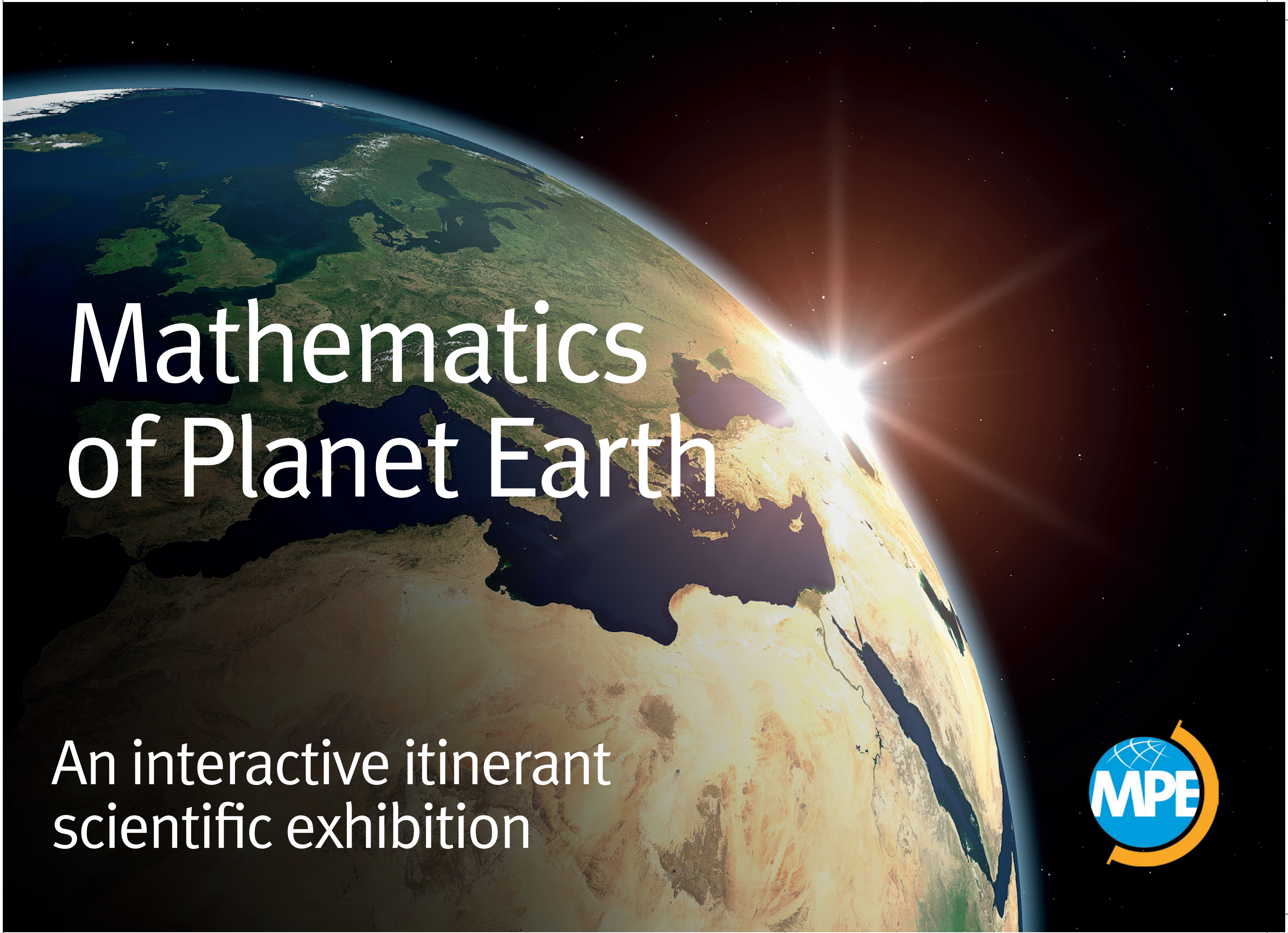

Mathematics of Planet Earth

An interactive itinerant scientific exhibition


OCEAN
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Introduction

Our planet is a complex system with multiple facets including the physical side (such as the atmosphere, the oceans, the soil and the ice sheets), biological side (e.g. biodiversity, aquaculture, carbon cycle) and the human side (e.g. urban climates, power systems and social media).

It is also a system at risk. The planet's capacity to support life as we know it is in danger. The stability of the systems supported by Planet Earth is threatened by rapid changes in the balance between climate and the Earth constituents: atmosphere, oceans, rivers, the chemistry of earth components and many others. Human activity has grown to the point that it influences directly the global climate. It now has a strong impact on the capacity of the planet to be self-sufficient and threatens the stability of the systems supported on it. The challenges that our planet and our civilisation are currently facing cannot be addressed only by separate scientific endeavours. Instead, our efforts must be multidisciplinary, and their common language is mathematics. The mathematical sciences play a leading role in the joint effort of researchers to understand and quantify the challenges we face, and look for solutions.

'Mathematics of Planet Earth' is an international exhibition displaying exhibits, videos and computer programs. Collectively these illustrate how mathematics plays a role in answering essential questions that concern our planet. In interactive graphics visualisations, videos and hands-on experiments you will discover the contributions that mathematics makes to topics such as astronomy, fluid dynamics, seismology, glaciology and cartography.

www.mathsofplanetearth.org

This exhibition is organised by the EPSRC Centre for Doctoral Training (CDT) in the Mathematics of Planet Earth (MPE).

www.mpecdt.org

The exhibit materials are provided by:

- IMAGINARY (project by the Mathematisches Forschungsinstitut Oberwolfach and supported by the Klaus Tschira Stiftung)
- Centre•Sciences (CCSTi Region Centre - Val de Loire)

Design and realization of the interactive hands-on exhibits:

- Centre•Sciences (CCSTi of region Centre - Val de Loire – Orléans) and ADECUM Association for mathematic culture development), under the auspices of UNESCO and international mathematical sciences organizations

Realization of the digital modules and the videos of the competition:

- Idea, Design, Program: participants of the competition MPE 2013 Jury chaired by Ehrhard Behrends (Freie Universität Berlin)

The virtual modules are available on the MPE Open Source Exhibition:

www.imaginary.org/exhibition/mathematics-of-planet-earth

Mathematisches
Forschungsinstitut
Oberwolfach

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1. Maps of the Earth

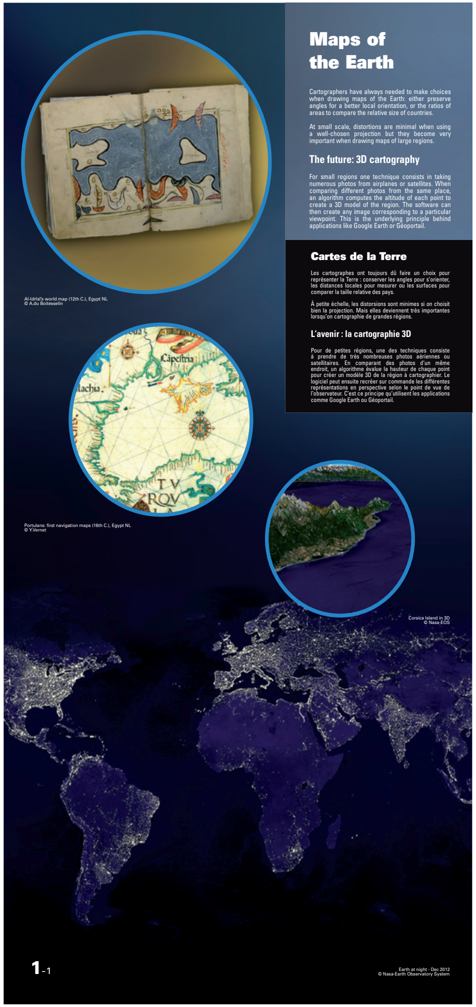
1-1 Maps of the Earth

Cartographers have always needed to make choices when drawing maps of the Earth: either preserve angles for a better local orientation, or the ratios of areas to compare the relative size of countries.

At small scale, distortions are minimal when using a well-chosen projection but they become very important when drawing maps of large regions.

The future: 3D cartography

For small regions one technique consists in taking numerous photos from airplanes or satellites. When comparing different photos from the same place, an algorithm computes the altitude of each point to create a 3D model of the region. The software can then create any image corresponding to a particular viewpoint. This is the underlying principle behind applications like Google Earth or Géoportail.



1-2 All maps are wrong!

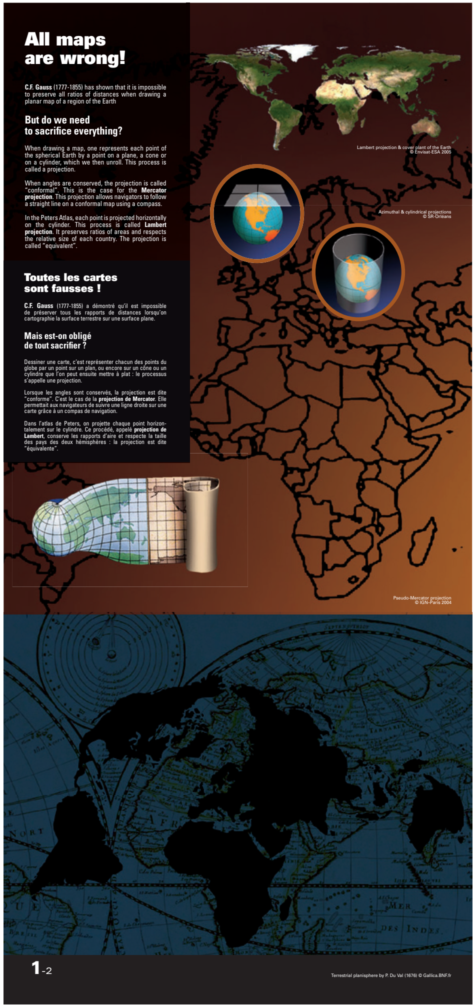
C.F. Gauss (1777-1855) has shown that it is impossible to preserve all ratios of distances when drawing a planar map of a region of the Earth.

The trouble with maps

When drawing a map, one represents each point of the spherical Earth by a point on a plane, a cone or on a cylinder, which we then unroll. This process is called a projection.

When angles are conserved, the projection is called “conformal”. This is the case for the Mercator projection. This projection allows navigators to follow a straight line on a conformal map using a compass.

In the Peters Atlas, each point is projected horizontally on the cylinder. This process is called Lambert projection. It preserves ratios of areas and respects the relative size of each country. The projection is called “equivalent”.



References:

An interactive module on the science of cartography and geometry of the sphere

- imaginary.org/program/the-sphere-of-the-earth (can be downloaded for free)
- imaginary.org/hands-on/all-maps-are-wrong
- www.ngi.be/FR/FR2-1-4.shtm
- accromath.uqam.ca/accro/wp-content/uploads/2013/08/Cartographie.pdf
- Maps That Prove You Don't Really Know Earthwww.youtube.com/watch?v=KUF_Ckv8HbE#t=110

Activities:

1. Paris – Vancouver –Tokyo
Using the string, find the shortest path joining two of these cities on the globe, and then on the flat map.

What should you find?
On a spherical surface, the shortest path between two points is the small arc of the great circle centred at the centre of the sphere and passing through the two points. Had you already realized that the shortest path from Paris to Vancouver was passing close to the North Pole? Not obvious when we look at the map...

Why are all maps distorted?
If we cut a cylinder or a cone we can flatten it on a plane. This is not possible with a sphere. Indeed, the curvature of the cylinder and of the cone is zero, while that of the sphere is positive. We cannot flatten a surface of positive curvature without multiple tears.

Source:

Centre•Sciences

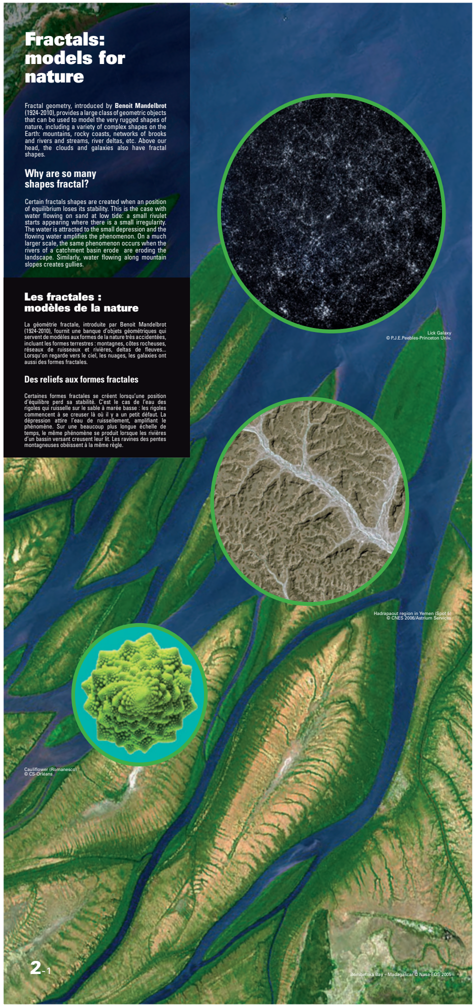
2. Fractals: Models for Nature

2-1 Fractals: models for nature

Fractal geometry, introduced by Benoit Mandelbrot (1924-2010), provides a large class of geometric objects that can be used to model the very rugged shapes of nature, including a variety of complex shapes on the Earth: mountains, rocky coasts, networks of brooks and rivers and streams, river deltas, etc. Above our head, the clouds and galaxies also have fractal shapes.

Why are so many shapes fractal?

Certain fractal shapes are created when a position of equilibrium loses its stability. This is the case with water flowing on sand at low tide: a small rivulet starts appearing where there is a small irregularity. The water is attracted to the small depression and the flowing water amplifies the phenomenon. On a much larger scale, the same phenomenon occurs when the rivers of a catchment basin erode the landscape. Similarly, water flowing along mountain slopes creates gullies.



2-2 Erosion and fractal coasts

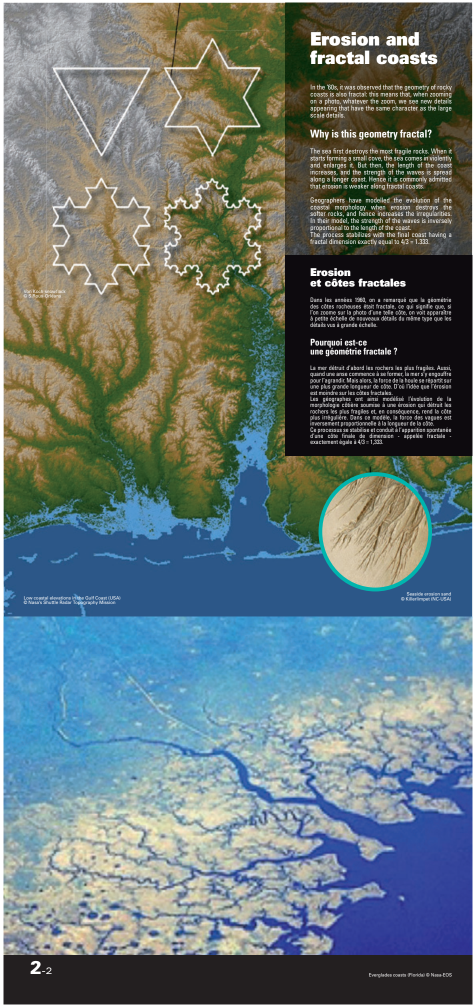
In the ‘60s, it was observed that the geometry of rocky coasts is also fractal: this means that, when zooming on a photo, whatever the zoom, we see new details appearing that have the same character as the large scale details.

Why is this geometry fractal?

The sea first destroys the most fragile rocks. When it starts forming a small cove, the sea comes in violently and enlarges it. But then, the length of the coast increases, and the strength of the waves is spread along a longer coast. Hence it is commonly admitted that erosion is weaker along fractal coasts.

Geographers have modelled the evolution of the coastal morphology when erosion destroys the softer rocks, and hence increases the irregularities. In their model, the strength of the waves is inversely proportional to the length of the coast.

The process stabilizes with the final coast having a fractal dimension exactly equal to 4/3 = 1.333.



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- accromath.uqam.ca/contents/pdf/fractales.pdf (Josiane Lajoie - UQTR - Automne 2006)
- geoffreyhistoire.pagesperso-orange.fr/fractales/géographie.html
- imaginary.org/hands-on/erosion-and-fractal-coasts

Activities:

2. Fractals, from large scale to small scale!
With each of the chains, measure the length of the more uneven coast, at the north of the island. In your opinion, why are the lengths so different?

What should you find?
The north coast, more uneven, is an example of fractal object: each small portion of the coast, after appropriate zooming resembles the entire coast.

The south coast is smoother than the north coast. The measured lengths vary very little.

Some coasts with infinite length!
If you were to measure a real coast with chains whose links got smaller and smaller, you would be able to take into account much smaller details, and the length of the coast would tend to infinity. Coasts like this can be found in Corsica, Iceland, Sardinia, etc.

Source:

Jean Brette (Paris) and S[cube]-CCSTi Île-de-France

3. The Water, from Rivers to Ocean

3-1 The melting of glaciers

Climate warming induces a thermal expansion of the oceans and the melting of glaciers.

Simulation software such as ”Flood map”, based on elevation data coming from satellite radars allows us to compute the potential consequences of sea-level rising on coastal regions and populations, including large cities like Rio de Janeiro, New York, Tokyo, Shanghai, etc.

How high can sea-level rise?

In the South Pole, the volume of the Antarctica glaciers is at least 22 million km3. In the North, the volume of Greenland glaciers is approximately 2,8 million km3. The total area of the Earth’s oceans is 335 million km2. If all the glaciers were to melt instantaneously, this could lead to a uniform rise of the sea-level of 75 meters!

Taking into account the limits of such models, the scientists are presently less pessimistic and forecast a sea-level rise somewhere between 20 cm and 60 cm before 2100, including seawater expansion

3-2 Permeable or impermeable?

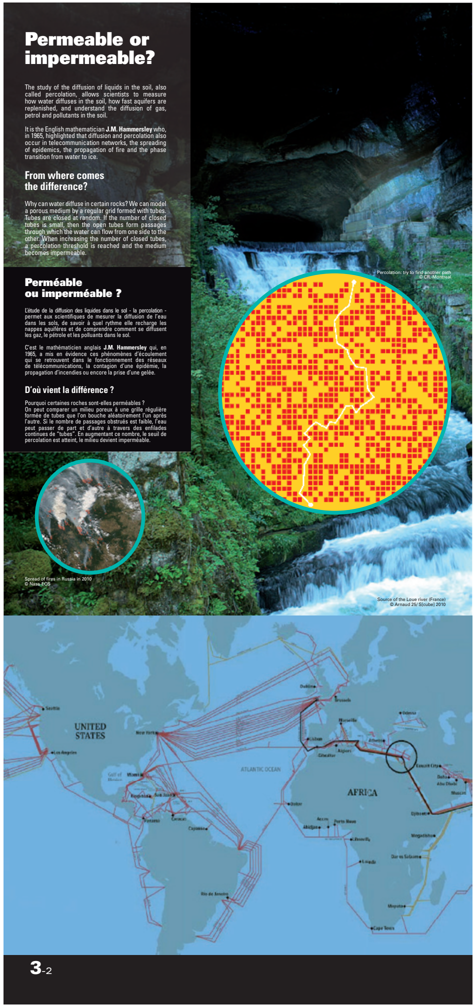
The study of the diffusion of liquids in the soil, also called percolation, allows scientists to measure how water diffuses in the soil, how fast aquifers are replenished, and understand the diffusion of gas, petrol and pollutants in the soil.

© Klaus Tschira Stiftung

In 1965, the English mathematician J.M. Hammersley, highlighted that diffusion and percolation also occur in telecommunication networks, the spreading of epidemics, the propagation of fire and the phase transition from water to ice.

Why can water diffuse in certain rocks?

We can model a porous medium by a regular grid formed with tubes. Tubes are closed at random. If the number of closed tubes is small, then the open tubes form passages through which the water can flow from one side to the other. When increasing the number of closed tubes, a percolation threshold is reached and the medium becomes impermeable.



References:

- imaginary.org/hands-on/the-melting-of-glaciers
- imaginary.org/hands-on/permeable-or-impermeable

Activities:

3-1 Why does the sea level rise?
With your fingers, heat the lower container. What happens? Why?

What should you find?
Global warming causes the melting of ice. When the ice comes from continental glaciers (mainly Antarctica and Greenland), this adds water to the oceans.

On the other hand, Archimedes' principle shows that the melting of the Arctic ice field and of the icebergs does not change the volume of the oceans.

Water dilates when warming
But the principal cause of the rising of the sea level is the thermal expansion of seawater. In the last century, the temperature of the Earth increased by 0.6 °C, inducing a warming of the oceans at a deepness of 1 km and causing a sea level rise of 15 cm. Global warming continues to increase the temperature of the oceans at greater depths.

3-2 "A porous abacus"
Place the abacus vertically and make the squares turn around the axes. Then put back the abacus on its support. Does there exist a path composed of red lines connecting two opposite sides?

What should you find?
The sides of the abacus contain squares representing links that can be created with neighbouring squares. When turning a square, the connections change. The experience shows a random drawing where there exist many paths from one side to the other. These are called "percolation paths".

A mathematical model of percolation phenomena
These paths are unstable and do not form at each try. It suffices to turn a few squares to have them appear or disappear.

This game helps explaining the permeability of irregular materials, like rocks, and also the electric conductivity. It provides a simple mathematical model of these physical phenomena.

Source:

3-1 Centre•Sciences

3-2 Xavier Guyon (University of Paris) and S[cube]-CCSTi Île-de-France

4. From the Core of Earth to Tectonic Plates

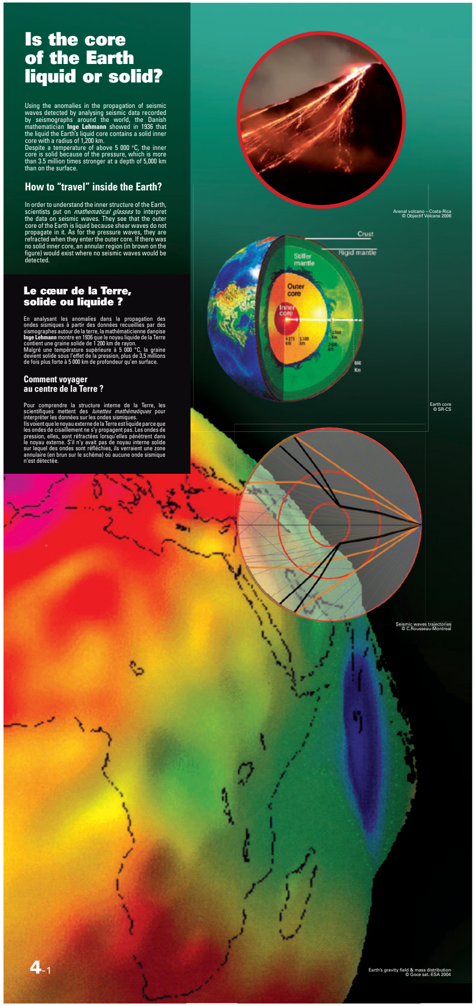
4-1 Is the core of the Earth liquid or solid?

Using the anomalies in the propagation of seismic waves detected by analysing seismic data recorded by seismographs around the world, the Danish mathematician Inge Lehmann showed in 1936 that the Earth's liquid core contains a solid inner core with a radius of 1,200 km.

Despite a temperature of above 5 000 °C, the inner core is solid because of the pressure, which is more than 3.5 million times stronger at a depth of 5,000 km than on the surface.

How to "travel" inside the Earth?

In order to understand the inner structure of the Earth, scientists put on mathematical glasses to interpret the data on seismic waves. They see that the outer core of the Earth is liquid because shear waves do not propagate in it. As for the pressure waves, they are refracted when they enter the outer core. If there was no solid inner core, an annular region (in brown on the figure) would exist where no seismic waves would be detected.



4-2 Tectonic plates

From the inner core to the surface the Earth temperature decreases. This induces convection movements in the Earth's mantle. The warmer magma moves closer to the surface where it cools down. It then sinks inside the mantle and a new cycle is started. This induces movements of the rigid tectonic plaques at the surface and creates shears.

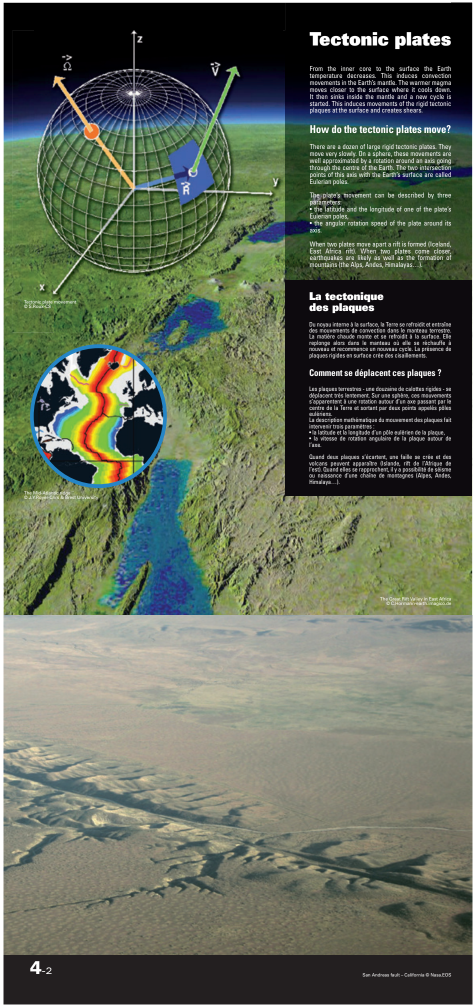
How do the tectonic plates move?

There are a dozen of large rigid tectonic plates. They move very slowly. On a sphere, these movements are well approximated by a rotation around an axis going through the centre of the Earth. The two intersection points of this axis with the Earth's surface are called Eulerian poles.

The plate's movement can be described by three parameters:

- the latitude and the longitude of one of the plate's Eulerian poles,
- the angular rotation speed of the plate around its axis.

When two plates move apart a rift is formed (Iceland, East Africa rift). When two plates come closer, earthquakes are likely as well as the formation of mountains (the Alps, Andes, Himalayas...).



References:

- imaginary.org/hands-on/is-the-core-of-the-earth-solid-or-liquid
- imaginary.org/hands-on/tectonic-plates

Activities:

4-1 The open heart of the Earth!

Two earthquakes just occurred in the Southern hemisphere. Observe the seismic waves trajectories in the open part of the Earth. Can you find where the earthquakes occurred?

What should you find?

To show that the inner core is solid, Inge Lehmann analysed the travel paths and arrival times of the seismic waves generated by large earthquakes located at antipodal points.

Calculate the travel time of seismic waves

It is easy to calculate the travel path and travel time of seismic waves if one knows the materials through which the wave has travelled. However, mathematicians want to reconstruct the inner structure of the Earth using the data on travel times of seismic waves recorded by different stations around the world. The model below was proposed by Inge Lehmann to explain the observed seismic data: the waves are reflected on the inner core.

Travelling through the Earth

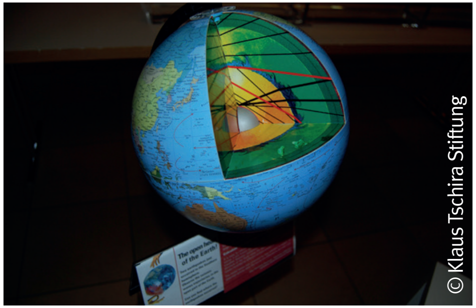
To study the core of the Earth, use your mathematical eyes: move the green laser to observe the propagation of seismic waves inside the Earth.

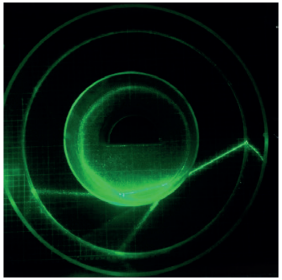
What should we find?

At the time of Inge Lehmann, it was admitted that a mantle would surround a core. However, some seismic waves seemed to indicate that this model has a flaw and must be correct to fit with the observed data. Inge Lehman deduced that the core was not homogeneous: there is a smaller inner core, surrounded by the outer core. Hence, the wave could be reflected if the angle of incidence is sufficiently large.

Source:

Christiane Rousseau (University of Montreal) and Centre•Sciences & Tryame





Activities:

4-2 Tectonic plates and subduction

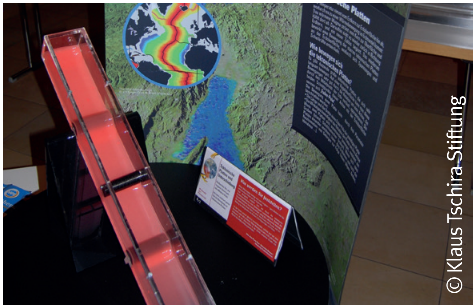
Observe how slowly this very viscous material flows. It simulates, in the laboratory, the moving of tectonic plates.

What should you find?

This silicon plate moves very slowly - similar to the tectonic plates of Earth's crust. A mountain range starts by the closure of an oceanic region. There is a lot of volcanic activity on the overlapping continent, which undergoes tectonic compression leading to a thickening of the crust and to the formation of mountain ranges with rifts, overlapping and folds. Once the ocean has completely disappeared, the continental margin enters in subduction under the thickened continent. This movement cannot last very long since the difference of density between the crust and the mantle is an obstruction to this subduction.

Source:

Jean-Pierre Brun (University of Rennes) and Centre•Sciences



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5. The Solar System

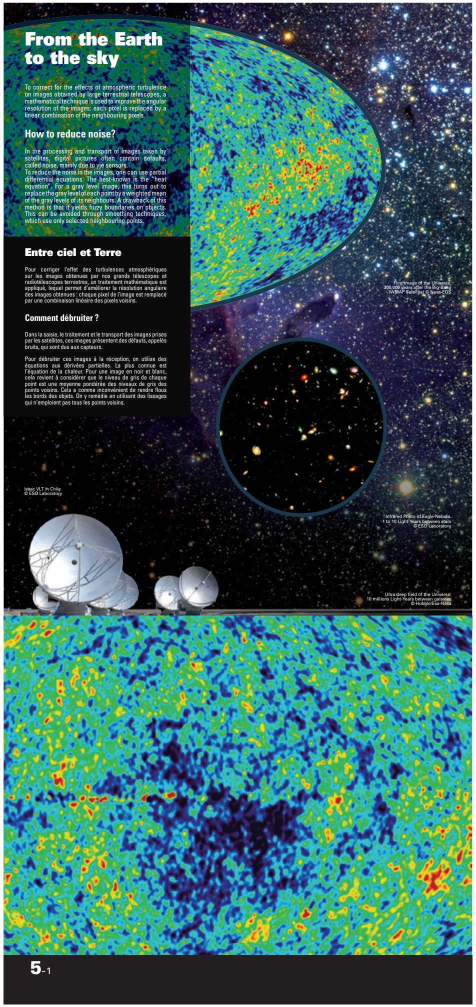
5-1 From the Earth to the sky

To correct for the effects of atmospheric turbulence on images obtained by large terrestrial telescopes, a mathematical technique is used to improve the angular resolution of the images: each pixel is replaced by a linear combination of the neighbouring pixels.

How to reduce noise?

In the processing and transport of images taken by satellites, digital pictures often contain defaults, called noise, mainly due to the sensors.

To reduce the noise in the images, one can use partial differential equations. The best-known is the “heat equation”. For a gray level image, this turns out to replace the gray level of each point by a weighted mean of the gray levels of its neighbours. A drawback of this method is that it yields fuzzy boundaries on objects. This can be avoided through smoothing techniques, which use only selected neighbouring points.



5-2 Where is the Sun at noon?

Maybe you have already noticed that the middle of the day (or solar noon) is not always at the same time on your watch? The sunset is sooner on December 10 than at the winter solstice.

If you observe the Sun at noon every day, then its position draws a figure eight curve, called analemna, with endpoints at the winter and summer solstices.

The equation of time

The equation of time is the difference between the solar noon and the official noon.

This difference is explained by two phenomena:

- Since Kepler, we know that the Earth's orbit is an ellipse with the Sun located at one focus. The Earth travels faster when it is closer to the Sun, i.e. during the boreal winter: real days are then shorter than official days.
- The Earth's axis is at an angle of 23.5 ° with respect to the axis normal to the plane where it rotates around the Sun (the ecliptic plane). Thus, solar days are then longer than the average days around the summer and winter solstices.

These daily differences add up during the year.

References:

- imaginary.org/hands-on/where-is-the-sun-at-noon

Activities:

5-1 The course of the Sun
Place the Sun on one of the seasons. Have it turn around the Earth. Imagine the shadow of the palm tree.

What should you find?
The sunrise is on the east and the sunset on the west. At noon, the Sun is at its highest position, and the shadow of a stick hammered in Earth is the shortest at midday. Outside of the tropics, the shadow is the shortest at the summer solstice.

A stick hammered in Earth is the oldest sundial. It allows dividing the day into two parts: before noon and after noon. If we could see the stars at the same time as the Sun, we would see them turning during the day, but, each month, the Sun would have changed constellation. The Sun visits the 12 constellations of the Zodiac, one per month of the year. This is the case since the Sun is the star closest to the Earth.

Source:

Centre•Sciences

6. The Space of Satellites and Communication

6-1 Where are you?

How many satellites orbiting around the Earth are needed to compute one's exact position at each instant?

The GPS system uses a set of satellites (at least 24) orbiting around the Earth and emitting signals.

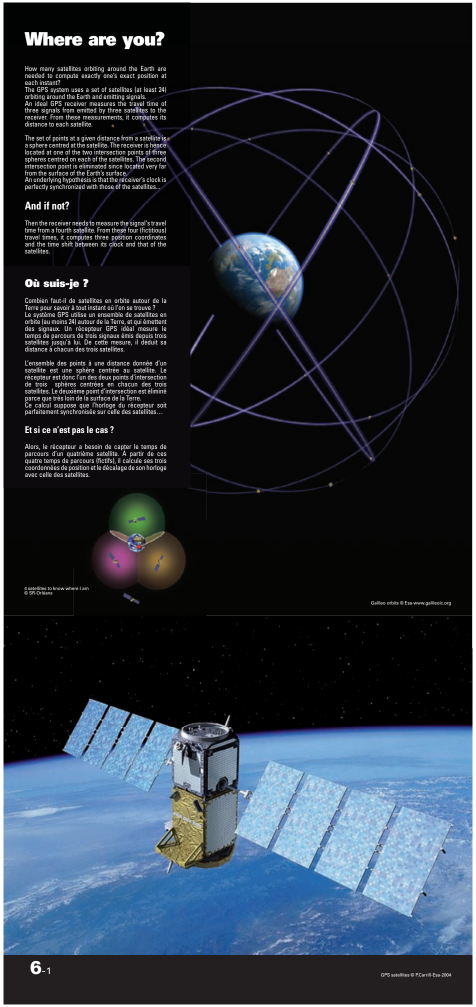
An ideal GPS receiver measures the travel time of three signals emitted by three satellites to the receiver. From these measurements, it computes its distance to each satellite. The set of points at a given distance from a satellite is a sphere centred at the satellite.

The receiver is hence located at one of the two intersection points of three spheres centred on each of the satellites. The second intersection point is eliminated since located very far from the surface of the Earth.

An underlying hypothesis is that the receiver's clock is perfectly synchronized with those of the satellites...

And if not?

Then the receiver needs to measure the signal's travel time from a fourth satellite. From these four (fictitious) travel times, it computes three position coordinates and the time shift between its clock and that of the satellites.



6-2 Satellites under control

Telecommunications, navigation, meteorology, etc., but also mobile phones, GPS and the Internet, provide numerous reasons to place satellites in orbits around the Earth. For this purpose, mathematicians select and optimise trajectories and orbits of spacecrafts with more sophisticated techniques.

Controlling the trajectory!

A satellite will remain on its orbit without the need of additional energy if it is given the right initial speed: this speed depends on the altitude of the orbit. There is only one possible altitude for a geostationary satellite: 36,000 km, the one for which the centrifugal force exerted on the satellite is exactly equal to the gravitational attraction of the Earth.

The same type of techniques can be used when designing interplanetary missions. In that case, the spacecraft is accelerated by passing close to celestial bodies. Moreover, to minimize energy spent in deceleration, its speed of approach should be small close to the celestial bodies it is visiting.

References:

- imaginary.org/hands-on/satellites-under-control
- imaginary.org/hands-on/where-are-you

Activities:

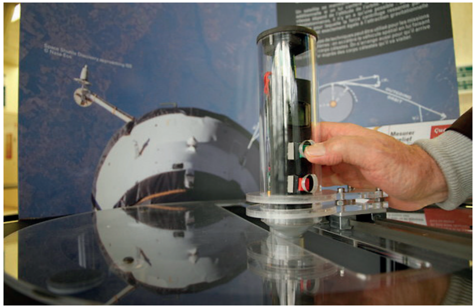
6-1 Measuring the height of the Earth
Move the satellite and find the highest point and the lowest one. To take a measurement, first press on the red button, then push on the green button to record the measurement.

What should you find?
The radar can see the height of the surface of the Earth by measuring the roundtrip travel time of a wave emitted by the satellite to the Earth. These waves cross the atmosphere and are reflected on the ground. This technique can be used day and night, as well as during cloudy periods.

A stereographic view of the oceans
These height measurements allow us to determine, with a precision of a few centimetres, the height of the Earth and the surface of the oceans, and to produce 3D maps. One can then observe the circulation of ocean currents and their variations.

Source:

Centre•Sciences & Tryame



Activities:

6-2 Orbiting satellites!

Take the tray in your hands. Make it turn as a gold panner would make. And make a marble turn around the Earth. Try to get it to move on an elliptical orbit. Can its orbit be circular? Why can it escape from the support?

What should you find?

The precise orbit of a satellite depends on its mission: observation, communication, GPS, etc. It will remain on its chosen orbit if it is given the right speed.

These orbits can be:

- geostationary for communication, remote sensing. The satellites are in orbit at an altitude of 36,000 km with the same angular rotation speed as the Earth.
- polar, with the satellite's orbit crossing the polar axis at an altitude between 700 and 800 km.
- almost circular. This is the case of the 24 satellites of the GPS system, which orbit at an altitude of 20,000 km with a speed of 14,000 km/h.

For determining these satellite orbits, mathematicians use tools developed since Kepler, but also recent results taking into account both special and general relativity.

Source:

Jean Brette (Paris) and Centre•Sciences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7. Chaotic Meteorology

7-1 Coriolis force

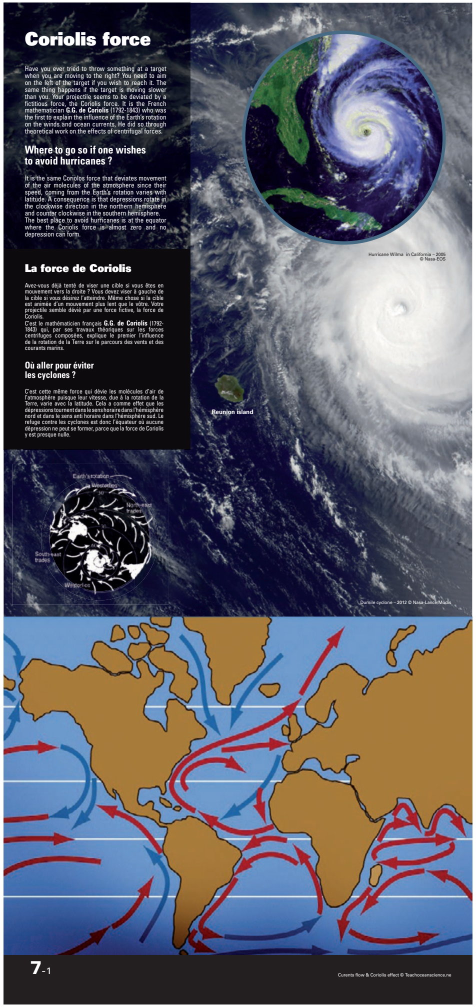
Have you ever tried to throw something at a target when you are moving to the right?

You need to aim on the left of the target if you wish to reach it. The same thing happens if the target is moving slower than you. Your projectile seems to be deviated by a fictitious force, the Coriolis force. It is the French mathematician G.G. de Coriolis (1792-1843) who was the first to explain the influence of the Earth's rotation on the winds and ocean currents. He did so through theoretical work on the effects of centrifugal forces.

Where to go if one wishes to avoid hurricanes?

It is the same Coriolis force that deviates movement of the air molecules of the atmosphere since their speed, coming from the Earth's rotation varies with latitude. A consequence is that depressions rotate in the counter clockwise direction in the northern hemisphere and clockwise in the southern hemisphere.

The best place to avoid hurricanes is at the equator where the Coriolis force is almost zero and no depression can form.



7-2 Turbulent weather!

Mathematical modelling plays a central role in all aspects of modern meteorological science. It is essential for describing and understanding the mechanisms of weather and climate and to make meteorological predictions.

Atmospheric motions are particularly turbulent, and turbulent flow is governed by the Navier-Stokes equations. However, we still do not know how to solve these equations.

Meteorologists must rely on numerical simulation, making use of the most powerful computers and the most sophisticated numerical schemes.

Controlling the trajectory at each instant!
Weather simulation uses the initial conditions provided by meteorological stations.

The meteorologist Edward Lorenz has shown that meteorological systems are chaotic: a very small imprecision in the initial conditions can lead to a very large error in the meteorological prediction. One must not then be surprised that predictions can be wrong in turbulent regime, even just a few hours ahead of time.

References:

- imaginary.org/hands-on/where-are-you
- nylander.wordpress.com/2007/10/27/magnetic-pendulum-strange-attractor/
- http://imaginary.org/hands-on/coriolis-force

Activities:

7-1 Tornados & hurricanes
Make the sphere rotate slowly and then stop it. Observe the movements of the fluid at the surface...

What should you find?
Coriolis force diverts the masses of air and water at the surface of the Earth. It induces the rotation movement of the wind inside the depressions: in one direction in the northern hemisphere and the other direction in the southern hemisphere, as illustrated. It also acts on the displacement direction of the trade winds, which are the winds of the intertropical regions. When two air masses come in contact, a warm one at sea or earth level that rises, and a cold one in altitude and going down, then an ascending stream is formed and the Coriolis force puts it in rotation. This induces eddies from a few meters (waterspouts) to several hundred kilometers (cyclones). This explains the violence of the cyclones with winds from 100 to 200 km/h and storm tides up to 5 m.

Source:

Centre•Sciences & Tryame

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Activities:

7-2 A chaotic pendulum

Swing the pendulum and try to guess at which magnet it will stop. Repeat the experiment so that it stops on the same magnet.

What should you find?

If the pendulum starts from a blue point, it will stop over the blue magnet. But near the boundary lines, any small discrepancy will completely modify the trajectory of the pendulum. The system is said to be chaotic. The famous butterfly effect: the flight of a butterfly in Brazil could cause a tornado in Texas several weeks later, is a metaphor introduced by Edward Lorenz in 1963 to illustrate the fact that meteorological systems are chaotic. When we make simulations of such systems, the discrepancy between the model and the real system grows exponentially with time, and no weather predictions are possible after 14 days. In perturbed meteorological regimes, meteorologists can produce wrong predictions, even a few hours ahead of time.

Source:

Centre•Sciences

8. Solitons and Tsunamis

Solitons are solitary waves observed for the first time by the Scottish mathematician and engineer J.S. Russel in 1834.

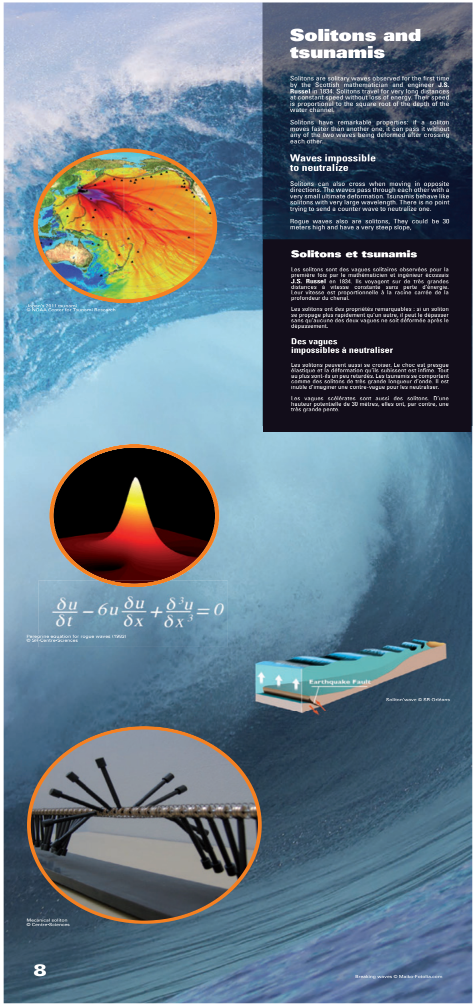
Solitons travel for very long distances at constant speed without loss of energy. Their speed is proportional to the square root of the depth of the water channel.

Solitons have remarkable properties: if a soliton moves faster than another one, it can pass it without any of the two waves being deformed after crossing each other.

Waves impossible to neutralize

Solitons can also cross when moving in opposite directions. The waves pass through each other with a very small ultimate deformation. Tsunamis behave like solitons with very large wavelength. There is no point trying to send a counter wave to neutralize one.

Rogue waves also are solitons. They could be 30 meters high and have a very steep slope.



References:

- imaginary.org/hands-on/solitons-and-tsunamis

Activities:

Make a tsunami

With the slab, send a short strong wave and observe its movement along the channel. Send a second wave just after and observe their shape when they cross or one passes the other.

8-1 What should you find?

Tsunamis are waves similar to solitons, but with 5 to 7 crests, 15 minutes apart from each other. They have an enormous mass and energy, and can travel a very long distance while keeping their energy.

Mathematical modelling allows us to improve prevention against tsunamis: by describing how tsunamis propagate in the oceans, and computing their speed depending on the mean depth of the ocean, it allows us to foresee when a tsunami will reach the land, simulating its impact on coastal regions, and designing emergency plans in case of earthquake.

The tsunami caused by the Sumatra earthquake in 2004 travelled at about 360 km/h towards the east, and about 720 km/h towards the west, since the ocean is approximately 4 times deeper on the western side.

8-2 Waves that keep the shape

Send a strong and short wave by making a quick turn of to the handweel. Watch it spread. Send a second wave so that it crosses the first one or overtakes it.

What should you find?

This experience highlights a very stable type of soliton.

If you send a single wave, it spreads along the cable and returns. Its form and speed are not changed.

If you wait, you will see the wave gradually slow down and stop without disappearing.

If you send two waves in succession, they cross or overtake each other without being disturbed. It is a characteristic of solitons.

Source:

8-1 Noureddine Mohammedi - (University of Tours) and Centre•Sciences

8-2 Gilbert Reinisch (Nice Observatory), Nice University & Centre•Sciences

Digital modules

1. Dune Ash
2. TsunaMath
3. The Future of Glaciers
4. The Sphere of the Earth

1. Dune Ash

On May 21st 2012 around 5:30 am the volcano Grimsvötn on Iceland erupted. The resulting ash cloud lead to the closure of airspaces across Scandinavia and Scotland and caused disruption of air travel throughout Europe.

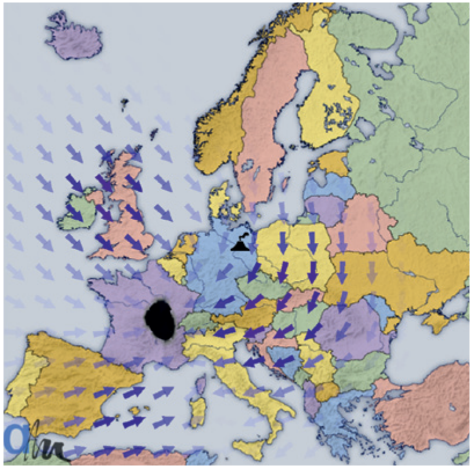
The program Dune Ash carries out realtime simulations of the dispersion of an ash cloud under the influence of given wind conditions,supplied by the user to the program. In this way it is possible to observe how the ash is distributed across Europe from the moment of eruption. The calculations are done in real time. This means it can take a while to complete even on powerful computers.

Partial differential equations

How can the dispersion of ash clouds be simulated mathematically?

The transport process of single ash particles is modeled mathemati- cally with a class of equations called partial differential equations, which are solved numerically by the program Dune Ash.

After the program starts you choose the position of the volcano, and with that the eruption site. Using the arrow button the program guides you through the next steps. Next you sketch the wind conditions (wind field), by drawing lines on the screen. In this way the main wind direction is chosen. The wind velocity is derived from the speed at which you draw the lines across the screen. Based on the lines you drew, the wind field will be calculated and displayed across the map of Europe. The blue arrows signal the wind strength; a light color means lower velocities and a darker color higher velocities.



Volcano in Berlin with ash cloud over France

During the simulation the wind velocity is kept constant, i.e. it does not change with time as expected in reality. Additionally, the dispersion of the ash is regulated by so called diffusion. The influence of diffusion on the simulation is regulated by a slider. With the wind field and diffusion constant as input, the program calculates the solution of the partial differential equation in real time. The spatial and temporal dispersion of the ash will then be drawn onto the screen.

With the slider at the bottom right corner of the screen, it is possible to display the solution at different times onto the screen. With the »start/ pause« button the animation can be halted and with the »stop« button the simulation is finished. Once the solution is calculated, the velocity field can be displayed at different resolutions as well as the grid that was used for solving the partial differential equation.

References:

This program has been developed mainly at the department of applied mathematics at the University of Freiburg and submitted by Tobias Malkmus.

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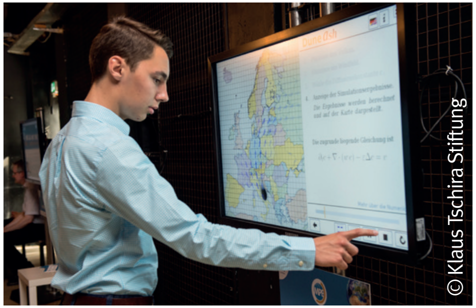
- Dune Ash uses the modular programming package »Dune« for solving partial differential equations. (www.dune-project.org
- imaginary.org/program/dune-ash

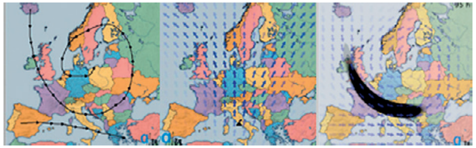


Activities:

Ash cloud from Iceland: Try to find a map displaying the ash cloud observed in 2011 and try to reconstruct it using the Dune Ash program. Do you observe differences between the model and reality?

Experiment with different wind fields and diffusion parameters. Under which condition does the ash cloud disappear after a given time and when does it accumulate in a given region on the map?

Volcano eruption: from the wind field to the ash cloud



2. TsunaMath

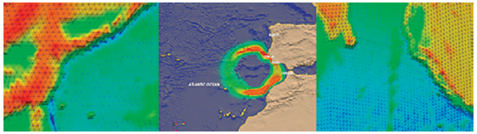
Tsunamis are big oceanic waves that collide violently with the coast. Most often, tsunamis are created by earthquakes that produce a sudden change on the topography of the ocean seabed. This exhibit explains how tsunamis are modeled mathematically, and recreates simulations of historical catastrophes.

Differential equations

How can we model tsunamis?

The relationship between the height of the tsunami wave and the magnitude of the seism that causes it is complex. The speed, acceleration, displacement and size of the tectonic plates affect the height of the wave and its initial velocity. Once the wave is traveling across the ocean, the depth of the sea influences also the height and velocity of the water, and once the wave hits the coast, the slope and the topography of the seabed and coastline modify significantly the wave.

Mathematically, the variables to be calculated are the height of the wave (water depth), and the velocity of the water at each point of the map. The data to be known beforehand is the height of the seabed at each point (bathymetry) and the local modification of this seabed due to the seism.



Simulation of tsunamis in Japan, Portugal and Indonesia

The so called Saint-Venant equations, or shallow water equations, are a set of partial differential equations that model the relations between these variables and data. These equations cannot be solved explicitly, but numerical algorithms can give accurate approximations that allow to obtain good simulations of the wave of a tsunami.

Using these methods, the exhibit shows simulations of great historical tsunamis, such as the Crete tsunami of 365 AD, the Lisbon tsunami of 1755, the Sumatra tsunami of 2004 or the Japan tsunami of 2011. These reconstructions are based on historical documents and descriptions or, in the more recent cases, can be checked against experimental observations to evaluate the validity of the model. Although earthquakes and tsunamis are difficult to predict, these simulations, together with measurements of active seismic zones, can help to reduce the potential impact of tsunamis on populated areas. Building realistic scenarios of affected areas can be used to plan infrastructures accordingly.

References:

imaginary.org/program/tsunamath

Credits: Raouf Hamouda, Emmanuel Audusse, Jacques Sainte-Marie (Developers). Project Numerical Analysis for Geophysics and Environment (Organisation). Institutional support by CEREMA - Compiegne Research Center, UPMC - University Pierre et Marie Curie (Paris), INRIA - Rocquencourt Research Center, France.

3. The Future of Glaciers

Alpine glaciers have been shrinking for more than one century. This trend is expected to continue if the global warming progresses. Thanks to mathematical models, the future evolution of glaciers can be estimated.

In this educational program, the background of glacier modelling is first explained in a short entertaining film. Then, the user is invited to choose among climatic scenarios for the 21st century and watch the corresponding evolution of the largest glacier of the European Alps. Finally, a second short film “glacial mystery” explains how glacier modelling allowed us to make a major advance in a police investigation started in 1926!

References:

This program has been developed and submitted by Guillaume Jouvet from Freie Universität Berlin.

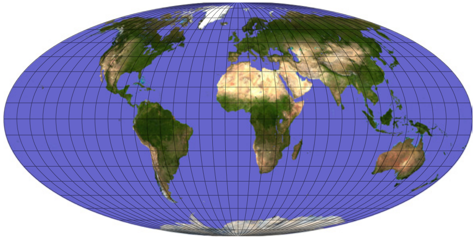
imaginary.org/program/future-of-glaciers-the-module

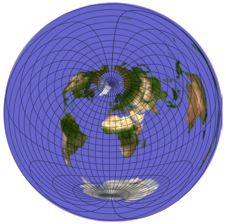
4. The Sphere of the Earth

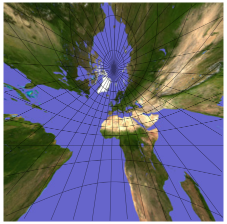
This program explores the science of cartography and geometry of the earth. Thegeometric properties of the sphere and the plane are essentially different, as the Mathematician

Carl Friedrich Gauß already proved: there is no map that faithfully represents the surface of the Earth without distortion. With this program it is possible to study these distortions by comparing six different map projections with different spatial properties. »The Sphere of the Earth« is part of a larger exhibit including posters, a globe, a flexible ruler, and other tools and activities.

In the upper part of the screen you can select the different map projections. When clicking on the map, a so-called Tissot's ellipse is shown. A Tissot's ellipse represents a real circle on the globe that is distorted due to the projection. A red ellipse indicates a distortion, but occasionally the ellipse's boundary or interior turns green, meaning that the shape or the area of the ellipse (respectively) are not distorted at this point.







Top: »Mollweide« projection.
Bottom: »Azimuthal Equidistant« and »Gnomonic« projection

Tissot's indicatrix

Can we measure distortion?

A map of the world is very different from a globe. Some continents appear larger, others smaller; even their shape does not always match. Why is that? From a mathematical point of view these distortions originate in the different geometry of the map and the globe. The sphere of the earth is curved while a map is flat. This curvature of the globe means that the laws of geometry that we know from planar geometry do no longer apply here! The angular sum of the triangle is greater than 180°, the area of a circle is smaller than $\pi \cdot r^2$ and so on. Thus it is not possible to project a triangle one-to-one from a globe to a plane.

Whatever you try, you either have to distort the angles of the triangle or the distance between the corners (or both). Likewise you have to distort either the radius or the area of a circle (or both). As a consequence there is no such thing as a perfect map: the surface of the earth can never be projected on a map without distortions. When creating a map you always have to decide on the type of projection, i.e. on how you display the proportions of the globe on the map. Many different types of projections were developed over the years, each with their own way of distorting angle and distance. To measure this distortion, a circle with arbitrary small radius around a set point is compared to its distorted counterpart in the projection. This image is called the Tissot's Indicatrix or ellipse of distortion. The more the ellipse differs from the original circle, the more distorted the map is in that point.

References:

The Sphere of the Earth Programme was designed and developed by Daniel Ramos of MMACA (Museu de Matematiques de Catalunya).

- You can find additional information on the different projections and other activities on the IMAGINARY web page. There are also posters of the different maps.
- This exhibit as well as »Dune Ash« and »The Future of Glaciers« are the three winners of the »Mathematics of Planet Earth 2013« competition. You can find more information about this initiative at www.mathofplanetearth.org.

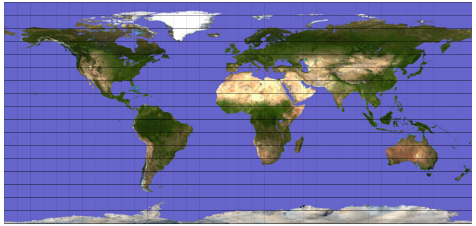
- Book recommendation: »Geometry and the Imagination« by David Hilbert and StephenCohn-Vossen, American Mathematical Society (October 1, 1999).
- imaginary.org/program/the-sphere-of-the-earth

Activities:

Size comparison:
Compare area and circumference of the continents to their image in the projections. What do you notice?

Projections:
Which of the projections shown are conformal/equal of area/equidistant?

More projections:
On the IMAGINARY web page you can also find the book » An Album of Map Projections« with more than 100 additional projections that could all be included in the program in the future.



The »Plate Carée« projection

Videos

1. The Future of Glaciers
2. Probing the Invisible, from the Earthquake to the Model
3. Sundials
4. Bottles and Oceanography

1. The Future of Glaciers

How can one predict the evolution of glaciers?

Alpine glaciers have been shrinking for more than a century. This trend is expected to continue if global warming progresses. This film shows how mathematicians and glaciologists work together to produce realistic estimates of the future evolution of glaciers.

Mathematicians calculate the movement of ice using complex equations, which can only be approximated by computer simulations. Glaciologists use precipitation and temperature data in order to calculate the accumulation and melting of ice.
Using all information available, that of the mathematician and the glaciologist, a method for simulating the evolution of glaciers over time is constructed. At the end of the film you can chose your own climate scenario for the 21st century and the simulate the future of the Aletsch Glacier situated in Switzerland. The possible scenarios are a temperature hike of 2°C or 4°C or a new ice age.

References:

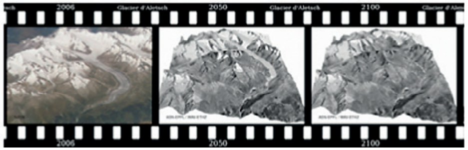
Authors: Guillaume Jouvet, Chantal Landry & Antonia Mey - Department of Mathematics and Computer Science, Freie Universität Berlin.

imaginary.org/film/the-future-of-glaciers

Activities:

Ice melting:

Can you calculate by how much the sea level would rise if all of the ice in Antarctica melted. Information: The ice in Antarctica covers an area of 14 million km2 and is on average 2 km thick. The earth can be assumed to be a sphere of radius 6371 km and is filled up to 70% with water. Water is denser than ice, 1 m3 ice are the equivalent of 0.9 m3 water.



2. Probing the Invisible, from the Earthquake to the Model

One of the main goals of wave propagation numerical modeling is to describe some earthquake phenomena as close as possible. The role of numerical wave propagation simulations is not to predict when and where there will be an earthquake, but to determine the strength and the trajectory of the seismic waves, depending on where the earthquake occur.

It is important to know the less exposed areas, in order to plan the locations of new buildings. Similarly, it is crucial for the authorities to know where to house the victims of an earthquake to protect them from aftershocks.

Numerical simulations are also used to attempt the reproduction of past earthquakes. By comparing the results obtained by the simulation with those recorded by seismographs, one can improve models of wave propagation, but one can also obtain information on the composition of the rocks illuminated by waves produced by an earthquake. Reproduction of these earthquakes can also help to a better understanding on how seismic fault lines break, which is a very important element for the determination of aftershocks. If dense enough receiver arrays are provided, better earth models can be obtained by using more sophisticated data inversion techniques.

The quality of the a priori information injected in the models is then a crucial point to have in mind. With the help of geologists and geophysicists, this difficulty can be constrained due to their data knowledge.

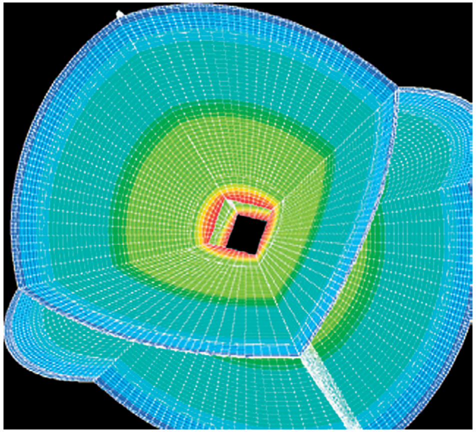
To perform these simulations, it is therefore necessary to gather the expertise of geophysicists, in order to obtain models of seismic wave propagation, of mathematicians, in order to propose numerical methods for solving the equations derived from the models, and High-Performance Computing specialists, in order to implement these methods on parallel computers.

References:

Presented by Inria and Interstices.
Author: Julien Diaz & Roland Martin

Film Director: Arnaud Langlois

imaginary.org/film/probing-the-invisible-from-the-earthquake-to-the-model



3. Sundials

The relationship between mathematics and astronomy goes back in time. An analysis, though superficial, of a few episodes in the history of Mathematics shows how this science is actually fundamental to the progress of the different branches of knowledge.

Greek astronomers, using elementary geometry, were able to determine the dimensions of the radii of the Earth, the Moon and the Sun and the measurement of time also worried the sages of antiquity. Questions like "What causes the variation in the direction of the object shadows during the day?" and "How can we take advantage of the variation in the direction of the shadows to measure time?" were certainly present in the invention of the sundial. With the advent of mechanical clocks, the question "What is the relationship between the hours indicated by a sundial and by a mechanical clock at the same location?" rose naturally.

The aim of this film is to explain how these questions have been answered over time.

References:

Authors: Suzana Nápoles (Univ. de Lisboa), Margarida Oliveira (Agrupamento de Escolas Piscinas Lisboa)

Supported by: Projeto Espiral. E.M.A. Estímulo à melhoria das aprendizagens Fundação Calouste Gulbenkian

imaginary.org/film/sundials

4. Bottles and Oceanography

Beneath the sea surface, there exists a vast network of ocean currents, which, like gigantic conveyor belts, transport huge water volumes. Although the real map of these currents is extremely complex, a couple of simple physical processes explain most of the large scale features.

We will look at the basic process leading to the displacement of huge water volumes. This process relies on the density difference between water masses. Computer experiments can help to grasp the concept, but we will see that a simple experiment can help too, which you can do at home with plastic bottles, straws and fruit syrup...

References:

Authors: Antoine Rousseau, Maëlle Nodet & Sebastian Minjeaud. Film Director: P-O Gaumin.

Presented by Inria and Interstices.

imaginary.org/film/bottles-and-oceanography

Posters

1. The Lorenz Attractor
2. Stereographic Projection
3. Solitons

1. The Lorenz Attractor

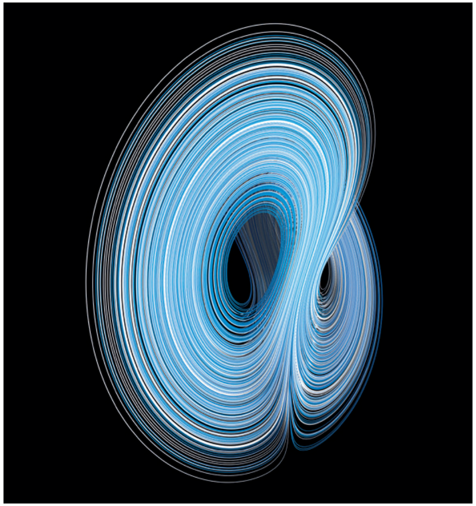
The Lorenz' atmospheric model is what physicists use to call a toy model: although it is so oversimplified that it does not have much to do with reality anymore, Lorenz soon realized that this model was very interesting.

If we consider two almost identical atmospheres (two points that are extremely close in Lorenz' model), we tend to quickly see the separation of the two evolutions in in a significative way: the two atmospheres become completely different. Lorenz saw on his model the sensitive dependence on initial conditions: chaos. Moreover, what is very interesting is that, starting from a large number of virtual atmospheres, even if they follow paths that seem a little bit crazy and unpredictable, they all accumulate on the same object shaped like a butterfly, a strange attractor.

References:

Author: Jos Leys

imaginary.org/gallery/the-lorenz-attractor



2. Stereographic Projection

Points on a sphere are projected onto a plane from the north pole onto a plane that is perpendicular to the axis through the poles, usually the plane through the south pole.

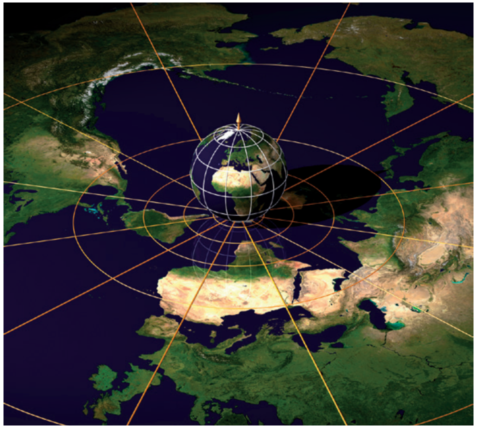
All points of the sphere can thus be projected onto the plane, except for the north pole itself, and a point at infinity is therefore associated with the north pole.

Stereographic projection will project circles on the sphere onto circles in the plane, and preserves angles.

References:

Authors: Aurélien Alvarez, Étienne Ghys, Jos Leys

imaginary.org/gallery/jos-leys-etienne-ghys-and-aurelien-alvarez



3. Solitons

Solitons are solitary waves observed for the first time by the Scottish mathematician and engineer J. S. Russell in 1834. Solitons travel very long distances at a constant speed without loss of energy. Their speed is proportional to the square root of the deepness of the channel.

Solitons have remarkable properties: if a soliton moves faster than another one, it can pass with without any of the two waves being deformed after the passing.

Waves impossible to neutralize

Solitons can also cross when moving in opposite directions. The waves pass through each other with very small ultimate deformation. Tsunamis behave like solitons with very large wavelength. There is no point trying to send a counter wave to neutralize one.

Rogue waves also are to solitons. They could be 30 meters high and have a very steep slope.

References:

Author: Jean Constant

imaginary.org/gallery/jean-constant-solitons

Soliton #1

Soliton #2

Soliton #1

Pseudo-spherical surface: 1 soliton, Breather plus soliton.

Soliton #2

4 soliton figure created as a wireframe in 3DXplorMath. The soliton wireframe was imported and manipulated in several 3D and 2D graphic programs.

The thread of the visualization was to outline the image development process — from the original wireframe depicted on the draft paper pinned on the frame, to the 3D treatment on the background, to the final figure in the foreground positioned in front of a virtual easel - as a model in an art class.

The original title of this composition was “the artist’s studio”.

About the MPE CDT

Mathematics of Planet Earth

EPSRC
Engineering and Physical Sciences Research Council

Imperial College London

University of Reading

The EPSRC Centre for Doctoral Training in the Mathematics of Planet Earth (MPE CDT) is an innovative fully funded PhD Programme in Mathematics, with strong emphasis on interdisciplinary links with weather, oceans and climate change.

The MPE CDT is jointly run by Imperial College London, Department of Mathematics and the University of Reading, School of Mathematics and Physical Sciences. It brings together:

- Two world class academic centres, Imperial College London and University of Reading, linked together via Grantham Institute for Climate Change and providing a pool of over 70 potential research supervisors.
- External partners including world-leading weather and climate services based in the UK (including the Met Office and ECMWF) and leading national and international centres of research and doctoral training in weather and climate science.
- Industrial partners from the energy, water, marine, and insurance sectors, with a potential interest in mathematics PhD projects with an interdisciplinary profile.

The MPE CDT students will graduate with substantial, interdisciplinary experience in applying cutting-edge mathematics to challenging and urgent problems, and will have developed the teamwork, communication, management and leadership skills needed for a successful career.

For more information, please visit: www.mpecdt.org

About the Mathematics of Planet Earth Exhibition and IMAGINARY

Mathematics of Planet Earth Exhibition dates back to an initiative of several mathematical research institutes in 2013.

From a public competition an exhibition was curated that premiered in UNESCO headquarters in Paris on 5 March 2013 and has been growing since. The first prizes were awarded to the digital modules „The Sphere of the Earth“, „Dune Ash“ and „The Future of Glaciers“. They are also shown in this exhibition.

All exhibits are open source and provided under Creative Common licenses on the platform „IMAGINARY - open mathematics“. The open source organization IMAGINARY constantly advances the exhibition. New modules and ideas are welcome

www.imaginary.org

Credits and references

The Exhibition is an outreach event organized by EPSRC Centre for Doctoral Training (CDT) in the Mathematics of Planet Earth (MPE) in collaboration with International Congress of Mathematicians (ICM), Mathematisches Forschungsinstitut Oberwolfach (MFO), Centre•Sciences (CCSTi Region Centre - Val de Loire). The project is funded by EPSRC Centre for Doctoral Training (CDT) in the Mathematics of Planet Earth (MPE), Imperial College London - Department of Mathematics (Research Engagement Impulse Award PE004DC), and University of Reading - Department of Mathematics and Statistics.

On the following pages you will find the credits for the MPE exhibition and all exhibits, and links to download the programs, 3D data, images or films.

The Scientific Committee:
Mireille Chaleyat-Maurel (University of Paris Descartes)
Michel Darche (Centre•Sciences & Adecum)
Andreas Daniel Matt (MFO & Imaginary)
Christiane Rousseau (Montreal University)

The experiments for the hands on modules were developed by:
Jean Brette – Paris
Jean-Pierre Brun - Rennes University
Michel Darche - Centre•Sciences
Etienne Guyon - ESPCi – Paris
Bernard Melguen - Nantes University
Noureddine Mohammedi - Tours University
Pierre Pansu - Paris-Sud University
Christiane Rousseau - Montreal University

and, the 'hands-on' modules were provided by:
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Centre•Sciences - CCSTi Region Centre - Val de France
S[cube]-CCSTi Île-de-France
Tryame - Le Perreux - Paris

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Gert-Martin Greuel
Andreas Daniel Matt
Antonia Mey
Christian Stussak
Bianca Violet
Robert Wöstenfeld

Local organizer:
Anna Radomska Botelho Moniz
(Centre Manager for the MPE CDT)

The exhibition was first presented on 2013 at UNESCO - Paris

Centre•Sciences and Imaginary are in charge of the implementation and coordination of the different actions.

Booklet, texts, layout and translations:
Christiane Rousseau - Montreal University
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Exhibits:

Dune Ash (MPE)
Dietmar Kröner, Martin Nolte, Theres Strauch, Tobias Malkmus (Organisation), M. Nolte, R. Klofkorn, D. Nies, J. Gerstenberger, T. Malkmus, A. Pfeiffer (Programming). Images: San Jose (Wikimedia Commons), H. Thorburn (Wikimedia Commons), J. Gerstenberger. Developed at the University of Freiburg, Department of Applied Mathematics. The exhibits marked with MPE are part of the Mathematics of Planet Earth initiative and exhibition (www. mathsofplanetearth.org). Licenses:Software: GNU-GPL-2 (www.spdx.org/licenses/GPL-2.0), images and data: CC-BY-NC-SA-3.0 (www.spdx.org/licenses/CC-BY-NC-SA-3.0)
Links:
www.dune-project.org
dune.mathematik.uni-freiburg.de/dune-ash
www.imaginary.org/program/dune-ash

TsunaMath (MPE)
Raouf Hamouda, Emmanuel Audusse, Jacques Sainte-Marie (Developers). Project Numerical Analysis for Geophysics and Environment (Organisation). Institutional support by CEREMA - Compiegne Research Center, UPMC - University Pierre et Marie Curie (Paris), INRIA - Rocquencourt Research Center, France.
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Links:
team.inria.fr/ange/software/
www.imaginary.org/program/tsunamath

The Future of Glaciers (MPE)
Guillaume Jouvet (Film, Author), Chantal Landry, Antonia Mey (Support). Glacier simulation: Guillaume Jouvet, Marco Picasso, Jacques Rappaz, Mathias Huss, Heinz Blatter, Martin Funk. Images: CWM GmbH and VAW ETH. Created at the Department of Mathematics, Freie Universitat Berlin and supported by Deutsche Forschungsgemeinschaft (Projekt KL 1806 5-1).

Links:
page.mi.fu-berlin.de/jouvet/
www.imaginary.org/film/the-future-ofglaciers

The Sphere of the Earth (MPE)
Designed and developed by Daniel Ramos of MMACA (Museu de Matematiques de Catalunya).

Links:
www.mmaca.cat
www.imaginary.org/program/the-sphereof-the-earth

Image gallery (printed)
Jos Leys, Aurélien Alvarez, Étienne Ghys, Jean Constant.
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Film gallery
Guillaume Jouvet, Chantal Landry, Antonia Mey, Julien Diaz, Roland Martin, Suzana Nápoles, Margarida Oliveira, Antoine Rousseau, Maëlle Nodet, Sebastian Minjeaud.
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