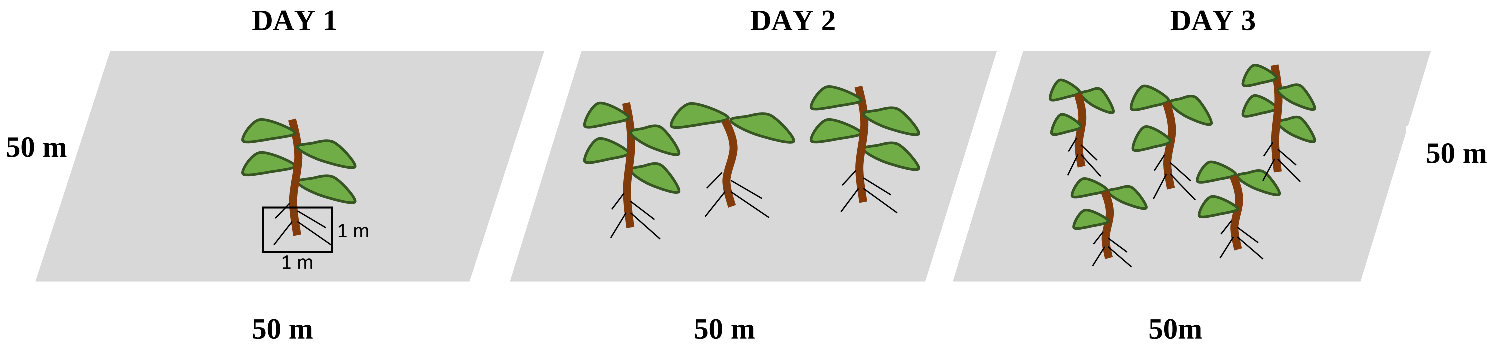
MATHEMATICAL EXHIBITION OF PLANT POPULATION GROWTH IN $50M^2$ GARDEN USING THE LOGISTIC EQUATION

BY

DAVID OMOGBHE



The above ecological phenomenon can be modelled by the logistic equation:

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right) \qquad (1)$$

where:

N=*population of the plant at a given time,*
K=*carrying capacity=50m²,*
r=*growth rate of the plant=2 per day,*
t= *time (days).*

Each plant occupies $1m^2$ of the garden.

The solution of the logistic equation is given by:

$$N(t) = \frac{KN_0}{N_0 + (K - N_0)e^{-rt}} \qquad (2)$$

Where: N_0 =initial population=1 plant.

Substituting the values of N, K and r in equation 2, we have:

$$N(t) = \frac{50}{1 + 49e^{-2t}} \qquad (3)$$

Prediction:

On the 10th day, the population of the plant in the garden is $N(10) = \frac{50}{1+49e^{-20}}$ ≈50 plants. This implies that, after the 10th day, the plants will start competing for space thereby causing a decline in the population growth.

MATHEMATICAL EXHIBITION OF THE PREY-PREDATOR (RAT AND CAT) MODEL USING THE LOGISTIC EQUATION IN 2Om^2 FARM LAND

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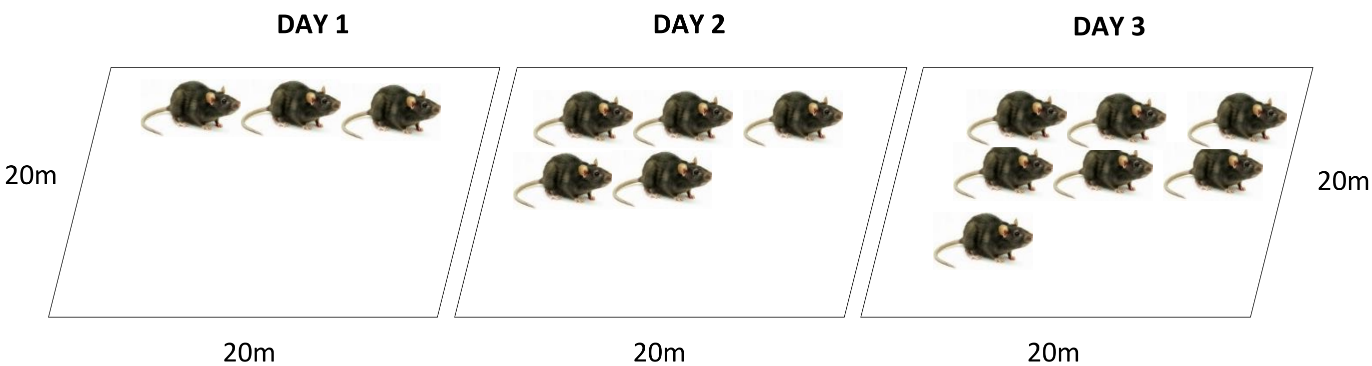
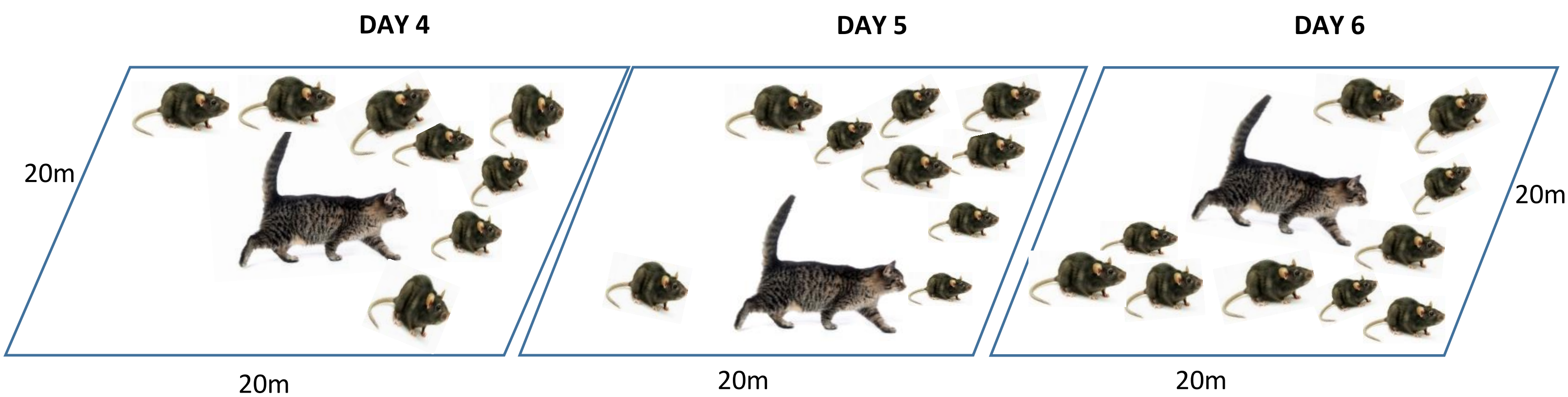
1. **Rat population in the absence of the cat**



2. **Rat population in the presence of the cat**



The above ecological system can be modelled by the logistic equation:

$$\frac{dx}{dt} = \alpha x\left(1 - \frac{x}{k}\right) - \beta xy \qquad (1)$$

Where:

$\boldsymbol{x}$ = *rat population at a given time,*

$\boldsymbol{y}$ =*cat population at a given time=1 cat,*

$\boldsymbol{k}$=*carrying capacity=20m^2,*

$\boldsymbol{\alpha}$=*rat population natural growth rate=2 per day,*

$\boldsymbol{\beta}$=*rat population death rate due to presence of the cat= 1 per day,*

$\boldsymbol{t}$ = *time(days).*

The cat and each rat occupies 1m^2 of the farm land.

On substituting the values α, β and k in equation 1 we have:

$$\frac{dx}{dt} = 2x\left(1 - \frac{x}{20}\right) - xy \qquad (2)$$

In the absence of the cat i.e y=0, the equation is reduced to

$$\frac{dx}{dt} = 2x\left(1 - \frac{x}{20}\right) \qquad (3)$$

Solving equation 3 we have:

$$x(t) = \frac{20x_0}{x_0 + (20 - x_0)e^{-2t}} \qquad (4)$$

substituting x_0=initial population=3 rats, we have:

$$x(t) = \frac{60}{3 + 17e^{-2t}} \qquad (5)$$

In the presence of the cat i.e when y=1, the equation is reduced to

$$\frac{dx}{dt} = 2x\left(1 - \frac{x}{20}\right) - x \qquad (6)$$

Solving equation 7 we have:

$$x(t) = \frac{10x_0}{x_0 + (10 - x_0)e^{-t}} \qquad (7)$$

substituting x_0=initial population=8 rats, we have

$$x(t) = \frac{80}{8 + 2e^{-t}} \qquad (8)$$

Predictions can be made from these results.