

The Barth Sextic with 30 Cusps

After Wolf Barth had constructed the sextic with the maximum possible number of singularities, 65 (see another surface in this gallery) and two of his Ph.D. students had also constructed new world record surfaces for higher degrees, he started to consider the question on the maximum number of cusps on surfaces of a given degree.

Barth's construction of the sextic with 65 singularities of type A_1^{+-} (double cones) can be adapted to cusps, this yields 30 of them:

$$P_6 - \alpha \cdot K^3 = 0,$$

where P_6 are the same symmetry planes of the icosahedron as for the other Barth Sextic wie bei der anderen Barth Sextik, and where K is again the equation of a unit sphere:



This is the current world record for the maximum number of real cusps on sextics, for complex cusps, it is 36.