Octagonal type of the quasiperiodic succession algorithm

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The *decagonal quasiperiodic succession algorithm* [1], related to *decagonal cluster cells* [2], generates the growth of an infinite *cartwheel-type tiling*, although it acts locally.
The paper presents a new version type, applicable for coverings of *octagonal clusters cells* $\boldsymbol{Q}$ (Fig. 1a) which have an equivalent relation to the *Gähler octagons* $\boldsymbol{\Omega}$ in a perfect *Ammann-Beenker tiling* [3]. The cell $\boldsymbol{Q}$ is based on the quasiperiodic *Ammann 8-grid* $\boldsymbol{\Gamma}$, a superposition of four 1D-grids $\boldsymbol{\Gamma}^{\mathbf{a}}$, $\boldsymbol{\Gamma}^{\mathbf{b}}$, $\boldsymbol{\Gamma}^{\mathbf{c}}$, $\boldsymbol{\Gamma}^{\mathbf{d}}$. The used substitution factor of $\boldsymbol{\Gamma}$ is $\boldsymbol{\lambda}^2$ (silver mean $\boldsymbol{\lambda} = 1+\sqrt{2}$). The growth process is controlled by the scale values $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$, $\mathbf{d}$ of the twin-scales $\mathbf{I}^{\mathbf{a}\pm}$, $\mathbf{I}^{\mathbf{b}\pm}$, $\mathbf{I}^{\mathbf{c}\pm}$, $\mathbf{I}^{\mathbf{d}\pm}$ (in general $\mathbf{I}^{x\pm}$) which are fixed on the cell grid $\boldsymbol{\Gamma}^{\boldsymbol{Q}}$ in a specific relation. On both scales $\mathbf{I}^{x+}$ and $\mathbf{I}^{x-}$ of a twin-scale $\mathbf{I}^{x\pm}$ two identical values $\boldsymbol{x}$, with $\boldsymbol{x} \in \{\boldsymbol{x}^{def}\}$, are synchronised by a sliding ruler. Its length, $\mathbf{L}^{aver}$, is the average of the *$\boldsymbol{q}$-line grid intervals* $\mathbf{L}^{\boldsymbol{q}}$ and $\mathbf{S}^{\boldsymbol{q}}$, with respect to the ratio $\sqrt{2}$:1 of their lengths and 1:$\sqrt{2}$ of their frequency rate in an infinitely expanded grid $\boldsymbol{\Gamma}^{\boldsymbol{q},\infty}$.
The *octagonal quasiperiodic succession algorithm* distinguishes 7 neighbour transformations $\boldsymbol{h}_{\boldsymbol{k}}(\boldsymbol{Q})$ with 4 specified equations each. The algorithm correlates the twin-scales of a cell $\boldsymbol{Q}$ with the parallel twin-scales of a cell $\boldsymbol{h}_{\boldsymbol{k}}(\boldsymbol{Q})$, converts their values and then verifies or falsifies the transformation. A *verified* transformation (e.g.: Fig. 1b) will be denoted $\boldsymbol{h}_{\boldsymbol{v}}(\boldsymbol{Q})$. Beginning with a *start-cell* $\boldsymbol{Q}_{\mathbf{0}}$ only cells of the form $\boldsymbol{Q}_{\mathbf{0}\ldots\boldsymbol{v}} = \boldsymbol{h}_{\boldsymbol{v}}(\boldsymbol{h}_{\boldsymbol{v}}(\ldots(\boldsymbol{h}_{\boldsymbol{v}}(\boldsymbol{Q}_{\mathbf{0}}))\ldots))$ are realized.
As a result we propose a recursive 7x4-*formula set* generating a flawless infinite step-by-step growth of an *octagonal Ammann-Beenker substitution tiling*, solely using local information.

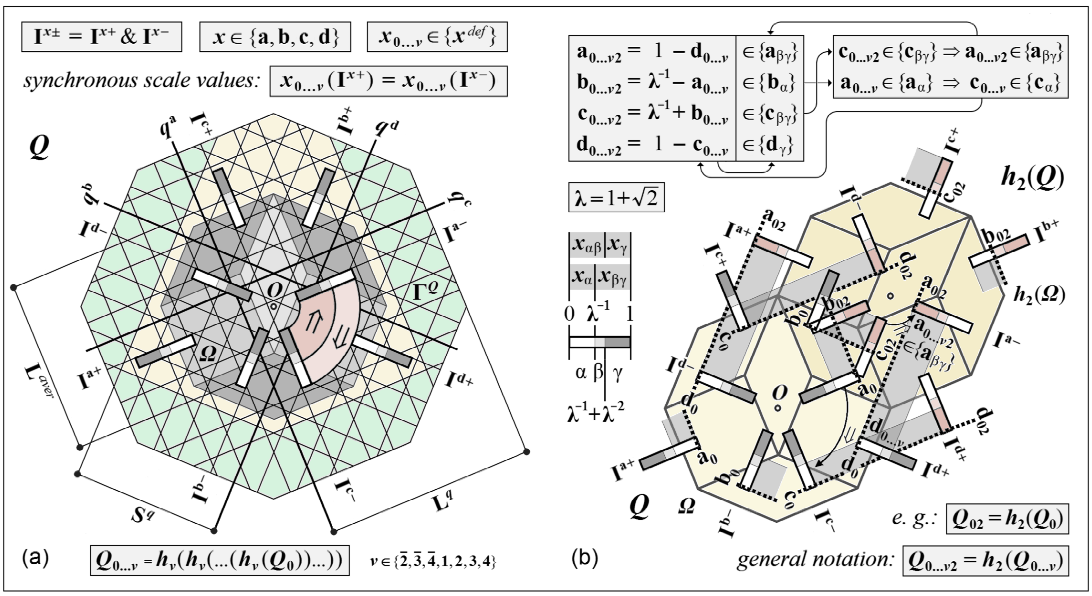

Figure 1. (a) Cluster cell $\boldsymbol{Q}$ with four twin-scales $\mathbf{I}^{x\pm}$, (b) Twin-scale correlation of cluster cells $\boldsymbol{Q}$ and $\boldsymbol{h}_2(\boldsymbol{Q})$.

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3. S. I. Ben Abraham, F. Gähler, *Phys. Rev*., B **60**, (1999), 860-864.