successively control the correct growth of the Fibonacci sequence.


Cartwheel with
Ammann-bars
& Rhombs
A
A-bar
L
S
L
S
L
L
S
L
A-bar
B
B-bars
C
C-bars
D
D-bars
E
E-bars
&
two
silver
Q-Cells
Five Fibonacci sequences define the rhombic cartwheel


Quasi-Cell Q
with twin-scales
a,
b,
c,
d,
e
a-scale
values of Q_{013}
value a_{013}
q_b
L_{aver}
Sliding ruler
q_a
a_{013}
L_q or S_q is possible
d
c
e
b
The scale-values are:
a, b, c, d, e
L
S
LS-flip
S
L



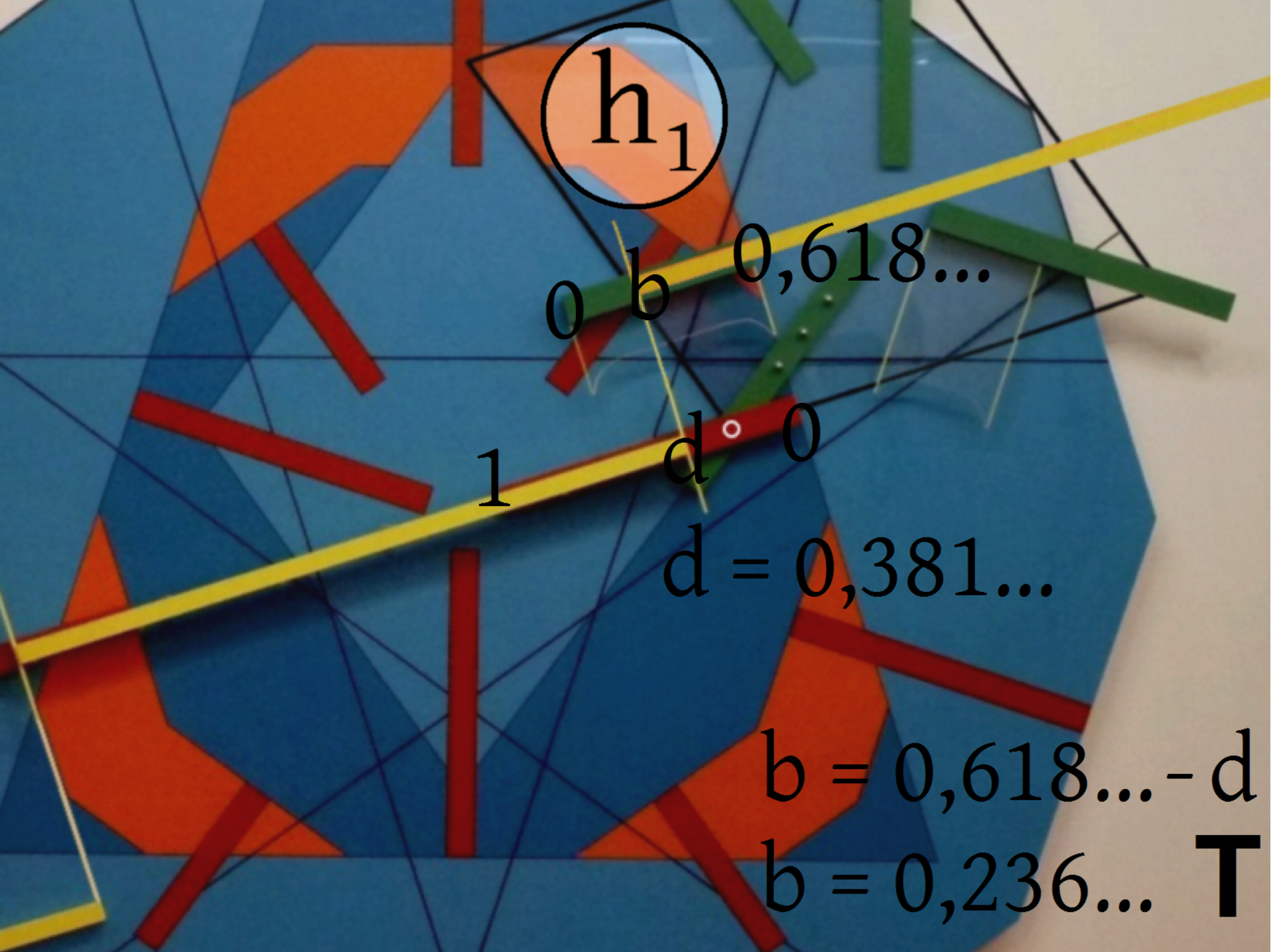
Quasi-Cell Q_{013} with Rhombus R_{013} in the center

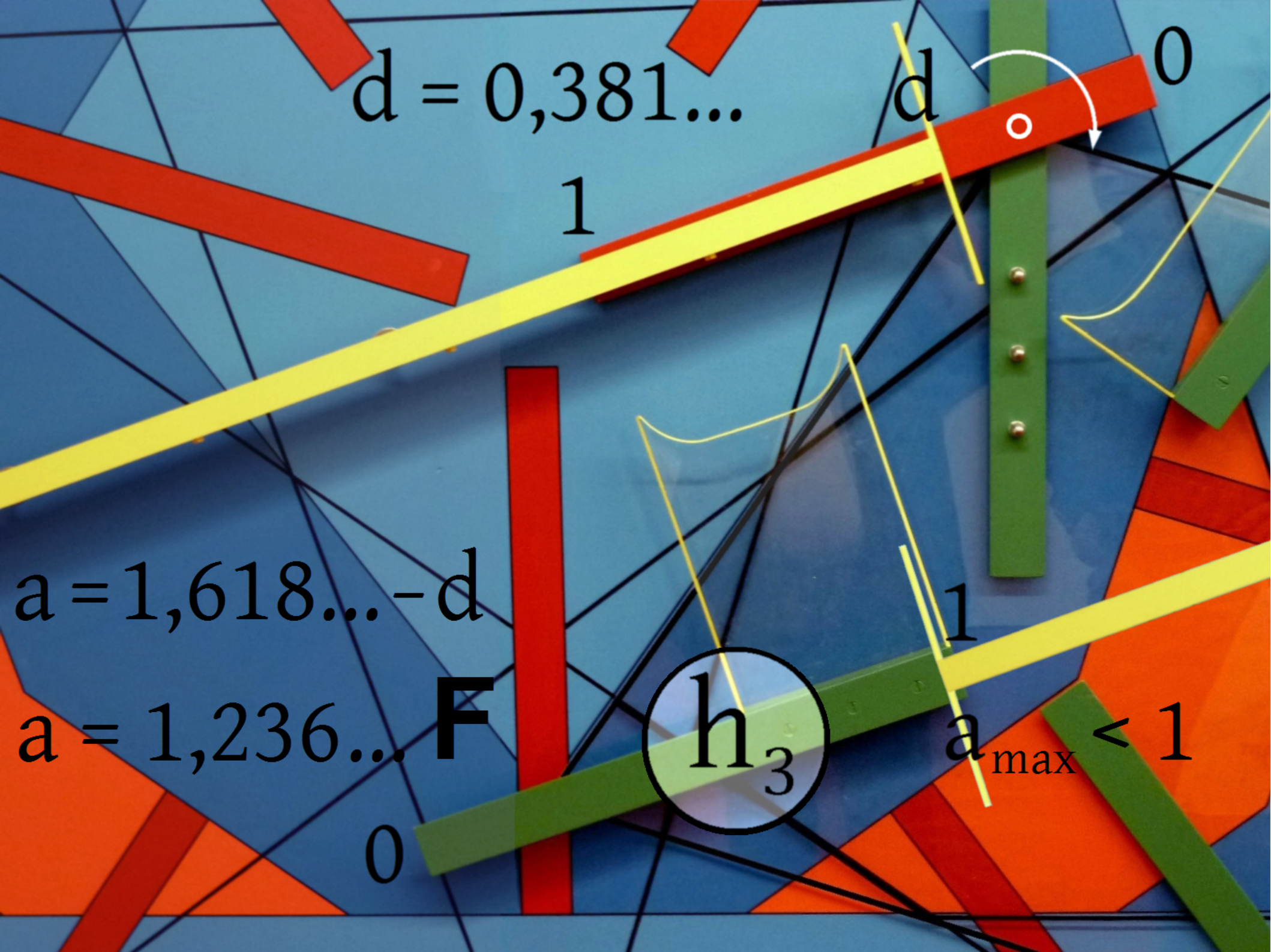
Rhombus R_{0131}

Cartwheel
R_0 (yellow)
R_{01} (orange)
R_{013} (blue)
R_{0131} (green)

Rhombus R_0
with five
Sliding Rulers

Decagonal
Quasiperiodic
Succession


h_1
0
b
0,618...
1
d
0
d = 0,381...
b = 0,618... - d
b = 0,236...
T


d = 0,381...
d
0
1
a = 1,618... – d
a = 1,236...
F
0
h3
1
amax < 1

$t(Q_{013}) = T\ (\text{True})$

$a_{013} = 0{,}618... + \mu_0$	$\in$	a^{def}
$b_{013} = \mu_0$	$\in$	b^{def}
$c_{013} = 0{,}618... + \mu_0$	$\in$	c^{def}
$d_{013} = 0{,}381... + \mu_0$	$\in$	d^{def}
$e_{013} = 0{,}381... + \mu_0$	$\in$	e^{def}

$0 < a^{def}(c^{def}, d^{def}) < 1$

$0 < b^{def}(e^{def}) < \tau^{-1} = 0{,}618...$

$i, j \in \{\bar{1}, \bar{4}, \bar{3}, \bar{2}, \bar{5}, 5, 4, 3, 2, 1\}$

j	$\bar{1}$	$\bar{4}$	$\bar{3}$	$\bar{2}$	$\bar{5}$	5	4	3	2	1
t_j								F	F	T

$h_{\bar{1}}$

$a_{013\bar{1}} = 1 - d_{013}$	$\in$	
$b_{013\bar{1}} = e_{013}$	$\in$	
$c_{013\bar{1}} = 1 - a_{013}$	$\in$	
$d_{013\bar{1}} = \tau^{-1} - b_{013}$	$\in$	
$e_{013\bar{1}} = \tau^{-1} - c_{013}$		

$h_{\bar{4}}$

$a_{013\bar{4}} = \tau^{-1} - b_{013}$	$\in$	
$b_{013\bar{4}} = -\tau^{-1} + c_{013}$		
$c_{013\bar{4}} = 1 - d_{013}$	$\in$	
$d_{013\bar{4}} = e_{013}$	$\in$	
$e_{013\bar{4}} = 1 - a_{013}$		

$h_{\bar{3}}$

$a_{013\bar{3}} = \tau - c_{013}$		
$b_{013\bar{3}} = 1 - d_{013}$		
$c_{013\bar{3}} = 1 - e_{013}$	$\in$	
$d_{013\bar{3}} = \tau - a_{013}$		
$e_{013\bar{3}} = b_{013}$	$\in$	

$h_{\bar{2}}$

$a_{013\bar{2}} = 1 - b_{013}$	$\in$	
$b_{013\bar{2}} = c_{013}$		
$c_{013\bar{2}} = 1 - d_{013}$	$\in$	
$d_{013\bar{2}} = \tau^{-1} + e_{013}$		
$e_{013\bar{2}} = \tau^{-1} - a_{013}$		

$h_{\bar{5}}$

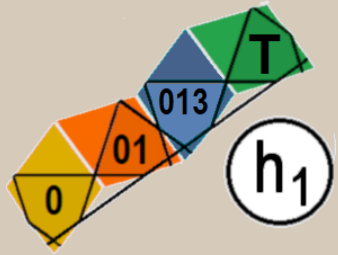
$a_{013\bar{5}} = \tau^{-1} + e_{013}$		
$b_{013\bar{5}} = -\tau^{-1} + a_{013}$		
$c_{013\bar{5}} = 1 - b_{013}$	$\in$	
$d_{013\bar{5}} = c_{013}$	$\in$	
$e_{013\bar{5}} = 1 - d_{013}$		

h_5

$a_{0135} = \tau^{-1} + b_{013}$		
$b_{0135} = 1 - c_{013}$		
$c_{0135} = d_{013}$	$\in$	
$d_{0135} = 1 - e_{013}$	$\in$	
$e_{0135} = -\tau^{-1} + a_{013}$		

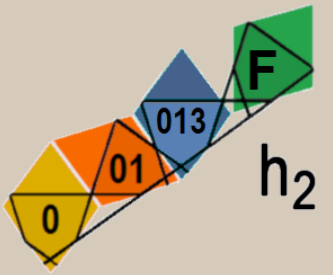
h_4

$a_{0134} = 1 - e_{013}$	$\in$	
$b_{0134} = \tau^{-1} - a_{013}$		
$c_{0134} = \tau^{-1} + b_{013}$		
$d_{0134} = 1 - c_{013}$	$\in$	
$e_{0134} = d_{013}$		



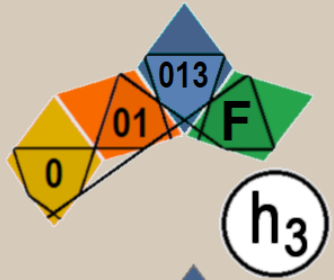
h_1

$a_{0131} = 0{,}381... - \mu_0$	$\in$	
$b_{0131} = 0{,}236... - \mu_0$	$\in$	
$c_{0131} = 0{,}236... - \mu_0$	$\in$	
$d_{0131} = 0{,}381... - \mu_0$	$\in$	
$e_{0131} = \mu_0$	$\in$	



h_2

$a_{0132} = \tau^{-1} - e_{013}$	$\in$	
$b_{0132} = 0{,}381... - \mu_0$	$\in$	
$c_{0132} = b_{013}$	$\in$	
$d_{0132} = 1 - c_{013}$	$\in$	
$e_{0132} = -0{,}236... + \mu_0$	$\notin$	



h_3

$a_{0133} = 1{,}236... - \mu_0$	$\notin$	
$b_{0133} = e_{013}$	$\in$	
$c_{0133} = 1 - \mu_0$	$\in$	
$d_{0133} = 1 - b_{013}$	$\in$	
$e_{0133} = 0{,}381... - \mu_0$	$\in$	

$t(Q_{0\ldots i}) = T\ (\text{True})$

$a_{0\ldots i} =$	$\in$	a^{def}
$b_{0\ldots i} =$	$\in$	b^{def}
$c_{0\ldots i} =$	$\in$	c^{def}
$d_{0\ldots i} =$	$\in$	d^{def}
$e_{0\ldots i} =$	$\in$	e^{def}

$0 < a^{def}\,(c^{def}, d^{def}) < 1$

$0 < b^{def}\,(e^{def}) < \tau^{-1} = 0{,}618\ldots$

$i, j \in \{\bar{1}, \bar{4}, \bar{3}, \bar{2}, \bar{5}, 5, 4, 3, 2, 1\}$

j	$\bar{1}$	$\bar{4}$	$\bar{3}$	$\bar{2}$	$\bar{5}$	5	4	3	2	1
t_j										

$h_{\bar{1}}$

$a_{0\ldots i\bar{1}} =$	$1 - d_{0\ldots i}$	$\in$
$b_{0\ldots i\bar{1}} =$	$e_{0\ldots i}$	$\in$
$c_{0\ldots i\bar{1}} =$	$1 - a_{0\ldots i}$	$\in$
$d_{0\ldots i\bar{1}} =$	$\tau^{-1} - b_{0\ldots i}$	$\in$
$e_{0\ldots i\bar{1}} =$	$\tau^{-1} - c_{0\ldots i}$	

h_1

$a_{0\ldots i1} =$	$1 - c_{0\ldots i}$	$\in$
$b_{0\ldots i1} =$	$\tau^{-1} - d_{0\ldots i}$	
$c_{0\ldots i1} =$	$\tau^{-1} - e_{0\ldots i}$	$\in$
$d_{0\ldots i1} =$	$1 - a_{0\ldots i}$	$\in$
$e_{0\ldots i1} =$	$b_{0\ldots i}$	$\in$

$h_{\bar{4}}$

$a_{0\ldots i\bar{4}} =$	$\tau^{-1} - b_{0\ldots i}$	$\in$
$b_{0\ldots i\bar{4}} =$	$-\tau^{-1} + c_{0\ldots i}$	
$c_{0\ldots i\bar{4}} =$	$1 - d_{0\ldots i}$	$\in$
$d_{0\ldots i\bar{4}} =$	$e_{0\ldots i}$	$\in$
$e_{0\ldots i\bar{4}} =$	$1 - a_{0\ldots i}$	

h_2

$a_{0\ldots i2} =$	$\tau^{-1} - e_{0\ldots i}$	$\in$
$b_{0\ldots i2} =$	$1 - a_{0\ldots i}$	
$c_{0\ldots i2} =$	$b_{0\ldots i}$	$\in$
$d_{0\ldots i2} =$	$1 - c_{0\ldots i}$	$\in$
$e_{0\ldots i2} =$	$-\tau^{-1} + d_{0\ldots i}$	

$h_{\bar{3}}$

$a_{0\ldots i\bar{3}} =$	$\tau - c_{0\ldots i}$	
$b_{0\ldots i\bar{3}} =$	$1 - d_{0\ldots i}$	
$c_{0\ldots i\bar{3}} =$	$1 - e_{0\ldots i}$	$\in$
$d_{0\ldots i\bar{3}} =$	$\tau - a_{0\ldots i}$	
$e_{0\ldots i\bar{3}} =$	$b_{0\ldots i}$	$\in$

h_3

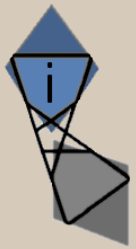
$a_{0\ldots i3} =$	$\tau - d_{0\ldots i}$	
$b_{0\ldots i3} =$	$e_{0\ldots i}$	$\in$
$c_{0\ldots i3} =$	$\tau - a_{0\ldots i}$	
$d_{0\ldots i3} =$	$1 - b_{0\ldots i}$	$\in$
$e_{0\ldots i3} =$	$1 - c_{0\ldots i}$	

$h_{\bar{2}}$

$a_{0\ldots i\bar{2}} =$	$1 - b_{0\ldots i}$	$\in$
$b_{0\ldots i\bar{2}} =$	$c_{0\ldots i}$	
$c_{0\ldots i\bar{2}} =$	$1 - d_{0\ldots i}$	$\in$
$d_{0\ldots i\bar{2}} =$	$\tau^{-1} + e_{0\ldots i}$	
$e_{0\ldots i\bar{2}} =$	$\tau^{-1} - a_{0\ldots i}$	

$h_{\bar{5}}$

$a_{0\ldots i\bar{5}} =$	$\tau^{-1} + e_{0\ldots i}$	
$b_{0\ldots i\bar{5}} =$	$-\tau^{-1} + a_{0\ldots i}$	
$c_{0\ldots i\bar{5}} =$	$1 - b_{0\ldots i}$	$\in$
$d_{0\ldots i\bar{5}} =$	$c_{0\ldots i}$	$\in$
$e_{0\ldots i\bar{5}} =$	$1 - d_{0\ldots i}$	



h_5

$a_{0\ldots i5} =$	$\tau^{-1} + b_{0\ldots i}$	
$b_{0\ldots i5} =$	$1 - c_{0\ldots i}$	
$c_{0\ldots i5} =$	$d_{0\ldots i}$	$\in$
$d_{0\ldots i5} =$	$1 - e_{0\ldots i}$	$\in$
$e_{0\ldots i5} =$	$-\tau^{-1} + a_{0\ldots i}$	

h_4

$a_{0\ldots i4} =$	$1 - e_{0\ldots i}$	$\in$
$b_{0\ldots i4} =$	$\tau^{-1} - a_{0\ldots i}$	
$c_{0\ldots i4} =$	$\tau^{-1} + b_{0\ldots i}$	
$d_{0\ldots i4} =$	$1 - c_{0\ldots i}$	$\in$
$e_{0\ldots i4} =$	$d_{0\ldots i}$	