

The Cayley Cubic

This cubic surface (surface of degree 3) is also contained in the gallery on simple surfaces. Altogether, it has four double cone singularities. It is named after Arthur Cayley who did a lot of research on cubics in the 19th century.

However, it was Ludwig Schläfli who first classified these surfaces in 1863 in a systematic way with respect to the possible singularities on them. E.g., in his article one can read why there cannot be more than 4 singular points on one cubic surface. This yields: $\mu(3) = 4$.

Around 1900, Felix Klein studied the possible shapes of real cubic surfaces; his idea was to answer this question starting from the Cayley Cubic by means of small deformations: By expanding double cone singularities, disconnecting or merging parts, he was actually able to find all possible shapes; here are a few of them:

