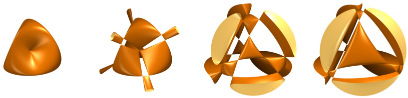
The Kummer Quartic

In 1875, Eduard Kummer was the first person who asked explicitly the question on the maximum number $\mu(d)$ of singularities on a surface of degree d, in the case of surfaces of degree 4, called *quartics*.

He showed that $\mu(4) = 16$. After that he studied quartics with 16 singularities in detail. A particularly beautiful family of such surfaces is given by:

$$\left(x^2 + y^2 + z^2 - \mu^2\right)^2 - \lambda\, y_0\, y_1\, y_2\, y_3,$$

where μ is a free parameter, and $\lambda = \frac{3\mu^2-1}{3-\mu^2}$; the y_i are the sides of a regular tetrahedron $y_0 = 1 - z - \sqrt{2}x$, $y_1 = 1 - z + \sqrt{2}x$, $y_2 = 1 + z + \sqrt{2}y$, $y_3 = 1 + z - \sqrt{2}y$ in order to make the surface symmetric. Not all members of this family have exactly 16 real singularities, although most of them do:



For some special values of the parameters, several of the singularities may coincide.