GRAVITY AND THE HUMAN MIGRATION

MATHEMATICS OF PLANET EARTH

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1 Introduction

Newton's law of universal gravitation, which states that two objects are attracted to each other by a force that is proportional to their masses and inversely proportional to the square of the distance between them, has been accepted as a physical law for over three centuries, but its extension and application to human life is relatively modern. Gravity models to explain human migration, mobility patterns and international trade have been successfully applied as a basic model in the social sciences.

Our project Mathematical Models of Migration in Africa has the following objectives:

- Show that the units of observation of migration are cities, since the vast majority of migration happens between different urban areas
- Use different cities in Africa to show that African cities have grown drastically over the past decades
- Display an image of urbanised Africa, by showing that the three largest metropolitan areas of the continent (Cairo, Lagos and Kinshasa) are larger than London in terms of population, and that there are nearly 40 cities in Africa larger than Lyon or Zürich, each with more than 2 million inhabitants
- Interactively demonstrate one of the most relevant models for human migration (the 'gravity model') by using physical gravity,

where mass (city size) and physical distance help predict the flow of migrants

- Show that city size is one of the most relevant predictors for the inflow of migration
- Show that the vast majority of migration is first internal (migrants move within the same country, from one city to another or from the countryside to a metropolitan area) and then regional (migrants tend to move to neighbouring countries or remain in the same continent)
- Through a hands-on exhibit of the gravity model, demonstrate to a young audience the power of mathematical models to explain or predict social behaviours; and highlight the interconnectedness of the subject with other spheres, e.g. Newtonian physics
- Challenge prevailing perceptions and the overriding dialogue of migration from Africa as being solely from that continent to Europe and highlight the importance of seeking out the relevant facts and statistics, rather than blindly trusting mass or social media.

Migration is a complicated topic to analyse, not least due to the lack of clarity regarding the concepts used. While the term *migrant* is often used for referring to international (and sometimes illegal) migration, here the term *migrant* is used for a person who has moved from one location to another with the object of settling permanently in the new location. It does not consider movements within the same metropolitan area.

A second relevant issue in the analysis of migration is the fact that debate often involves a sensationalist approach to news reporting, with no reliable data or a reporting of absolute figures that fails to put them into the overall context. Here we use *Trends in International Migrant Stock: Migrants by Destination and Origin* published by the United Nations Department of Economic and Social Affairs (DESA) in 2015

(United Nations Department of Economic and Social Affairs (DESA), 2015) which provides the most reliable information in terms of international migration.

2 Cities as units of observation

Migration is often considered at a national (or even regional) level, so frequently the number of migrants that have moved from a country or continent to another is reported. However, data on migration shows that in the US, for instance, less than 10% of the people who arrive at a metropolitan area come from another country and the vast majority of migration occurs at a national level, that is, from the countryside to a city (or vice versa), or from one city to another (United States Census Bureau, 2016).

What is more, in the case of Africa, its level of urbanisation is often ignored. However, African cities are growing at a fast rate (Moriconi-Ebrard and Heinrigs, 2015), with the largest cities of the continent (Cairo, Lagos and Kinshasa) being larger than London or Paris. Thus, cities and secondary towns should be the units of observation of migration models, rather than larger areas such as countries (Prieto Curiel et al., 2017).

3 Gravity model of migration

Based on Newton's Law, the gravity model is frequently used to model human migration and trade (Anderson, 2010), where the predicted movement of people from city i to j is expressed as X_{ij}, the population of cities i and j is expressed as P_i and P_j respectively, and the Euclidean distance between them as d_{ij}. The gravity model then predicts that the movement from i to j is given by

$$X_{ij} = \alpha \frac{P_i P_j}{d_{ij}^{\beta}}, \tag{1}$$

where α allows for the control of the total migration flux and β the impact of distance.

Although there are some relevant and important problems with the gravity model—such as the fact that the flux of migration from i to j is the same as the flux from j to i or that it gives an unbounded estimate for the number of migrants—the model has been used to model international migration (Lewer and Van den Berg, 2008); trade between 138 countries (Burger et al., 2009); migration to the United States (Karemera et al., 2000), and more. Although other models for migration exist, such as the radiation model (Simini et al., 2012), Markov models (McGinnis, 1968; Kelley and Weiss, 1969) and others, the gravity model gives a baseline which allows one to compare the observed migration against its expected value: differences between the two often the result of natural or political frictions.

4 Hands-on exhibit

The deformation that physical gravity created on the map of Africa allowed us to develop a hands-on exhibit in which our audience (more than 50 people divided into groups of different sizes and a wide variety of ages) could see the impact of city size. Hence the link between patterns in social behaviour and the physical laws of nature, a concept that perhaps would have been difficult for especially a younger audience to grasp, was easily visualised and became a natural step in the flow of ideas exchanged during the interaction with the audience.

Also, the audience was encouraged to play with the map, stretch it, observe its deformation both from above and below and, importantly, move the particles on the map so as to simulate migration and trade patterns.

A wave of migration moved mostly from regions representing the countryside of Africa to its largest cities, so as a result, Cairo, Lagos, Kinshasa and Johannesburg attracted most of the migration while smaller

cities such as Harare, Dakar, N'Djamena or Mbuji-Mayi attracted a limited amount of migration. Furthermore, particles dropped south of the Sahara desert largely failed to make it across to North Africa (and vice versa), providing some indication of the importance of natural barriers as limiting facotrs on migration. Similarly, large conglomerations of cities (Abidjan, Accra and Lagos) were strong pools of attraction for the simulated migrants. A list of the 23 cities used, along with their populations and the weight used to represent them is provided in table 1.

One noted also that some particles remained in the 'countryside' regions of Africa (i.e. did not move to the cities included): discussion on this with the audience also led to the conclusion that this was realistic, as not everyone moves away from home.

5 Challenging the perspective of migration

Unfortunately, migration is dominated by opinions rather than facts. Standing in front of a captive and engaged audience, talking about African migration, was a good opportunity to guide them towards a better understanding of migration patterns and statistics.

A rough estimate of the dimensions of internal migration (people who move within their own country) and international migration reveals that from one year to the next:

- Roughly 3% of the population will migrate.
- Out of those who migrate, more than 98% of them will do so within their own country (internal migrants) and only 2% will migrate to another country (Prieto Curiel et al., shed).
- Out of those who move from one country to another, the majority remain within the same continent. After all, gravity matters!

Worldwide, data shows (United Nations Department of Economic and

	City	Country	Population (m)	Weight (g)
1	Cairo	Egypt	18.8	500
2	Lagos	Nigeria	17.6	469
3	Kinshasa–Brazzaville	Congo	13.5	359
4	Johannesburg	South Africa	13.1	349
5	Luanda	Angola	7.5	198
6	Khartoum–Omdurman	Sudan	5.7	152
7	Dar es Salaam	Tanzania	5.5	147
8	Nairobi	Kenya	5.4	143
9	Abidjan	Ivory Coast	5.2	139
10	Accra	Ghana	4.7	125
11	Casablanca	Morocco	4.2	113
12	Algiers	Algeria	3.8	100
13	Addis Ababa	Ethiopia	3.5	93
14	Kampala	Uganda	3.2	85
15	Yaoundé	Cameroon	3.0	80
16	Bamako	Mali	3.0	80
17	Ouagadougou	Burkina Faso	2.5	67
18	Harare	Zimbabwe	2.4	63
19	Maputo	Mozambique	2.3	60
20	Mogadishu	Somalia	2.0	53
21	Mbuji–Mayi	Congo	2.0	53
22	Niamey	Niger	2.0	53
23	Ndjamena	Chad	1.3	33

Table 1: List of the 23 cities used in the model, along with their country, population and the weight used to represent them. Population data is taken from `wikipedia.org/wiki/List_of_metropolitan_areas_in_Africa`. Note that these are not the 23 largest cities in Africa (Ndjamena, for example, is 42nd) as some were removed as they were too close to others already on the map.

Social Affairs (DESA), 2015) that from the 250 million international migrants, 63% of them remained in their continent of origin in a different country. From Africa there are roughly 33.2 million international migrants, out of which 16.4 million moved to another country inside Africa (for example, from Burkina Faso to Ghana). Thus, only 16.7 million Africans are currently living outside Africa.

We asked the audience to give us an estimate of the magnitude of migration. Two questions were asked: *"for every 100 Africans, how many do you think are currently living outside Africa?"*; and, to set a comparison point, we asked the same question for Europe, *"for every 100 Europeans, how many do you think are currently living outside Europe?"*

The answers we obtained were surprisingly (or perhaps unsurprisingly) bad. The vast majority of the audience believed that Africans migrated with a much higher frequency than Europeans, and the majority also assumed that the dimensions of migration were much higher than those observed.

Data shows (United Nations Department of Economic and Social Affairs (DESA), 2015) that for every 100 Africans, 1.4 currently reside outside Africa, whilst for every 100 Europeans, 3.2 currently reside outside Europe.

6 Overall discussion

The project Mathematical Models of Migration in Africa was very well received by the audience. Firstly, its construction and implementation (painting the map of Africa, locating the cities and assigning a weight to each) is easily reproducible somewhere else and another setting (for instance, cities in Europe or North America) could be considered.

In terms of science communication, being able to touch, play and interact with the exhibit provides the opportunity to move away from the formalism of written text and create an engaging exhibit for the

audience, in which various topics can be discussed.

In terms of the model, although the gravity model for migration has its caveats, it provides an intuitive way of quantifying social aspects, such as migration.

Finally, we had the opportunity of encouraging a data-driven debate of social issues among the audience, aiming to challenge the current worrying trend that denigrates science and statistics and its proper application in favour of "post-truth" politics and populist policies. We aimed to highlight the important role mathematical modelling has to play in understanding the world around us: both in terms of natural phenomena (gravity) and social behaviours (migration and trade).

Finally, and due to the nature of the event which we attended, in which many of our audience were considering their future options of study, we showed how mathematics permeated through many other subjects, even the most unexpected, hopefully encouraging many of them to take up mathematics in the future.

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