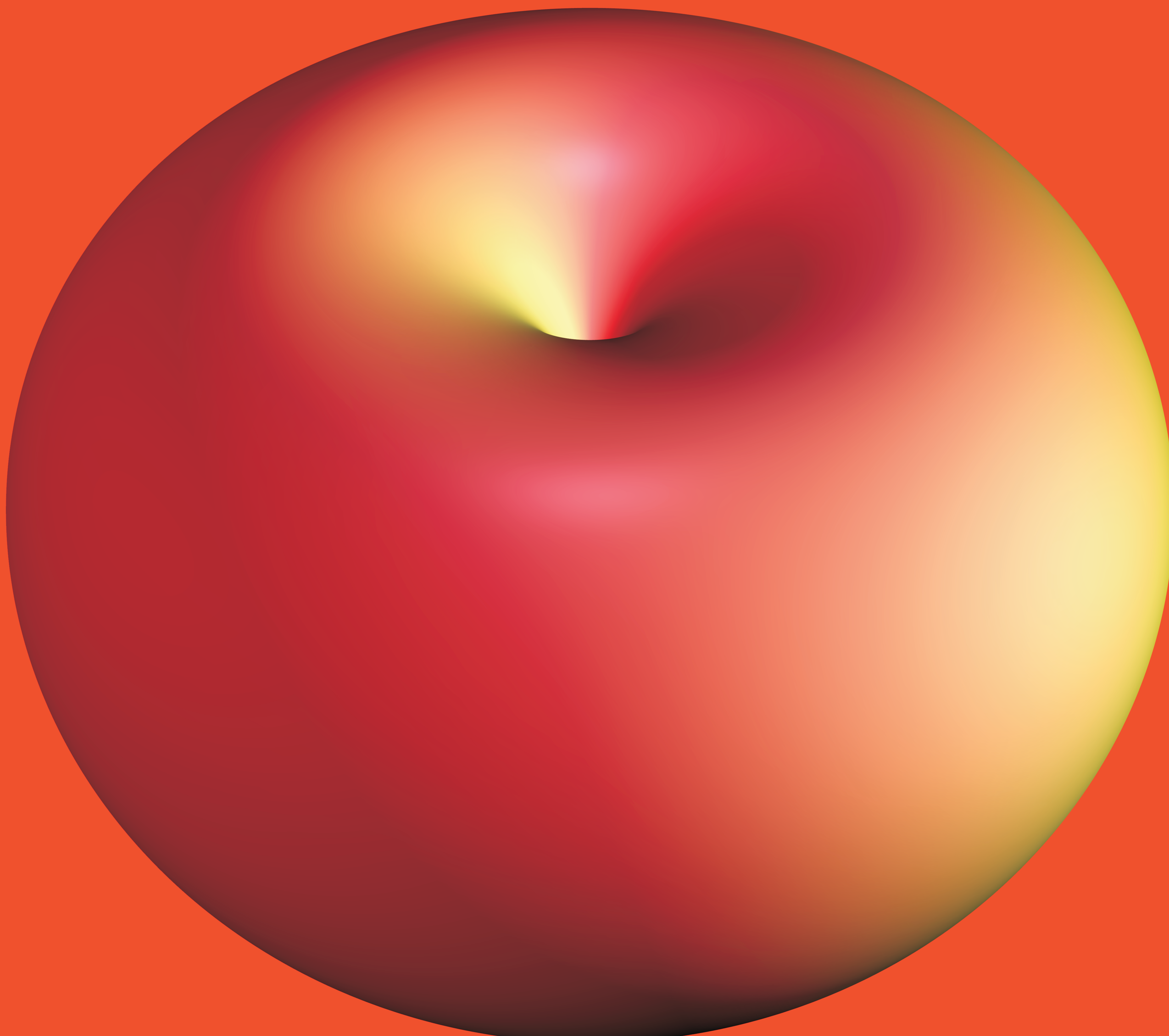


Dullo $(X^2+y^2+Z^2)^2-(X^2+Y^2)=0$

Dullo

If the audience in an oval stadium scream about a score (typically of the favoured team), the sound spreads like a quickly inflated floating tyre. After some split seconds the tyre meets itself at its centre – the opening has closed – and that is what exactly happens at the kick-off spot. At that point sound waves from all directions meet simultaneously and are boosted accordingly. This is why referees are advised to always stay level with the ball. Thus, when a goal is scored they do not stand in the middle circle and get a buzzing in their ears.

决斗

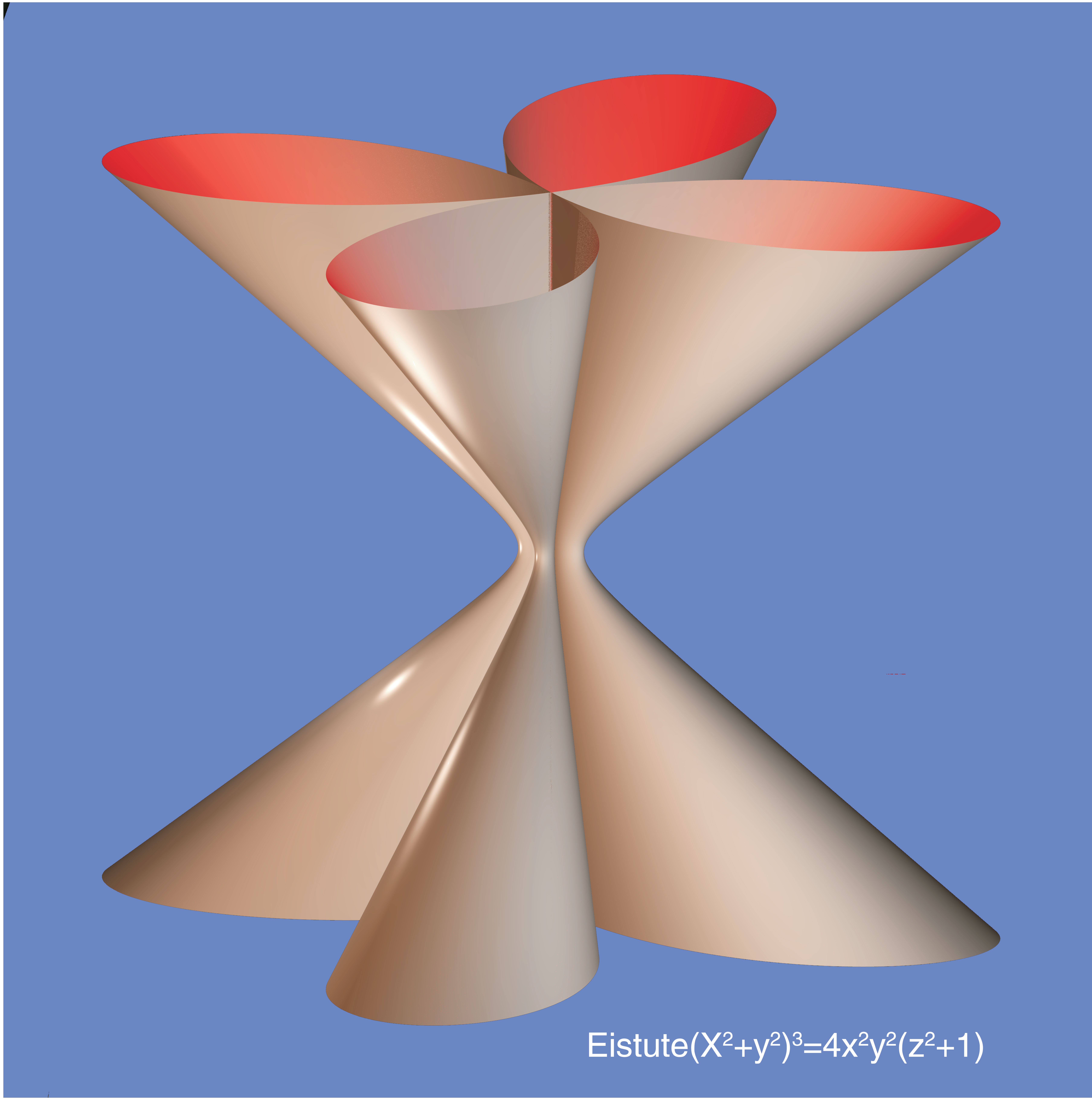
如果在一个椭圆形体育场中的观众为队伍（尤其是为了自己喜欢的队伍）得分而尖叫，声音的传播就像一个快速膨胀的轮胎。几秒钟轮胎就充满了气，也就是一开始就结束了，而这就是刚开始就发生的事情。各个方向来的声波同时相遇并因而相应增强。这就是为什么裁判员被忠告要始终和球保持在同一水平。这样，当球队得分时他们才不会位于圆环的中心并因而耳朵被震得嗡嗡响。

Ding Dong

This surface described by the equation $x^2+y^2+z^3 = z^2$ was one of the very first visualizations we tried. Equation and shape are simple: A vertical alpha-loop rotates around the z-axis. But there was the problem with the colouring. Green is generally rather tricky in three-dimensional visualization of surfaces and, in addition, tends to be matt or yellowish. The lights and reflexions must be well tested. Note the light blue hard shadow intensifying the spatial effect.

叮咚

由方程$x^2+y^2+z^3 = z^2$所描述的这个曲面是我们最早努力可视化的成果之一。方程和形状都很简单：一个垂直的阿尔法环绕着z轴旋转。但是在涂色上存在问题。绿色通常在曲面的三维可视化中是比较厚重的，并且会倾向于缺乏光泽或者发黄。灯光或者反射波必须经过良好的测试。注意轻微的蓝色阴影会增强空间感。



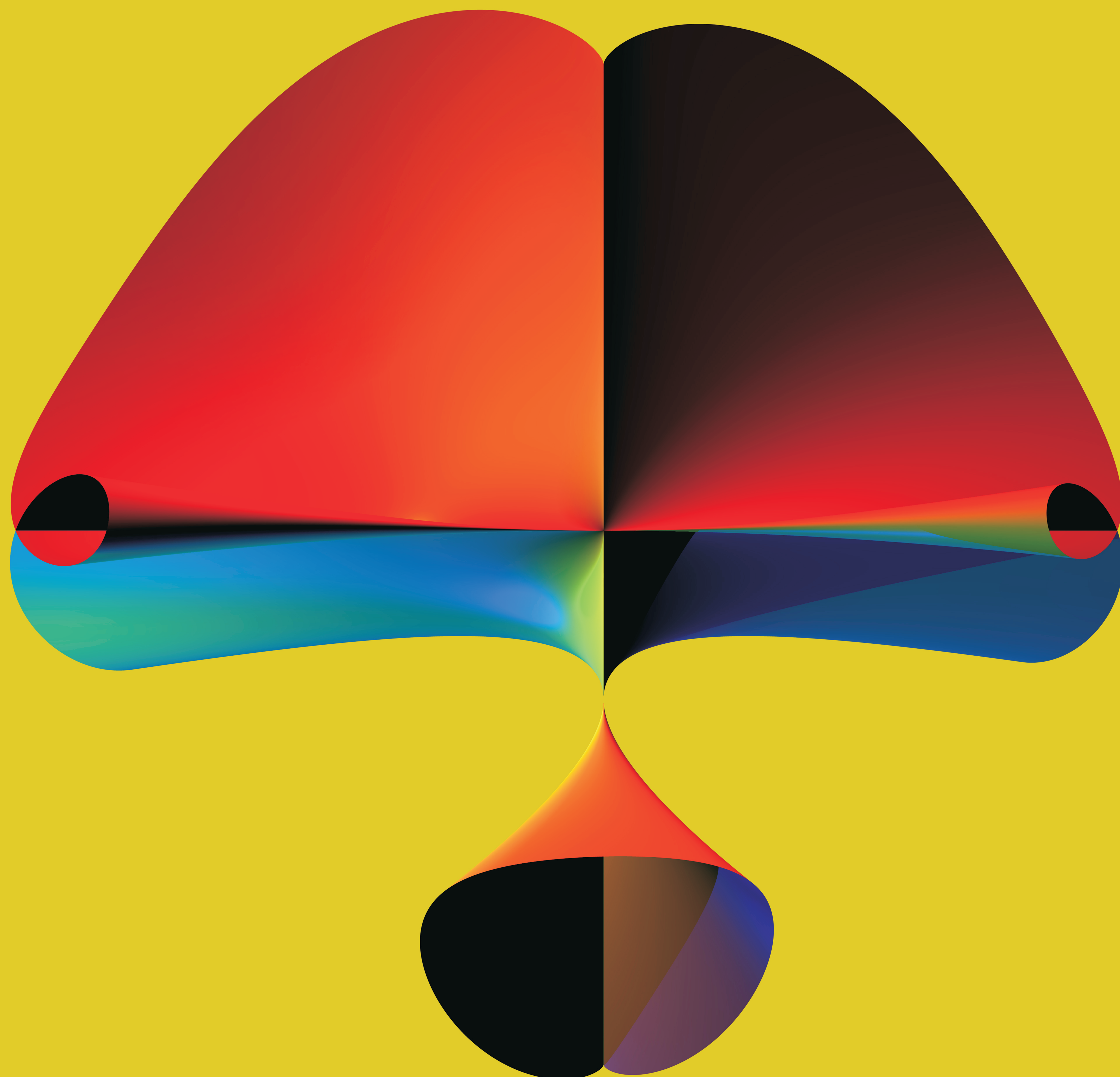
Cone

The horizontal section through Cone is a so-called rosette curve: A small wheel rolls round the interior of an annular body while a pencil fixed to the wheel is drawing a curve. Different curves are formed depending on the ratio of the two radii. The curve closes if the ratio is a rational number. In our case it is the four-leaf clover. Our cone has the advantage that four scoops fit in. You must hurry licking otherwise the ice cream is dripping down.

锥面

锥面的水平截面被称作玫瑰型曲线：玫瑰线是由一根铅笔固定在轮子，让轮子绕环面内部旋转而画出的曲线。根据两个半径的比例不同会做出不同的曲线。如果比例是一个有理数就会做出一个闭合的曲线。在我们的例子里做出了一个四叶的三叶草。

我们的锥形的优点是嵌入了四个勺子。你必须快一点舔掉否则冰淇淋就会滴下来了。



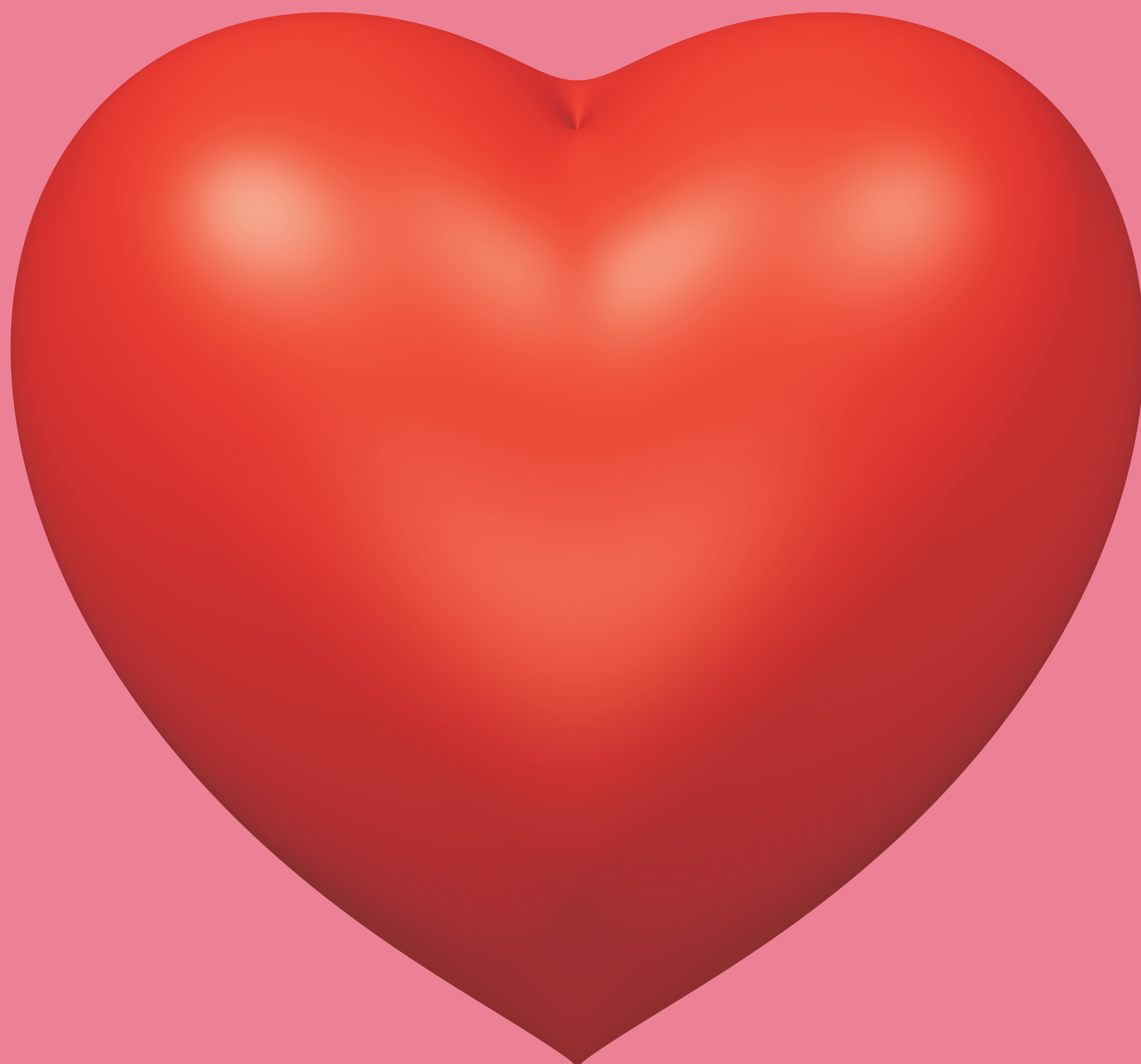
Geisha $x^2yz+x^2z^2=y^3z+y^3$

Geisha

This surface is, in fact, rather complicated, even if it features a regular mirror symmetry. Its name is due to its inward-turned identity: discrete, quiet and graceful. The loop-shaped bow makes one think of a kimono bow. The colours and the image result from ray tracing: A ray is traced from each point in the image and where it hits an object the colour value is calculated by means of light source models. The grace of Geisha is underlined by special lighting.

艺妓曲面

艺妓曲面是整齐的镜面对称的，但事实上它并不简单。它的名字是源于它的内旋特征：离散，安静并且优美。环形的鞠躬让人想起和服鞠躬礼。色彩和图片来自于光线追踪：一根光线从图片上的一个点开始追踪，当它遇到一个物体时，就用它的光源模型计算出色彩值。特别的照明更加强调了这个曲线的优美。



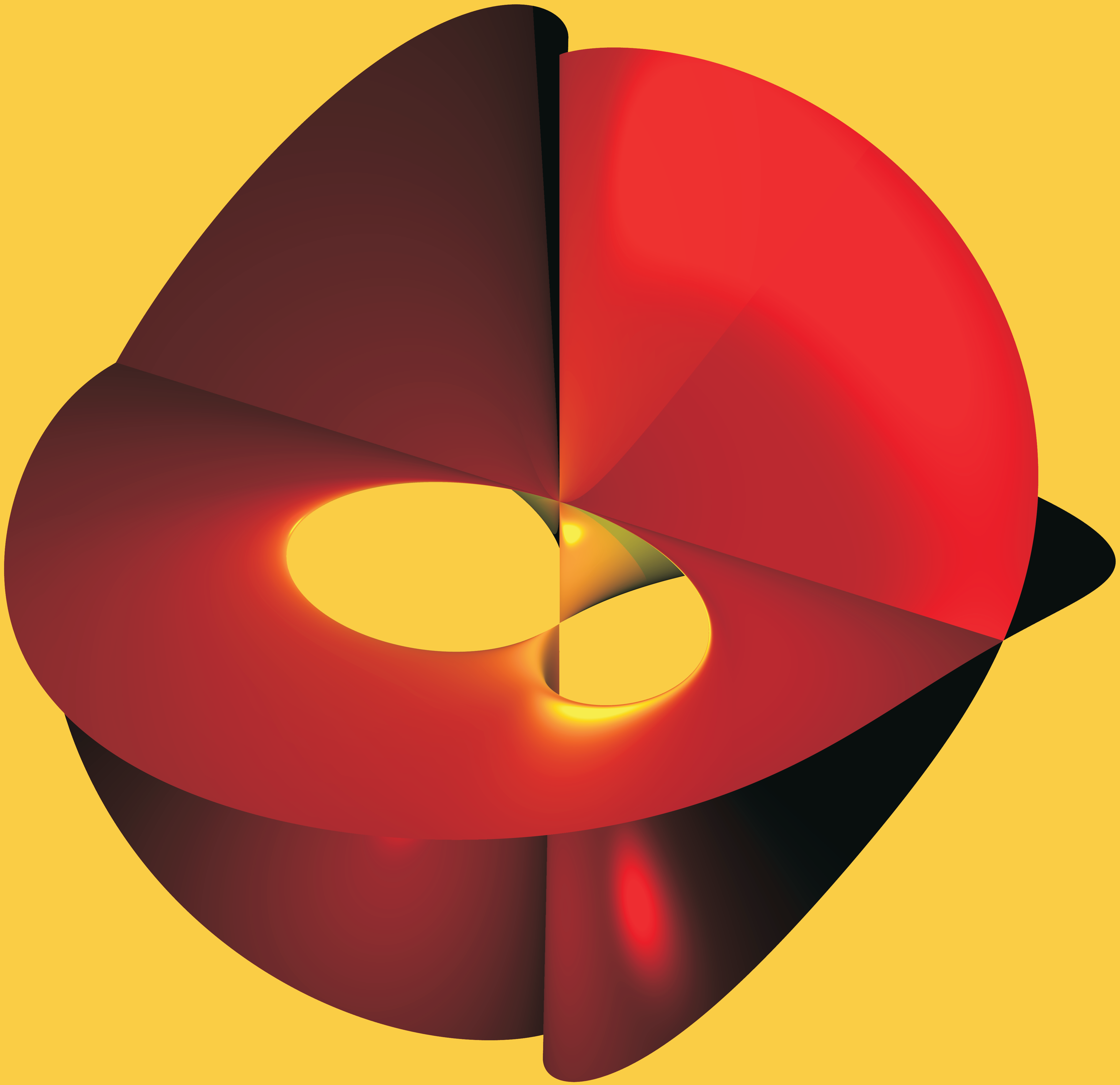
Suss $(x^2+9/4y^2+z^2-1)^3-x^2z^3-9/80y^2z^3=0$

Sweet

Notes from algebraic geometry: Vertices or singularities are very interesting from the mathematical point of view. These points behave extraordinarily. Tiny changes in the equation have strong effects on these points.

甜心曲面

来自代数几何：从数学角度来看顶点和奇点是非常有趣的。这些点行为异常。方程中微小的改变可以对这些点产生巨大的影响。



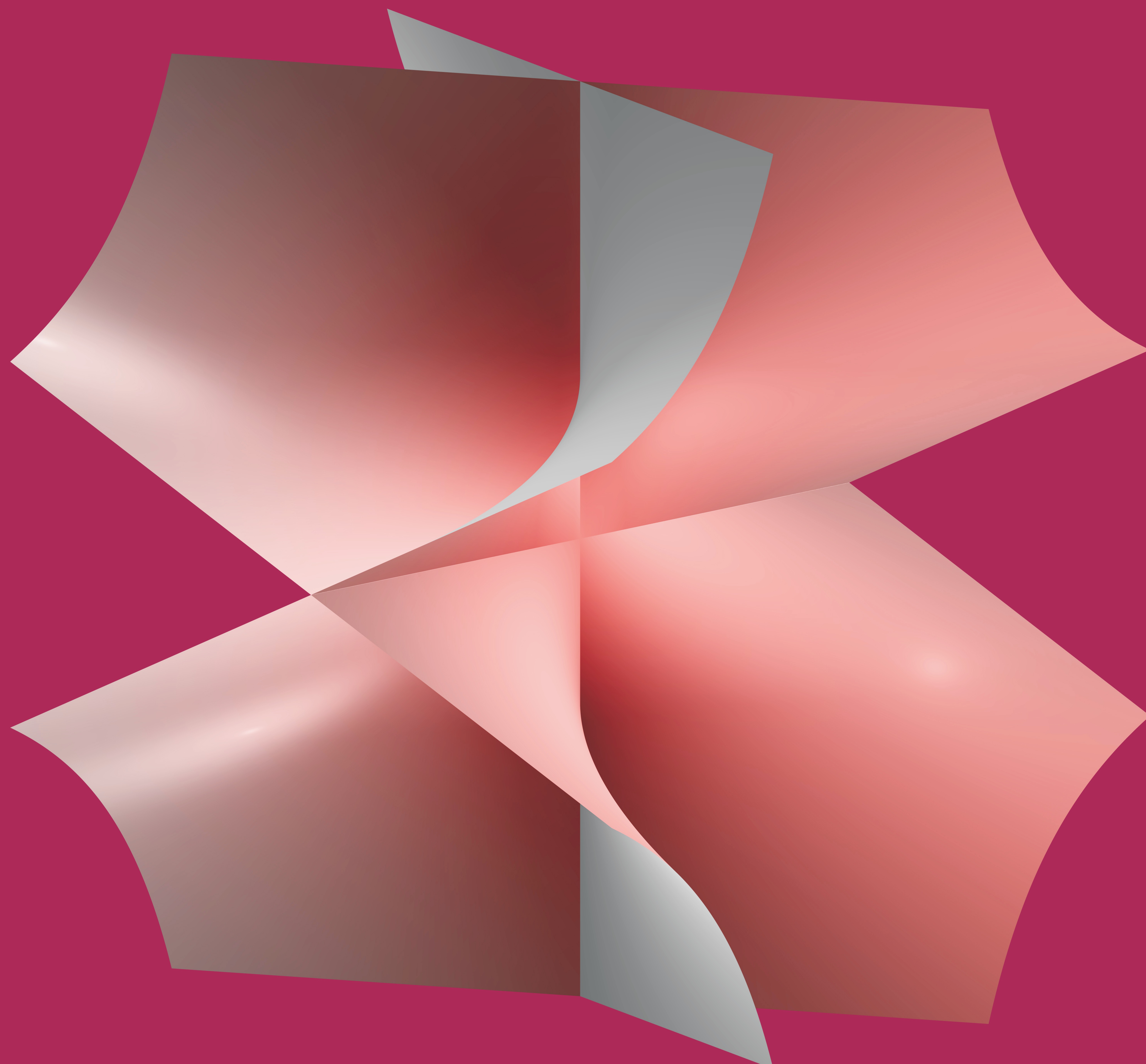
Miau $x^2yz+x^2z^2+2y^3z+3y^3=0$

Miau

this surface was developed by coincidence on a drab train ride (Working with algebraic visualization makes time elapse quite fast). But the constellation isn't a coincidence at all, on the contrary, if one wanted to systematically derive the algebraic equation for this surface one would face unsurmountable problems. The challenge of Miau is, of course, the double opening with embedded singularity. For mathematicians this is a treasure trove exploring the relationship between equation and form.

妙悟曲面

这个曲面是在一次乏味的火车旅行中偶然发现的（做代数可视化工作，时间消耗得很快）. 但是这个系列的发现并不完全是偶然的，相反，如果想系统地导出这个曲面的代数方程将面临无法克服困难。当然妙悟曲面面临更困难的挑战，在于其双开口外的嵌入奇点上。对于数学家来讲，探索方程和形式（方程对应的曲面）之间的关系就像一个寻宝的过程。



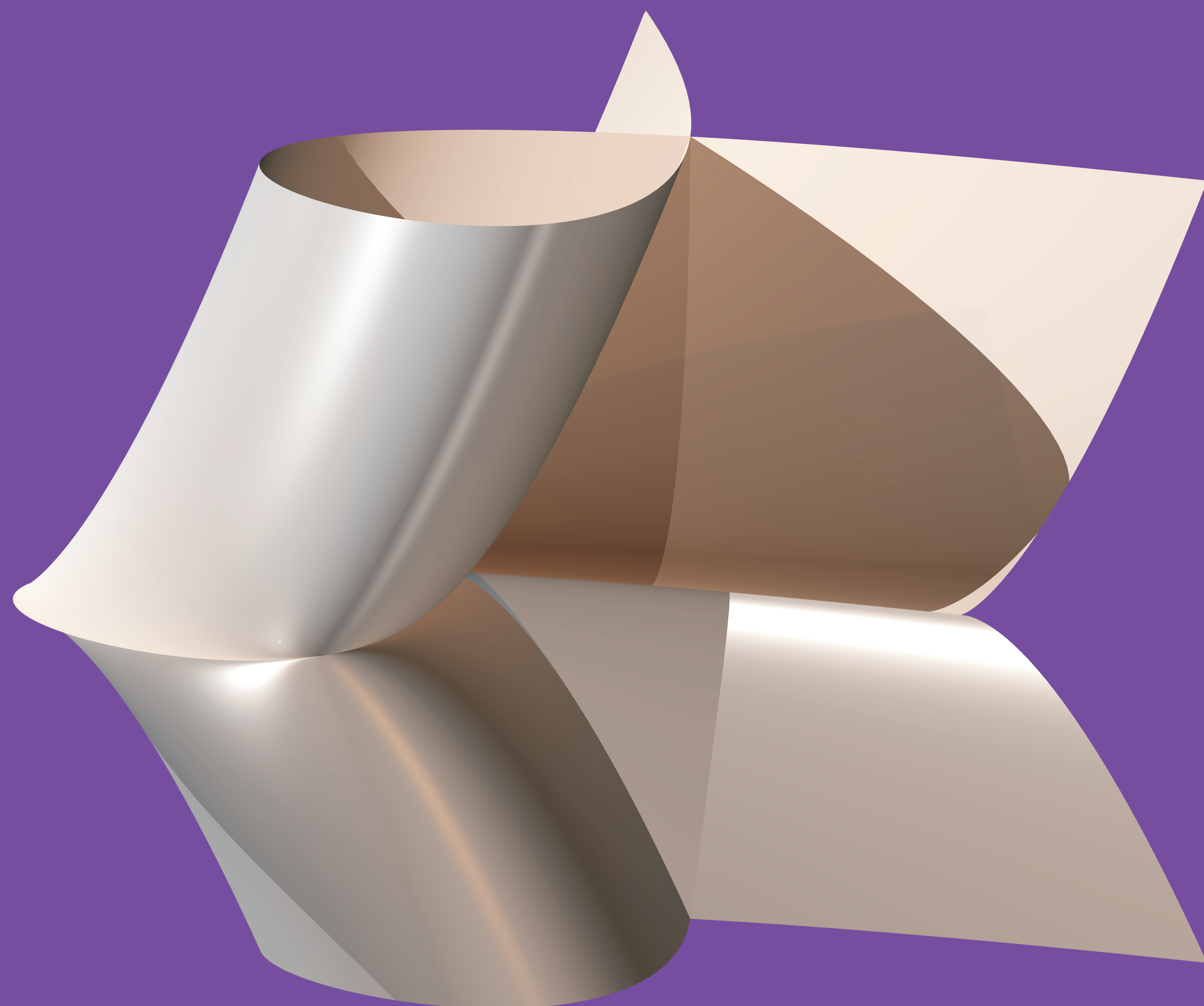
Himmel und Holle $x^2 - y^2 z^2 = 0$

Heaven and hell

A piece of paper is folded and is held from beneath such that you can put your four fingers in the four corners so formed. By spreading your fingers the figure opens in two different ways so that two of the four inner sides can be seen at a time, the blue ones for heaven, the red ones for hell. Children guess which colour will show up. Our figure reminds us of this game, hence the name. By adding up the squares at y and z you get the highest exponent 4. This is called an equation of the 4th degree. The higher the degree the more complicated it is to calculate the surface.

天堂与地狱

曲面的名称来自于一个儿童猜色游戏。把一张纸折叠然后把你的四个手指放在四个角从下面捏紧就会形成这样一个曲面。伸开你的手指，曲面会由两个不同方式展开，可以同时看到四个内面中的两个，蓝色的表示天堂，红色的表示地狱。将y的次数和z的次数相加得到方程的最高阶数为4阶，所以这个方程称为四阶方程。方程的阶数越高推断曲面就会越复杂。



Quaste

$$8z^9-24x^2z^6-24y^2z^6+36z^8+24x^4z^3-168x^2y^2z^3+24y^4z^3-72x^2z^5-72y^2z^5+54z^7-8x^6-24x^4y2-24x^2y^4-8y^6+36x^4z^2-252x^2y^2z^2+36y^4z^2-54x^2z^4-108y^2z^4+27z^6-108x^2y^2z+54y^4z-54y^2z^3+27y^4=0$$

Quaste

The upper border of this surface is a loop in the shape of the Greek letter Alpha, whereas the border to the right consists of two parallel so-called cuspidal curves with a spike each. By moving the horizontal loop downwards, leading its two endpoints on the right along the curve, the surface Quaste will come out. On the other hand, you can also shift one of the curves along the loop and you will get the same figure. Surfaces with this property are named Cartesian products after the French mathematician René Descartes.

凯斯特曲面

曲面的上边界是一个像希腊字母α的环形，而右边界由两个具有尖点的所谓尖点曲线组成。向下移动水平环，同时带领其右侧的两个端点沿曲线移动，就可以形成凯斯特曲面。另一方面，你还可以沿着环移动其中的一个曲线，从而得到一个相同的图形。为了纪念具法国数学家勒内.笛卡尔，这个曲面也被被称为笛卡尔乘积。

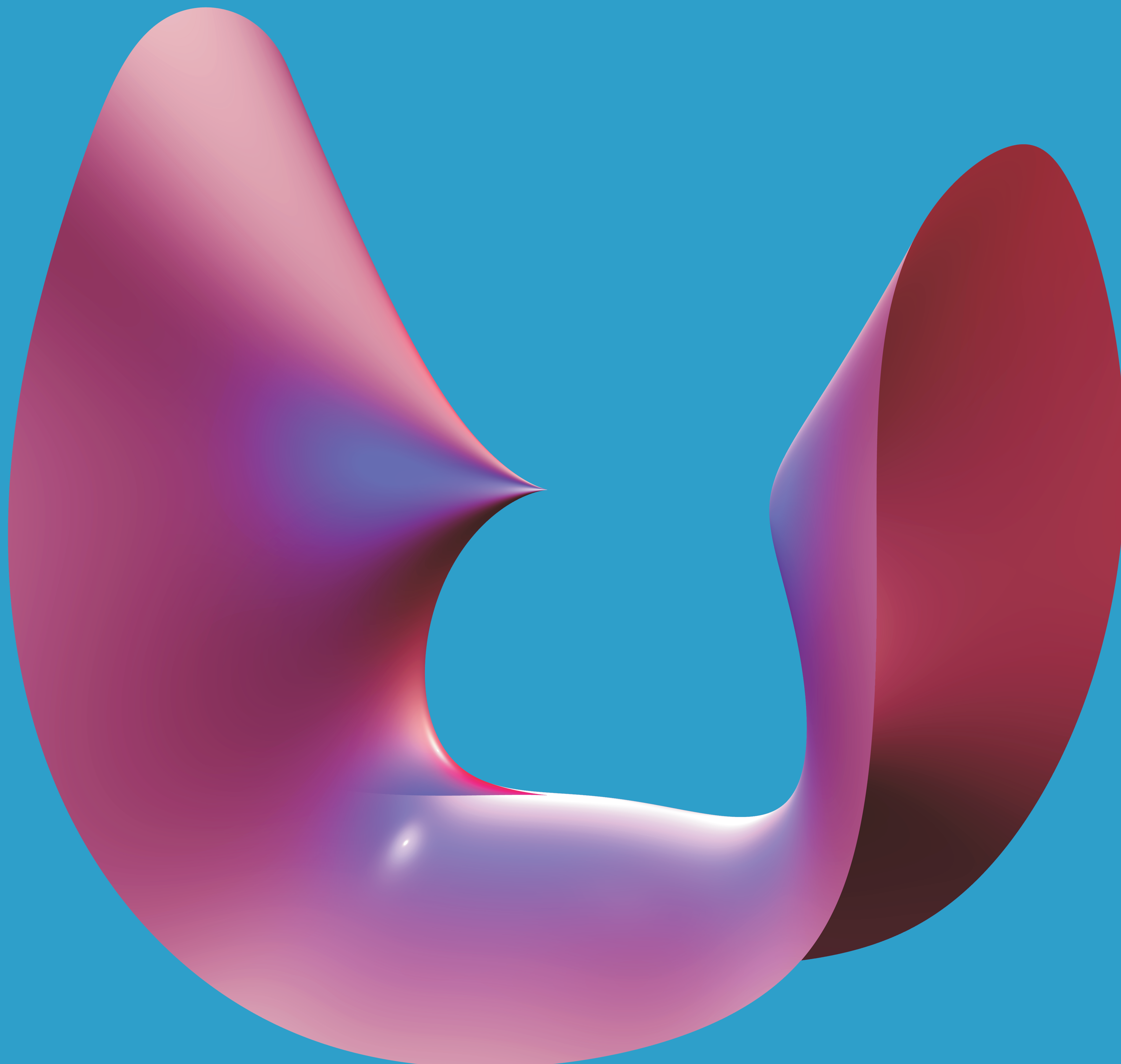
Seepferdchen $(X^2-y^3)^2=(X+y^2)z^3$

Seahorse

If you want to find the equation of this surface it would take strong efforts. The soft tangential contact is not easy to achieve. It vanishes as you only slightly change the formula. The elegance of the sea horse is an illusion: If you look at it from behind or from the side, it appears quite clumsy. Sea horses live worldwide in tropical and temperate climate zones. Its Latin name is Hippocampus, you can find the equation next to it.

海马曲面

找到这个曲面的方程非常困难。软切向接触并不容易得到。你只要稍微改变公式，它就会消失。优雅的海马是一个错觉：从后面或从侧面看上去，它显得很笨拙。海马广泛地生活在世界各地的热带和温带气候区。你可以在方程旁边找到它的拉丁名称Hippocampus.



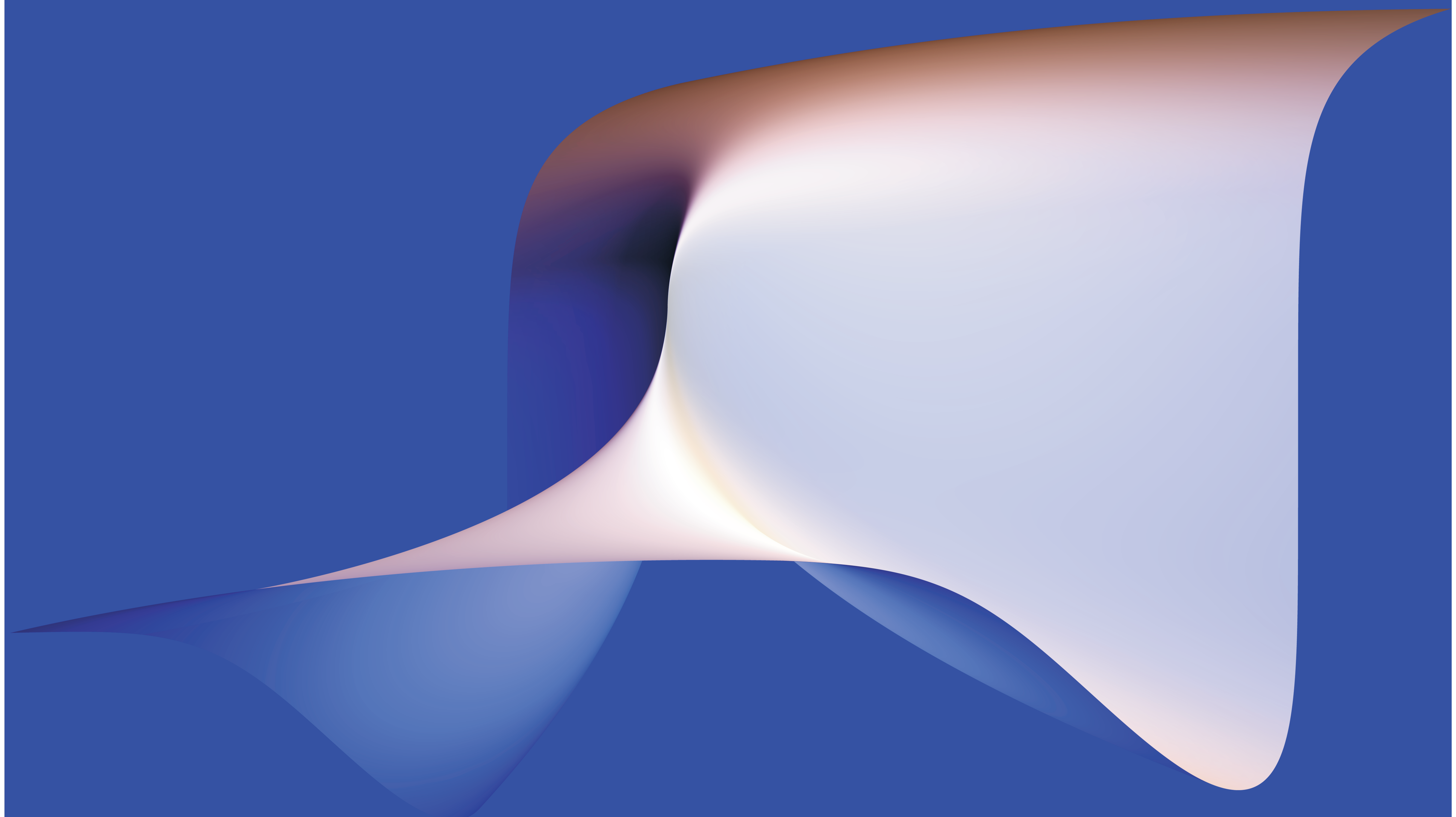
Visavis $X^2-x^3+y^2+y^4+z^3-z^4=0$

Vis à Vis

Vis à Vis means opposite – and, here, two essential phenomena of algebraic geometry stand opposite each other. The singular tip on the left looks at a curved but smooth hill on the right. This singularity is more exciting, because various changes to the equation can result in unpredictable changes to the figure, which does not happen at smooth points. By using the SURFER program, which is available for free at surfer.imaginary2008.de such surfaces can be generated and modified quite easily and intuitively. The comparison of form and formula, i.e. of equation and corresponding surface, becomes an interactive experience which is intriguing to understand.

面对面

Vis à Vis的意思是面对面，体现了代数几何中的两个互相对立的基本现象。从左边看，可以看到奇异点，望向右边可以看到弯曲而光滑的山。这个奇点非常令人兴奋，因为方程的不同变动会导致图形发生不可预测的变化，这在光滑点是不可能发生的。我们可以利用surfer.imaginary2008.de上的免费程序SUFFER，非常容易和直观地生成或改变这个曲面。在这个软件中，比较曲线与公式，或者说方程和对应的曲面，会有一种引人入胜的交互式的体验。



Sofa $x^2+y^3+z^5=0$

Sofa

Though an algebraic surface is called “Sofa” sitting on it is not necessarily comfortable. What we have here, rather, is a seating for two separated from each other by a singularity. This singularity is called E8 in mathematical language and is perhaps the most famous of all singularities. It combines, among others, the theory of the symmetry groups of Platonic solids (E8 is part of the icosahedral symmetry group) and the theory of the Lie groups. The real image of this singularity excels through its elegance, though it does not reveal its mathematical complexity, which only becomes apparent as you include the imaginary part.

沙发曲面

虽然这个代数曲面称为“沙发”，坐在它上未必舒适。相反，这里我们面对的是一个由奇点分割开的两个座位。这个在数学上称为E8奇点也许是所有奇点中最著名的一个了。它还结合了柏拉图立体的对称群理论(E8是二十面体对称群的一部分)和Lie群理论.虽然你仍无法揭示数学的复杂性，但你可以凭借想象力清晰地想象出其真实情况。

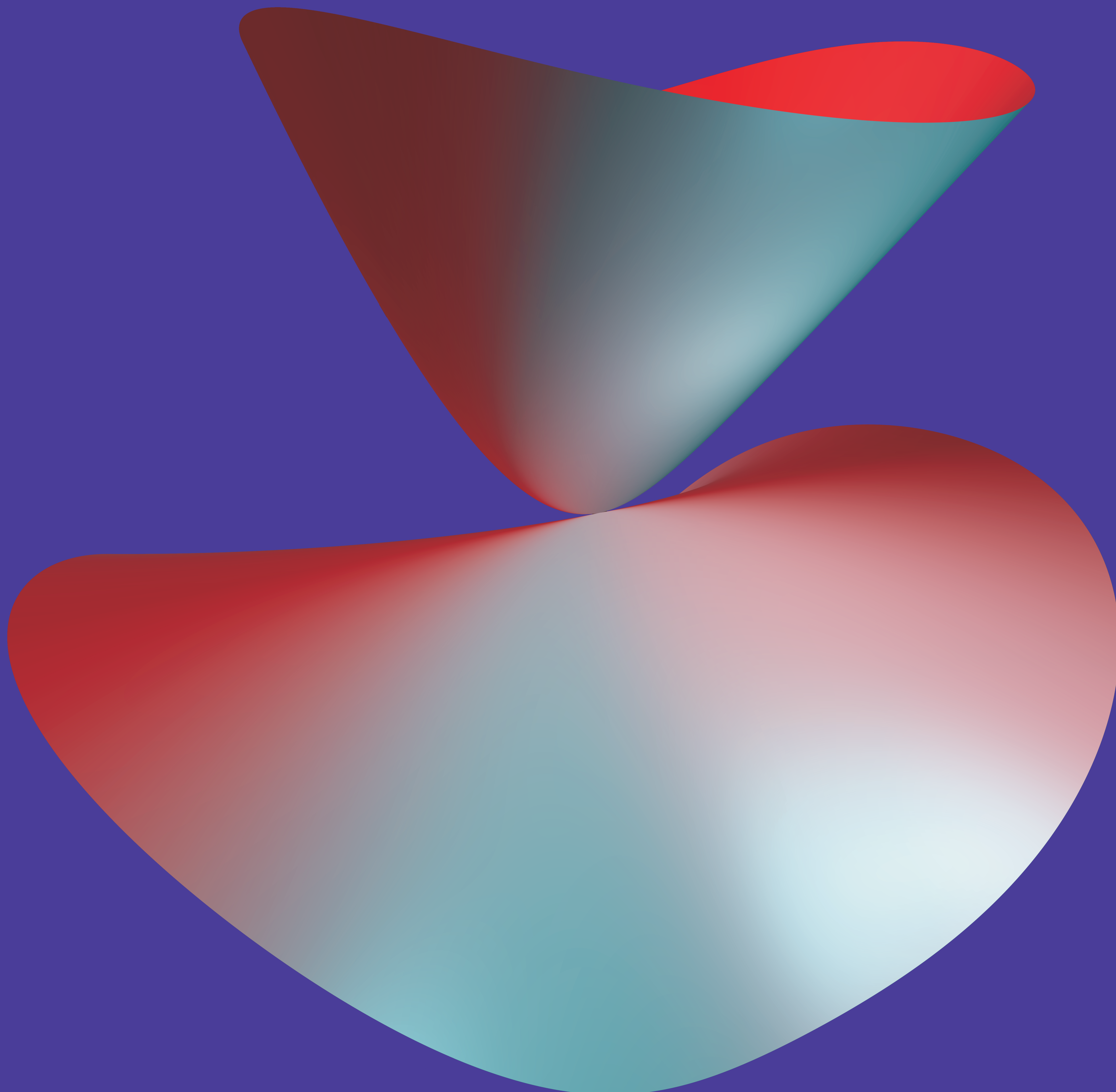
Zitrus $x^2+z^2=y^3(1-7)^3$

Citric

The equation $x^2+z^2 = y^3(1-y)^3$ of Citric appears as simple as the figure itself. Two cusps mirror-symmetrically arranged rotate around the traversing axis. The equation $x^2+z^2 = y^3$ simplified by omitting $(1-y)^3$ provides for exactly one cusp, and $x^2+z^2 = (1-y)^3$ yields the mirror image. Both are infinitely extending surfaces. The product on the right side of the initial equation ensures that Citric remains bounded. You may consider the following: If the absolute value of y is getting larger than 1 the right side becomes negative and the equation does not admit real solutions of x and z.

柠檬曲面

柠檬的方程$x^2+z^2 = y^3(1-y)^3$和它的图像一样简单。两个尖点绕其对称轴旋转成镜面对称，就会得到柠檬曲面。等号右侧省略掉$(1-y)^3$ 后得到$x^2+z^2 = y^3$，这个方程表示曲面只有一个尖点，另一个方程$x^2+z^2= (1-y)^3$产生其镜面对称图像。两个曲面都是无限延伸的曲面。将这两个初始方程的右侧相乘保证柠檬曲面有界。可以考虑以下问题：如果y的绝对值大于1，右侧的符号为负，则方程没有x,z的实数解。

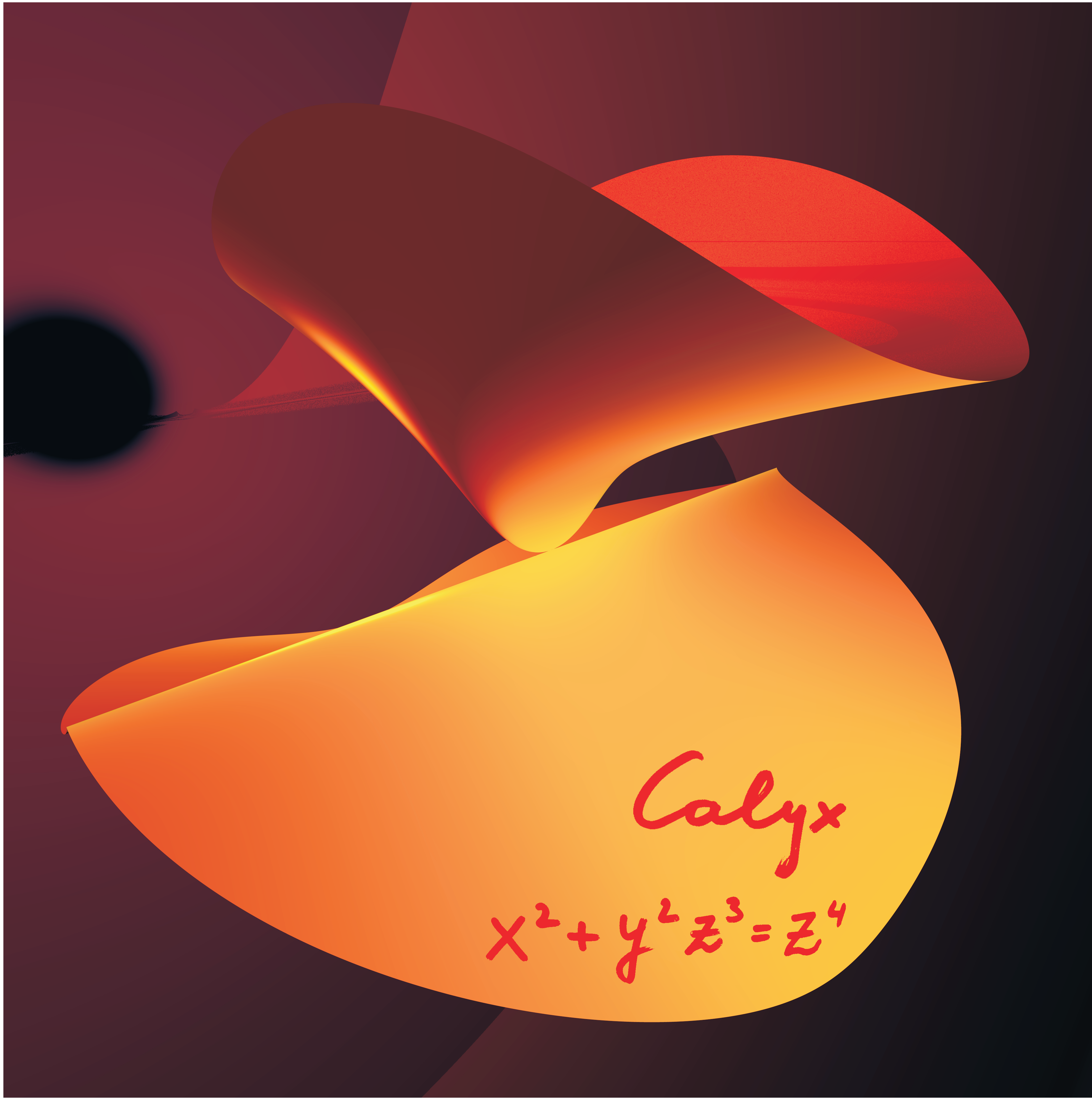


Calypso

The surface Calypso with equation $x^2+y^2z = z^2$ contains three straight lines. The horizontal straight line is clearly visible, it passes through the origin (zero) where the upper and lower part meet. The two other straight lines lie in a vertical plane, they also pass through 0 and intersect each other at that point. The section of the surface with this plane shows the two straight lines. If you shift this plane a bit forward the section curve turns into a hyperbola. This can be easily checked by calculation. You set either y=0 or y=1. In the first case the result is $x^2 = z^2$ or $(x-z)(x + z) = 0$, the equation of two straight lines in the plane. In the second case you get $x^2+z = z^2$. This can be rewritten in $-x^2+(z-1/2)^2 = 1/4$, the equation of a hyperbola with centre at (0, 1/2).

卡利普索曲面

方程$x^2+y^2z = z^2$对应的Calypso曲面包含3条直线。其中水平的直线是清晰可见的，它通过上部和下部的交汇处——原点。其他两条直线在垂直面内，他们也通过原点，并在该点彼此相交。曲面和这个垂直面相交的交线为两条直线。如果把这一平面稍微向前移动，交线就会变成双曲线。这个结果可以很容易验证。设y=0或y=1.在第一种情况下，我们得到方程$x^2 = z^2$或 $(x-z)(x + z) = 0$，这是xOz平面内的两条直线方程。在第二种情况下，我们得到方程$x^2+z = z^2$，将该方程改写为$-x^2+(z-1/2)^2 = 1/4$，这恰是一个以(0, 1/2)为中心的双曲线。

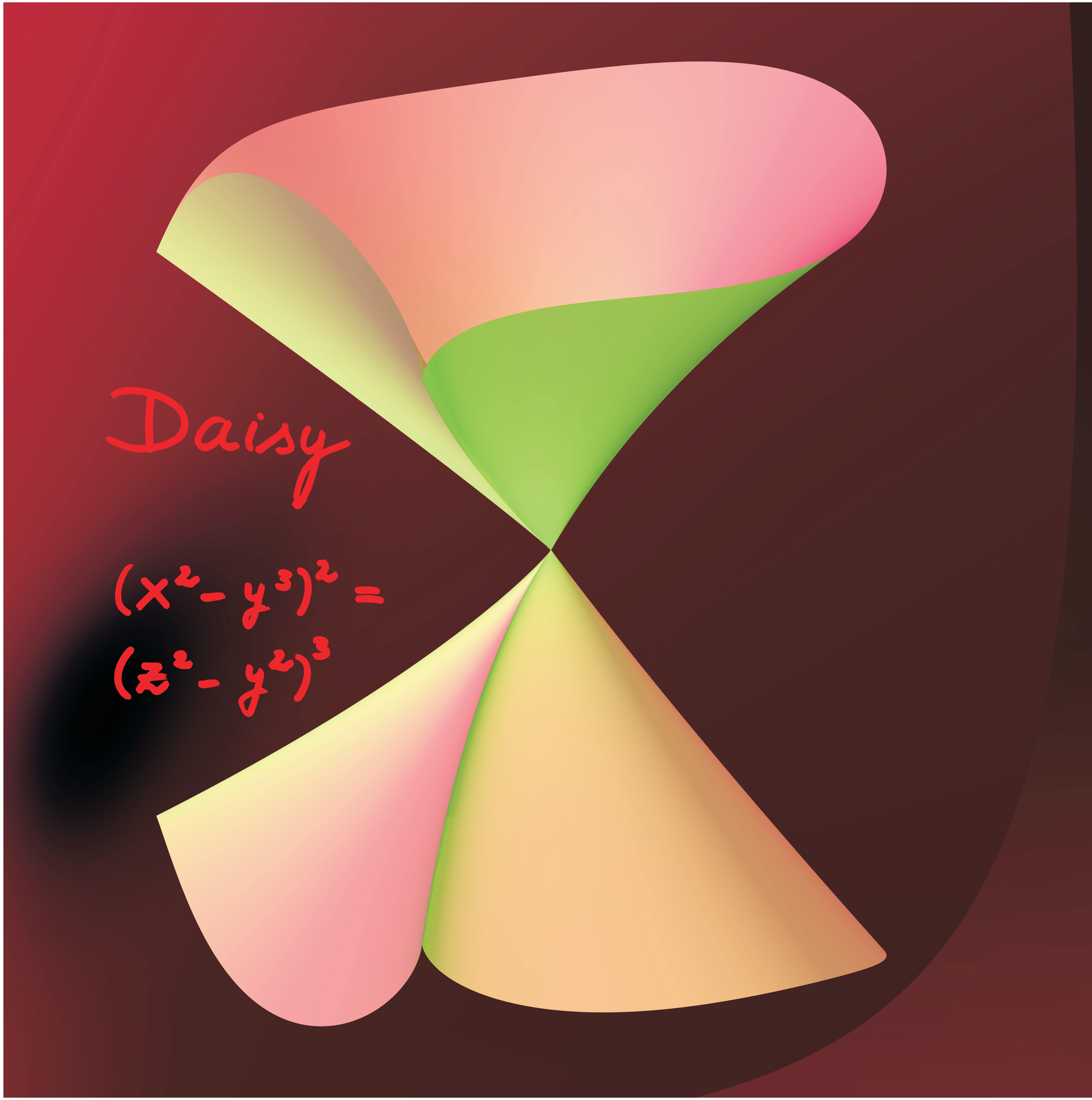


Calyx

The surface Calyx with equation $x^2+y^2z^3=z^4$ has a straight line as its singular locus. The lower part of the surface has cusp-shaped singularities alongside the straight line, whereas the upper part tangentially touches the straight line at a point, the origin. The real image is misleading insofar as the defining polynomial is irreducible and the surface, as a result, consists just of one algebraic component (not of two components as the figure suggests). It can be shown that Calyx is an appropriate projection of Calypso. In a three- dimensional space a cylinder surface is contracted to the singular straight line of Calyx. Algebraically, the mapping is defined by the requirement $(x, y, z) \to (xz, y, z)$. It is very simple. The respective substitution in $x^2+y^2z^3=z^4$ and subsequent reducing of z^2 yields the Calypso equation $x^2+y^2z=z^2$.

花萼曲面

方程$x^2+y^2z^3=z^4$表示的曲面——花萼曲面有一条直线的奇异轨迹。曲面的下半部分沿着这条直线有尖角形奇异点，而曲面的上半部分在原点与这条直线垂直相切。因为定义曲面的多项式是不可约的，这个图像有时候会有误导作用。且仅包含其中一个代数组成（而不是图中应有的两个部分）。可以证明Calyx是Calypso的投影。在三维空间，圆柱体表面收缩为Calyx的奇异直线。曲面从代数上说，这是非常简单的映射：$(x, y, z) \to (xz, y, z)$。在方程$x^2+y^2z^3=z^4$中将对应变量替换为映射右侧的对应变量，即x替换为xz，y、z不变，并约去z^2就得到Calypso 方程 $x^2+y^2z=z^2$.



Daisy

The equation $(x^2-y^3)^2 = (z^2-y^2)^3$ of Daisy implies by differentiation that the singular locus consists of two (plane) curves which transversally meet at their common singular point. In order to better understand singularities the geometrician constructs their resolution by means of blowups. In finitely many steps they provide a surface without singularities (a manifold) together with a projection map onto the original surface which interprets it as a shade of manifold.

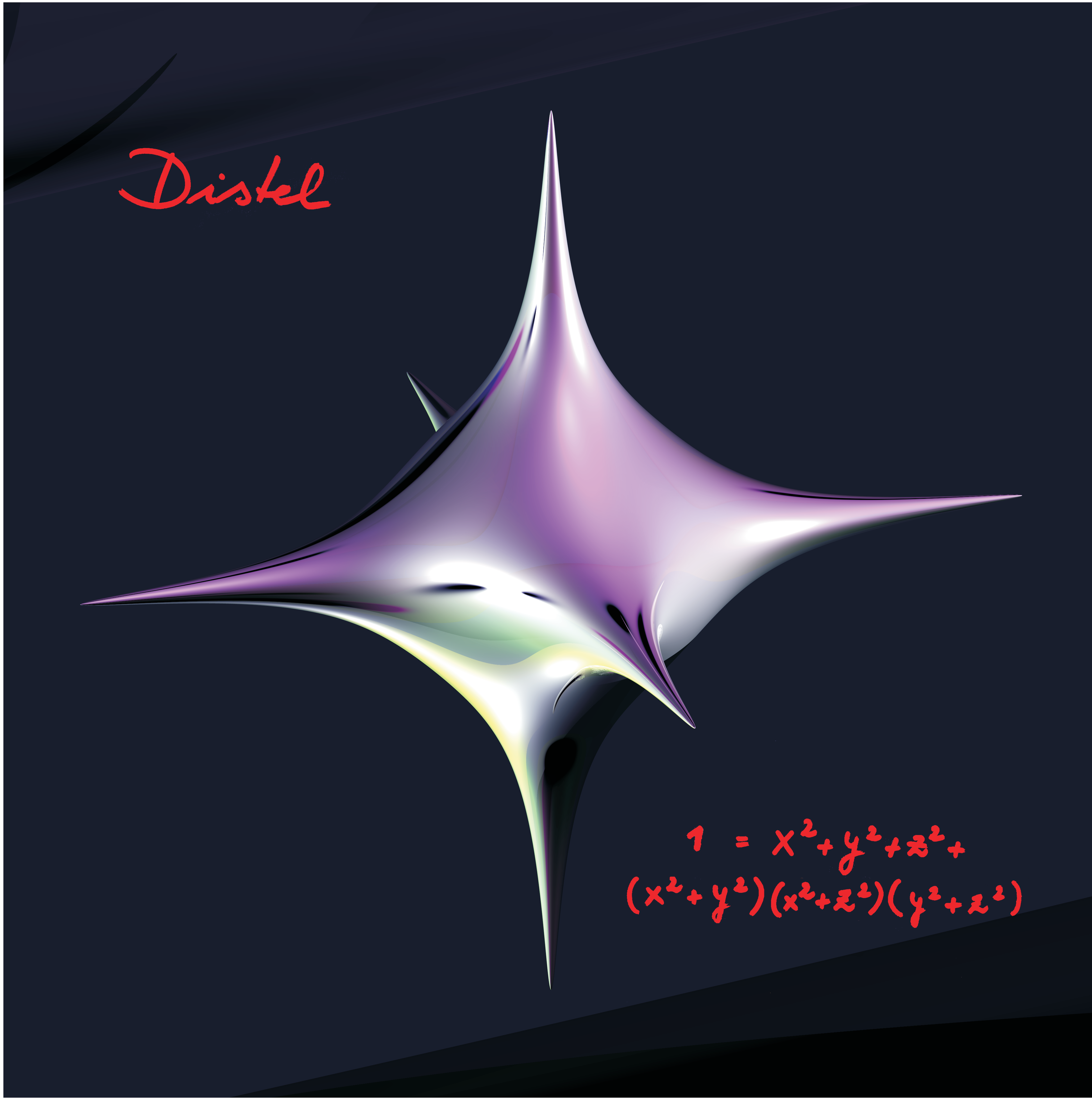
维菊

通过求导可知，维菊方程。$(x^2-y^3)^2=(z^2-y^2)^3$
奇异点的轨迹是由两条（平面）曲线组成，这两条曲线横向相交于它们公共的奇异点。

为了更好地理解奇异点，几何学家通过胀开的方式来构造它们的分解。

有限步以后，他们得到了一个无奇点的曲面（流形）和映满到原曲面的一个投影映射。

原曲面可以理解为这个流形的底影。



Thistle

Ithe surface Thistle with the equation $x^2+y^2+z^2+c(x^2+y^2)(x^2+z^2)\ (y^2+z^2) = 1$ excels through its extraordinary symmetry. The real image was provided with a very big coefficient c. The six spikes are located on the three coordinate axes of the Euclidian space. Each twist permuting the three axes leaves Thistle unchanged. The symmetry group, as a result, is that of the cube and the octahedron which is dual to it, two of the five Platonic solids. Surprisingly, it is not possible to construct totally regular stars such as Thistle having any number of spikes. This results from considerations of group theory. There can be only four, six, eight, twelve or twenty spikes according to the sides of the Platonic solids. We leave it to the curious viewer to find the appropriate equations for all these stars.

Thistle

由方程$x^2+y^2+z^2+c(x^2+y^2)(x^2+z^2)(y^2+z^2)=1$表示的Thistle曲面具有非常好的对称性。下面提供的实图对应的方程带有一个非常大的系数c。六个尖峰位于欧氏空间的三个坐标轴上。任意置换三个坐标轴的变换保持Thistle曲面不变。Thistle曲面的对称群（也叫自同构群）与立方体和八面体 的对称群同构。立方体和正八面体互为对偶，属于五个柏拉图体（正四面体，立方体，正八面体，正十二面体和正二十面体）中的两个。令人惊讶的是，基于群论的角度考虑，不能构造出像Thistle曲面一样带有任意个尖峰的完正则的星形。 类似柏拉图体，只可能存在四，六，八，十二和二十个尖峰。我们把它留给好奇的读者去寻找这些星形适合的方程。

Helix

The Lemniscate is the plane curve with the equation $y^4+z^2 = y^2$. It results from the circle $y^2+z^2 = 1$ by substituting z by z/y. Geometrically, the substitution corresponds to the one-time torsion of the circle to a figure-eight loop. If $y^4+z^2 = y^2$ is conceived as equation with three variables x, y and z (where x is a stowaway) then the solution set is a surface in the three-dimensional space, i.e. the cylinder above the lemniscate. The Helix equation $y^4+x^2z^2 = y^2$ is yielded by substituting z by xz. From the geometrical view, this construction is a kind of folding. The symmetry with respect to x and z is clearly visible. For the final formula we added the factors 2 and 6 to slightly stretch Helix. The singular locus is a cross of straight lines. The sections of Helix with the planes x = c or z = c are lemniscates for $c \neq 0$, whereas the sections y = c are hyperbola pairs

双纽线

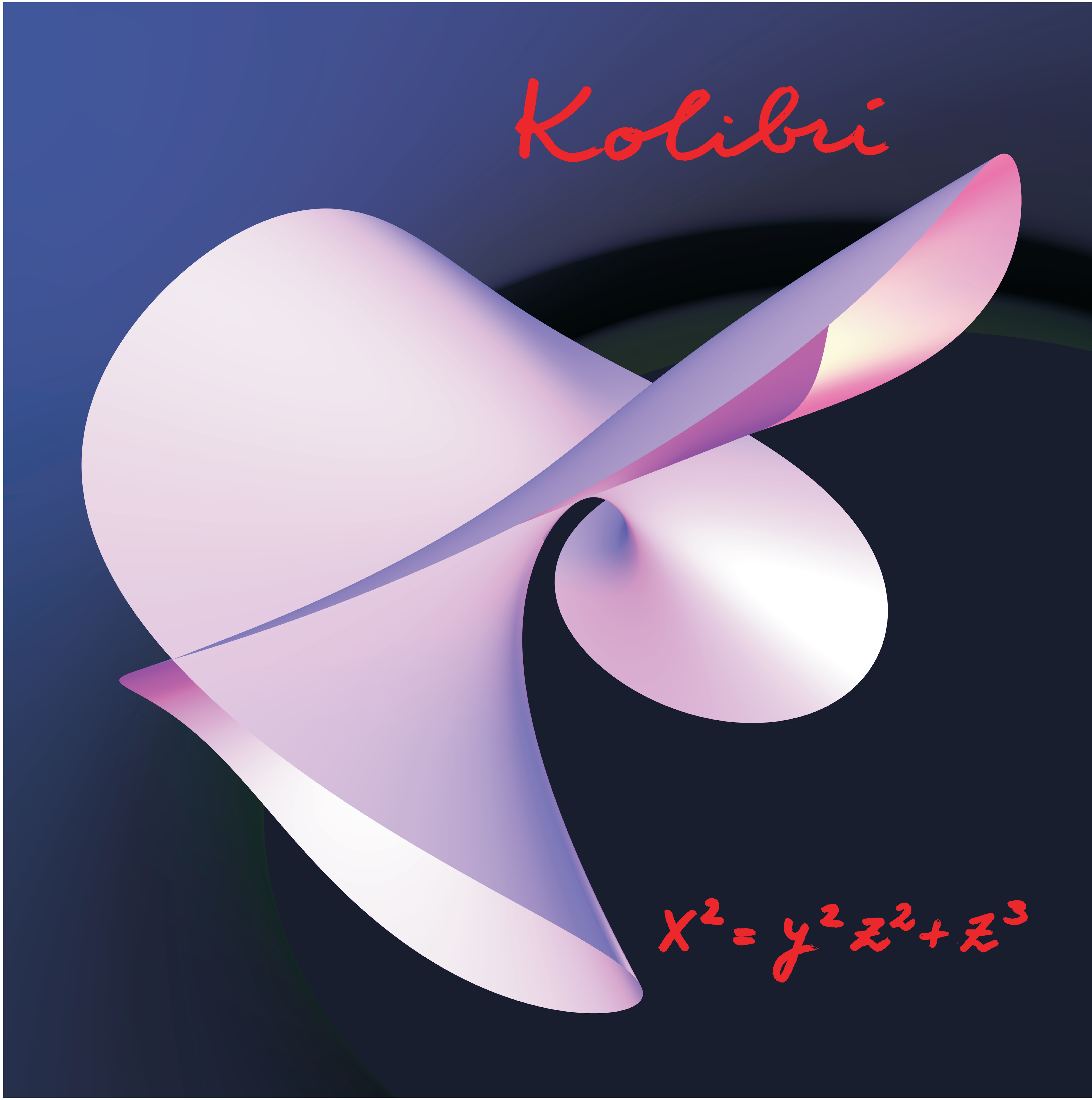
双纽线是一条平行面曲线，其方程式为$y^4+z^2=y^2$。它是在圆的方程$y^2+z^2=1$中用$\frac{z}{y}$替换z而得到。几何上，这个代换对应于圆到‘8’字环的一次扭转。如果将$y^4+z^2=y^2$看作是关于x，y，z的方程（其中x省略），那么它的解集就是三维空间中的曲面，即双纽线上的柱面。在双纽线方程中用xz替换z即得螺旋线方程$y^4+x^2z^2=y^2$.从几何上看，这个构造可以看成是一个折叠。关于x和z的对称性是显而易见的。为了得到最后的公式，我们增加因子2和6来轻微地拉伸螺旋线，奇异点的轨迹是两条交叉的直线。当$c\neq0$时,平面x=c和平面z=c与螺旋面相截为双纽线,平面y=c与螺旋面相截为双曲线对。

Heart

Despite the simple equation $y^2+z^3 = z^4+x^2z^2$ the surface Heart possesses a subtle local and global structure. The singular locus is a straight line alongside which the surface intersects itself. The origin 0 is the interesting point: We intersect Heart with planes $x = c$ orthogonal to the singular straight line. The result is a loop which contracts like a knot if c tends to 0. A funnel is formed. Viewed from the distance we see a circle shaped opening in the surface. The section with the vertical xy-plane is in fact a circle. The simplicity of the surface results in that we can capture its form and memorize it. With closed eyes we recall the figure at once in every detail. What is much more difficult is to verbally communicate the geometrical pattern to a third party. The adequate terminology is lacking.

心脏曲面

尽管心脏曲面方程$y^2+z^3=z^4+x^2z^2$非常简单，但心脏曲面却拥有一个微妙的局部和整体结构。它的奇异点的轨迹是一条直线，沿着这条直线，曲面自相交。原点0在这里是一个很有趣的点，心脏曲面与平面x=c的交线垂直于奇异点的直线。在此循环中，若c趋于0，则结果收缩至一个节点，一个漏斗就此形成。从远处看，我们看到一个圆形开口，与竖直的xy平面的交线实际上是一个圆。这个曲面非常简单使得我们能够抓住它的形态并且记住它。闭上眼睛，我们可以立刻记起它的每个细节。困难的是将他的几何图形用另外一种几何图形联系起来，找到合适的术语来描述它。



Hummingbird

As you can easily see Hummingbird is a closeup view turned upside down of Heart near the origin. The equation is $x^3+x^2z^2 = y^2$, compare with the Heart formula $y^2+z^3 = z^4+x^2z^2$. If you substitute x by z and z by –x in the latter the monomial x^4 can be eliminated by a change of coordinates so that the Hummingbird equation comes out. The Hummingbird as one of the smallest birds is equipped with impressing abilities. Its wings are able to beat up to 200 times in a second so that it can stand still in the air. This activity needs a lot of energy, that is why the hummingbird must eat twice its body weight in food per day. Without constant food supply it would starve within a few hours. It lowers its body temperature substantially at night so as to save energy.

蜂鸟

正如你很容易看到的那样，蜂鸟曲面是在原点附近倒置的心脏面的特写视图.考虑方程$x^3+x^2z^2 = y^2$和心脏面方程$y^2+z^3 = z^4+x^2z^2$相比较。如果在前一方程中将z替换x，将–x替换z，然后通过坐标变换消去x^4，便可得到蜂鸟方程。蜂鸟随时最小的鸟类，却拥有非凡的能力。它的翅膀可以在1秒内扇动达到200次之多，因此它可以在空中不动。要做到这点需要大量的能量，这就是为什么蜂鸟要在一天中吃掉它两倍体重的食物。没有持续的食物补充，蜂鸟会在很短的时间内感到饥饿.蜂鸟也会在夜晚降低体温以保存能量。

Solitude

The Solitude equation $x^2yz+xy^2+y^3+y^3z = x^2z^2$ does not reveal its hidden geometrical diversity. Similarly, the image shows only part of the phenomena. What is informative, however, is the camera drive round the surface as it can be experienced in the film “ZEROSET– I spy with my little eye”. There are obviously two openings, a larger well visible one and a smaller one, which you would not suspect from the first perspective. The view from above shows the vertical singular straight line alongside which horizontal sections force a sharp curve. The example of Solitude shows the complexity of the problem to deduce the visible real geometry from the equation. Similarly, one can question the underlying complex or number-theoretical geometry.

Solitude

Solitude方程$x^2yz+xy^2+y^3+y^3z = x^2z^2$ 没有透露其隐含的几何多样性。同样的，它的图形也仅仅显示了一部分此种现象.然而真正提供信息的是绕在表面的摄像头驱动，正如电影“零位调整–我用我的小眼睛侦察”中应用的那样。它有明显的两个开口，一个大的可以看到，一个小的不能够一下找到。从上而下的视图显示了一条竖直的奇异点的直线，沿着这条直线的水平截面是一条尖锐的曲线。这个例子说明了要从方程本身演绎可视的真实几何性征是非常复杂的。同样，任何人都可以质疑其中的复杂性和数值理论的几何形态

Dance

By setting z = 0 the Dance equation $2x^4 = x^2 + y^2z^2$ yields the equation $2x^4 = x^2$. This is the section of Dance with the horizontal xy plane. If you rewrite the equation you will get $x2(\sqrt{2}x+1)(\sqrt{2}x-1) = 0$ of three parallel straight lines. We, here, observe a typically real phenomenon: The straight line x = z = 0 belongs to the solution set of the Dance equation $2x^4 = x^2+y^2z^2$, but is an isolated one-dimensional component, because if x is near 0 there are only the solutions x = y = 0 or x = z = 0, that means the cross consisting of the y and z axis. As straight lines are infinitely thin they are missed by the visualization program or are not indicated. Therefore, the existence of one- dimensional components in the solution set must be cleared up by computation beforehand and then they must be added to the image as thin cylinders, if necessary.

舞蹈

令z=0，舞蹈方程$2x^4 = x^2 + y^2z^2$ 导出$2x^4=x^4$这是舞蹈方程与水平的XY–平面的截面，此方程可以改写为（$\sqrt{2}x+1$）（$\sqrt{2}x-1$）=0，即三条平行直线方程。这里，我们观察到一个真实的现象，直线x=z=0属于舞蹈方程的解集，但是是孤立的一维分支。因为当x接近0时，只有两组解x=y=0，x=z=0，就是y轴和z轴。因为直线可以无限细，可视化时容易被忽略或者没有标明。因此，解集中的一维分支的存在性必须提前通过计划算确定，然后，有必要的话把他们用细线柱体的形式添加到图像中。

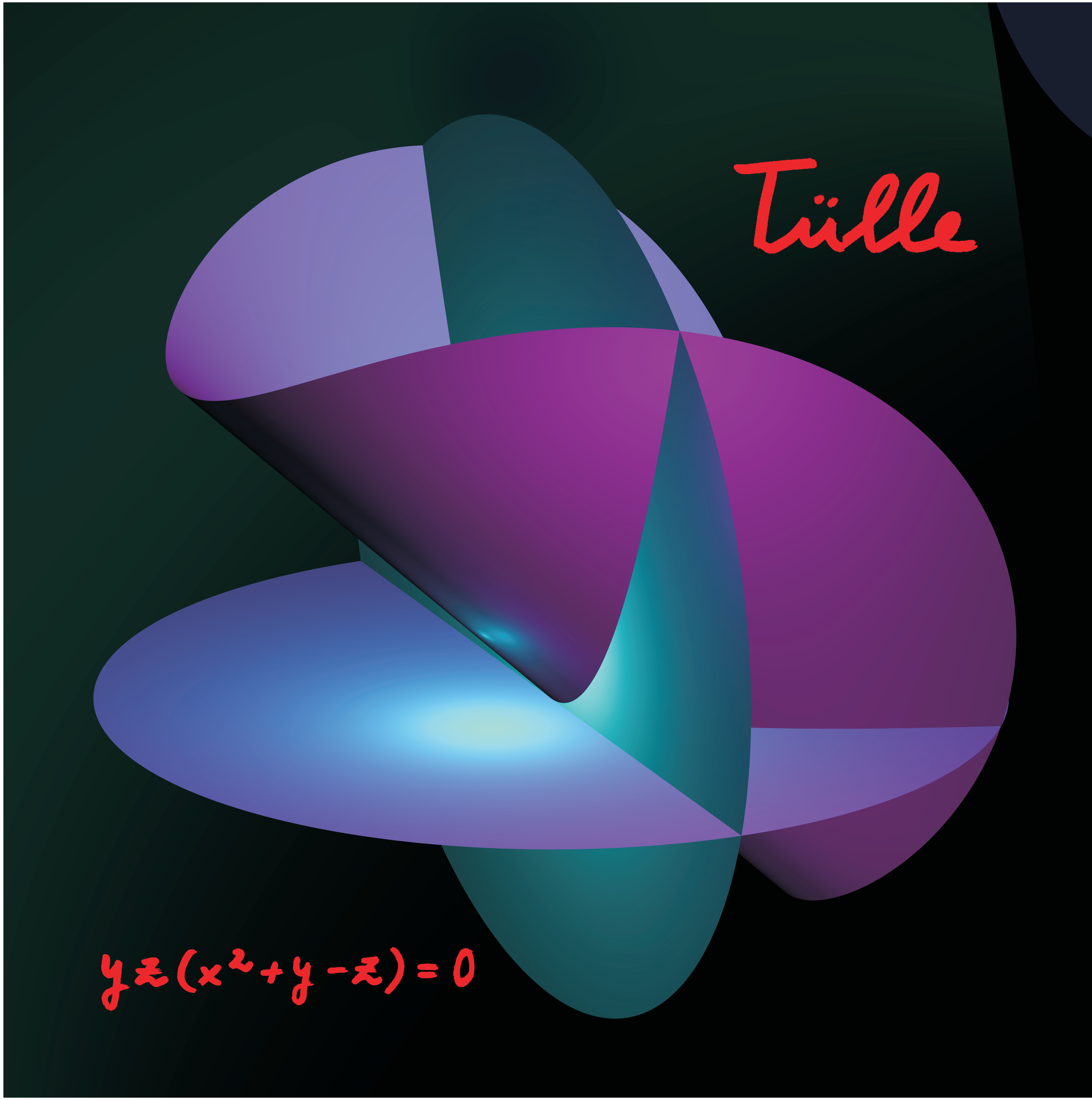
Dove

Dove possesses the amazing formula $256z^3 - 128x^2z^2+16x^4z+144xy^2z-4x^3y^2-27y^4 = 0$. The coefficients are not incidental. On the contrary: The equation results from another more general formula, the so-called discriminant. It describes the shade of a surface or variety which emerges along with the projection on a surface or a linear space of higher dimension. The contour line is clearly defined by the surface and the projection and, from the algebraic point of view, also the form of the equation.

鸽子

鸽子拥有神奇的方程 $256z^3-128x^2z^2+16x^4z+144xy^2z-4x^3y^2-27y^4=0$，它的系数并非偶然，相反，它的方程源于一个更一般的公式即所谓的判别式。他描述了曲面的阴影，或者是投影到一个曲面或者更高维的线性空间而出现的曲面簇。等高线

可由曲面和投影精确定义，并且从代数角度，也可以给出方程的确切形式。



Nozzle

The Nozzle surface is constituted by three smooth components, which intersect each other pairwise in a (likewise) smooth, plane curve. That way, three section curves are obtained, i.e. one straight line and two parabolas. Note that these curves touch each other tangentially at the origin. We here have the simplest example of a surface with three pairwise transversal components so that the section curves of every two of them do not intersect transversally. The surface, thus, is no Mikado Variety. The transversal intersection of two smooth components of a surface – a fundamental concept of geometry - can, by way of the so-called ideal theory, be put into formulas in an algebraically precise manner. It can perfectly be used for calculation and proofs. In the case of singular components a correct definition of transversality is still to await.

Nozzle

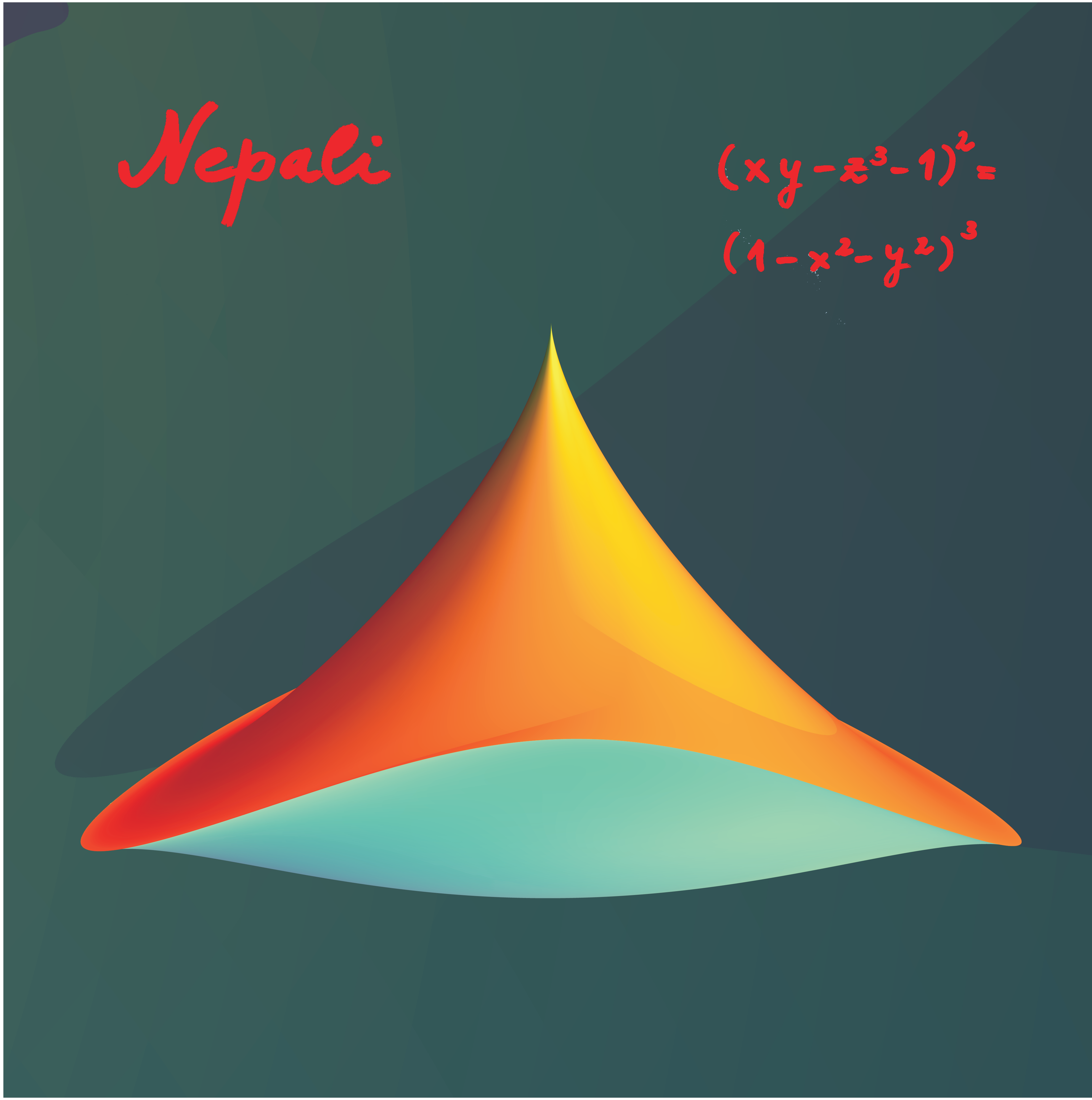
Nozzle曲面有三个光滑分支组成，它们分别在光滑平面曲线上两两相交。 这样就得到了三条曲线，即一条直线和两条抛物线，注意到这些曲线在原点处相切，这里我们得到一个最简单的曲面实例：这个曲面有三个两两横截的分支，每两个分支的相交曲线都不横截相交。因此，这个曲面不是Mikado簇。一个曲面的两个光滑分支横截相交（几何学的基本概念）可以利用所谓的理想理论用精确的代数公式表达。它可以完美的用于计算和证明。单一分支的情形还没有准确的横截定义。

TICK

The simple Tick equation $x^2+y^2 = z^3(1-z)$ fully dictates the geometry just as with the other surfaces; that means, both the singular points and the outer shape, the curvature and the extension are clearly defined by the four monomials x^2, y^2, $-z^3$und z^4. As a result, the formula is a very efficient way to codify forms which appear complicated. However, the geometric information cannot always be read from the formula. The local shape of the surface near a given point can be explicitly defined, in most cases; the techniques of local analytic geometry have a good effect. Defining the global structure requires much more efforts and cannot always be satisfactorily accomplished.

滴答曲面

就像其他曲面一样，滴答曲面的简单的方程$x^2+y^2 = z^3(1-z)$ 充分表现了其几何特征； 也就是说不管是奇艺点还是外形，抑亦是其曲率以及拓展都清晰的由四个单项式x^2, y^2, $-z^3$和z^4清晰地展示出来。 总之方程是表达看起来复杂形状的很有效的方式。但是几何信息也不总是都可以从方程中看出来。 大部分情形是曲面在给定点附近的形状会清晰的定义出来；也就是局部解析几何技术总是起到很好的表达效果。但是定义整体的结构即使花费更大的力气也未必能取得满意的表达效果。



Nepali

Let us look at the defining equation $(xy-z^3-1)^2 = (1-x^2-y^2)^3$ of Nepali. The symmetry between x and y is enforced by the quadratic polynomial x^2+y^2, which is rotation symmetrical in contrast to the monomial xy. Sections with horizontal planes $z = c$ yield closed curves being almost circles. The simultaneous occurrence of squares and third powers produce the tapering at the top. The lateral boundary curve of Nepali is no exact circle but is arching up and down like the brim of a hat. Its projection to the horizontal xy plane, however, is a circle as can be seen from the top view. The surface shown is bounded; hence there was no need of any cube or sphere intersection. This fact can be directly derived from the formula by accurate analysis.

尼泊尔曲面

让我们看看由Nepali定义的方程$(xy-z^3-1)^2 = (1-x^2-y^2)^3$二次多项式 x^2+y^2规定了x与y之间的一个对称关系，与单项式xy规定的对称不同，这是旋转对称。Nepali曲面与水平面 $z = c$的截面图形接近于圆。方程两边的平方和三次方产生曲面上方的尖锥形。Nepali曲面的外侧边界曲线不再是圆了，而是像帽檐一样向上和向下拱的形状。从上面看，它在水平坐标面xy面上的投影是个圆。所示的曲面是有界的，因此没必要,给出所有的立方截断和球截断。这个事实可以由精确分析公式直接得到。