Around the world in …N fractals for Mathlapse!

Abstract.
In the video, we wander around the world using fractal geometry. A fractal is mathematical set that exhibits a repeating pattern that displays at every scale. We connect some fractal figures to the symbols of some countries. We use Iterated Function System (IFS) fractals to represent the Eiffel Tower in Paris, temples in Bagan, Cathedral in Prague, dome in Milan and so on. The video was created during a mathematical activity in the classroom with my students in Liceo Scientifico "L. Siciliani" in Catanzaro Italy.
We would like that, our video can be a symbol of peace and friendship in the world in the name of Mathematics.

Introduction

Digital technologies are impacting all aspects of personal, social and professional life now, spreading out at an incredible speed. We should take into account all these changes also in the teaching and learning processes of mathematics, taking on new challenges and responsibilities. It is necessary that the new technology (ICT) has a relevant role in the classroom to improve the educational approach to mathematics. The video *"Around the world in …N fractals for MathLapse!"* could be an expression of this new educational approach, where theoretical content is combined and modified through multimedia tools. The video includes the following mathematical contents: affinity transformations, some well-known fractals, matrices that the students applied to create images in the video, using some free digital resources on the web.

The context

My school Liceo Scientifico "L. Siciliani" in Catanzaro (Italy) is a place where you can experiment with new ways of teaching and learning mathematics with students. In fact, the Liceo has been participating for eight years with a pool of teachers (me included) in a national project, called Matematica & Realtà (M&R), managed by the department of mathematics at the University of Perugia. The project deals with making proposals to develop innovative and educational relationships between mathematics and the real world.
M&R is an innovative approach to mathematics education; the mathematical models are the central focus of the project. It includes some different activities: mathematical modelling and best maths presentation competitions.
The main actions of the project are mathematical laboratories, organized in each school taking part in the projects, during it the teacher proposes different themes, related to M&R, depending on the age of the pupils and the compulsory mathematical contents. For instance:

1. Linear mathematical models of reality (for 15-16 years old students);
2. Iterative functions and fractal geometry (for 16-17 years old students);
3. Nonlinear mathematical (exponential and trigonometric) models of reality (for 17-18 years old students).

My Maths Laboratory

My mathematical laboratory is concerned about geometry, in particular fractal geometry, geometric properties of fractal figures and affine transformations. The laboratory requires 20 hours (two hours per week in a single lesson) and it is an extra-time activity. Students attend the course as volunteers, they want to be involved in different educational methods about geometry.

The activities, during the laboratory, are divided into two parts:

a) The presentation of geometric contents

- Iterative processes on geometrical figures;
- Processes on figures generated by geometrical transformations;
- Geometrical transformations and matrices;
- Affine transformations, isometries, composition of geometrical transformations;
- Well-known fractals as: Koch' fractal, Sierpinski' gasket, Barnsley' cathedral;
- Genetic code of a fractals;
- Dimension of a fractals.

b) The final group-works

Some group of students design and create the multimedia objects concerning the mathematical contents, guided by the teacher. They use free software and resources on the web as Powtoon, Go Animate, Moovly and so on for sound and for images.

The final goal of every group is to take part in "The Best maths presentation competition", that is held every year at the University in Perugia.

The video *"Around the world in… N fractals!"* has been presented in the competition on the last April.

"Around the world in… N fractals!"

Idea

During the activities in maths laboratory, for including Martina, a student with Italian, English, Czech origins, we designed to wander around the world through geometry, linking a fractal figure to each foreign country. We chose some countries (France, Italy, Czech Republic, India …) and connected them to a fractal figures, that in our opinion, could represent different countries.

Mathematical contents

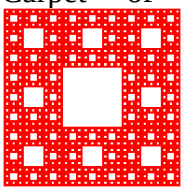
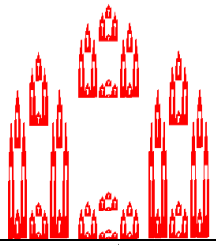
We studied the fractal codes and the affine transformations. In one case, we changed the fractal codes to generate a new fractal for example the fractal for the temples in Bagan.

In our video we use two different forms to indicate the transformation in IFS fractals:

a) in equation form $T:\begin{cases} x' = ax + by + c \\ y' = dx + + ey + f \end{cases}$ with $\begin{vmatrix} a & b \\ d & e \end{vmatrix} \neq 0$

b) in matrix form $T\begin{bmatrix} a & b & d & e & c & f \end{bmatrix}$

Fractals codes & Countries

Fractal	Code	Symbol &Country
	$T_1\begin{cases} x'=\frac{1}{2}x \\ y'=\frac{3}{10}y \end{cases}$ $T_2\begin{cases} x'=\frac{1}{2}x+\frac{1}{2} \\ y'=\frac{3}{10}y \end{cases}$ $T_3\begin{cases} x'=\frac{1}{2}x+\frac{1}{4} \\ y'=\frac{7}{10}y+\frac{3}{10} \end{cases}$	Tour Effel France
Carpet of Sierpinski 	$T_1:\begin{cases} x'=1/3x \\ y'=1/3y \end{cases}$ $T_5:\begin{cases} x'=1/3x+1/3 \\ y'=1/3y+2/3 \end{cases}$ $T_2\begin{cases} x'=1/3x \\ y'=1/3y+1/3 \end{cases}$ $T_6:\begin{cases} x'=1/3x+2/3 \\ y'=1/3y \end{cases}$ $T_3\begin{cases} x'=1/3x \\ y'=1/3y+2/3 \end{cases}$ $T_7:\begin{cases} x'=1/3x+2/3 \\ y'=1/3y+1/3 \end{cases}$ $T_4\begin{cases} x'=1/3x+1/3 \\ y'=1/3y \end{cases}$ $T_8:\begin{cases} x'=1/3x+2/3 \\ y'=1/3y+2/3 \end{cases}$	Persian Carpet Turkmenistan
Cathedral of Barnsley 	$T_1=\begin{cases} x'=\frac{1}{3}x \\ y'=\frac{4}{5}y \end{cases}$ $T_2=\begin{cases} x'=\frac{1}{3}x+\frac{2}{3} \\ y'=\frac{4}{5}y \end{cases}$ $T_3=\begin{cases} x'=\frac{1}{3}x+\frac{1}{3} \\ y'=\frac{1}{5}y \end{cases}$ $T_4=\begin{cases} x'=\frac{1}{3}x+\frac{1}{3} \\ y'=\frac{23}{50}y+\frac{4}{5} \end{cases}$	Dome in Milan Italy
	$\begin{bmatrix} 0.33 & 0 & 0 & 0.71 & 0 & 0 \\ 0.33 & 0 & 0 & 0.71 & 0.6 & 0 \\ 0.33 & 0 & 0 & 0.28 & 0.28 & 0 \\ 0.33 & 0 & 0 & 0.46 & 0.33 & 0.71 \end{bmatrix}$	Temple Myanmar
	$\begin{bmatrix} 0.25 & 0 & 0 & 0.25 & 0 & 0.50 \\ 0.72 & -0.61 & 0.61 & 0.72 & 0.41 & -0.19 \end{bmatrix}$	Bush in landscape
	0.797881 0.000000 0.000000 0.833333 0.097828 0.168721 -0.131628 -0.366787 0.399144 -0.140643 0.500098 0.180655 0.099649 0.403824 0.408196 -0.100330 0.364628 0.157770 0.049180 0.000000 0.000000 0.333333 0.398795 0.009695	Pine Tree
	$\begin{bmatrix} 0.25 & 0 & 0 & 0.25 & 0 & 0.50 \\ 0.82 & -0.47 & 0.47 & 0.82 & 0.30 & -0.17 \end{bmatrix}$	Spiral

Multimedia Tools

We used the following resource on the web:

a) *Moovly* is a free browser-based platform and does not require any installation, it combines visuals, sounds, voice and video in clear explanations or engaging stories. We used it to create the video.
b) *IFS Construction Kit by Larry Riddle* was, used to design and draw fractals based on iterated function systems. This software may be freely used. Copyright is maintained by Larry Riddle.
c) Audacity® is a free, open source, cross-platform software for recording and editing sounds.
d) Paint.NET is free image and photo editing software for computers that run Windows

Methodology in group- works

After some theoretical lessons about fractal geometry, I used *flipped classroom* technique to work with the specific group. In particular, in the group of *"Around the world.."* Martina and Giovanni studied fractals and created many fractals imagines by using different software and Mariarosaria used moovly to create the video.

The mathematics aims

During the lessons the most important mathematical aims are:
For students

- Improving the study of geometrical transformations;
- Building known fractals (Sierpinski gasket, cathedral of Barnesley…) identifying the geometrical transformations and their codes;
- Making conjectures and individual simulations to create new fractals, for stimulating their creativity through the geometry;
- Learning to model the real world using affinity transformations as models;
- Using new technologies to improve the knowledge of mathematical contents.

For teacher

- Developing a mathematics education opened to the external world, remaining anchored to the epistemology of the discipline;
- Giving students more autonomy in geometric knowledge;
- Using new technologies to better address the specific needs our students may have;
- Involving students in a new educational approach to geometry

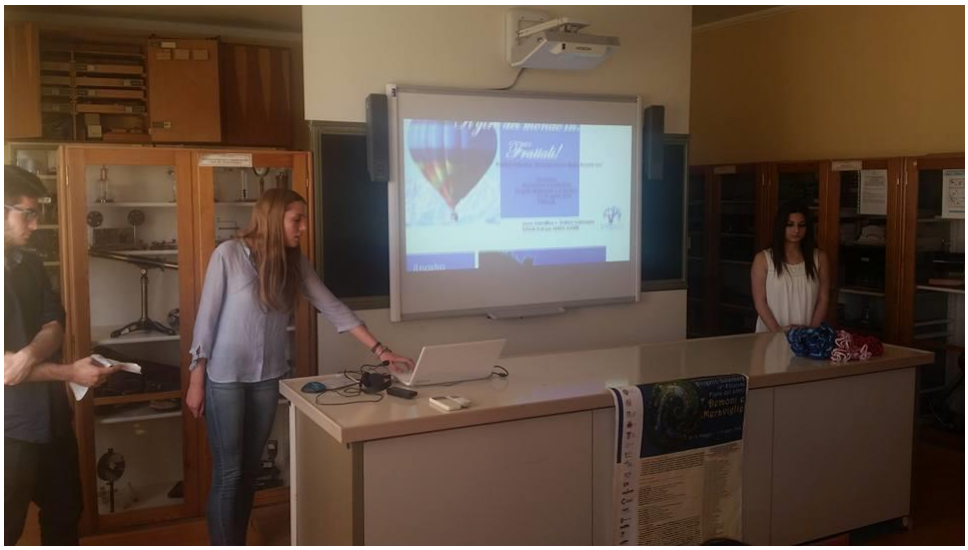
Conclusion

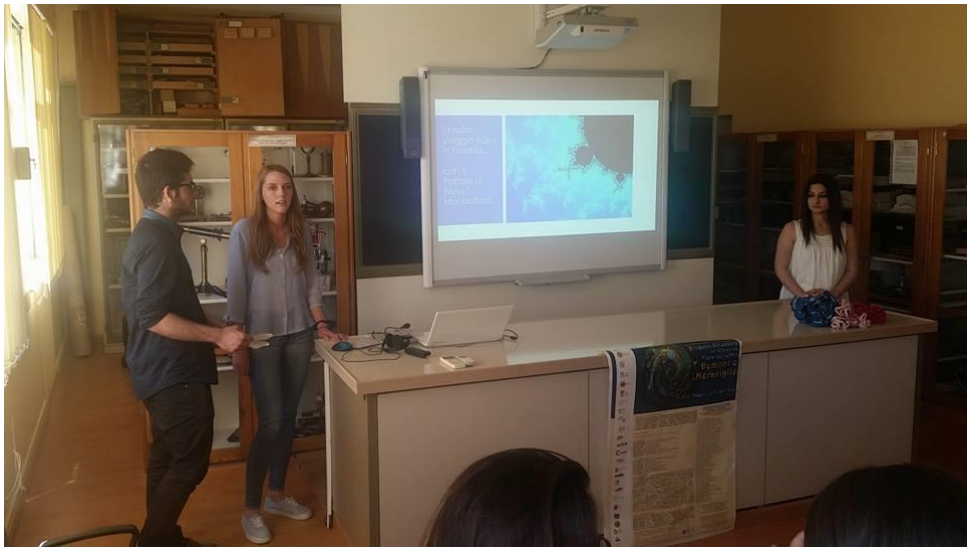
This educational activity is based on an inclusive approach between students and the mathematical contents. It is necessary to stimulate their curiosity, to foster their development of logical skills and attitudes, to support them in their educational and emotional weakness, to increase their sense of self-confidence.
It is also an activity to encourage the respect of the difference under a common denominator: making Mathematics together.

https://youtu.be/67ZM3MkOsKg

Some Images

During activity in the school





During the Competition in University of Perugia