Calibration of Flood Models – Details and Mathematical Description

Modes of Presentation/Use

The interactive program is intended to be used on a large touch screen monitor. Users should be able to drag around the floodplain with their hands to help with spatial awareness of the terrain.

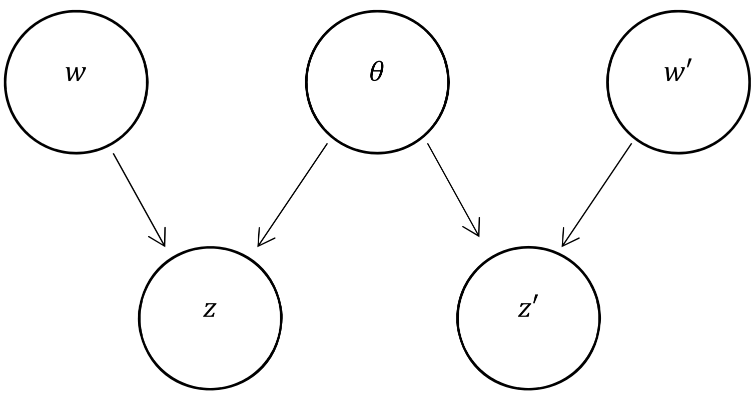
Scientific Content

To model the physical world mathematically we generally need to make assumptions to simplify the real world process into something that we can encode in mathematics and something that a computer can simulate in a finite time.

Floods are a very complicated real world process and we want to be able to predict them very quickly in order to plan flood defences and provide early warnings to those affected. The flow of water is determined by the Navier-Stokes equations, some flood models use full 3D solutions of these equations but these models are far too slow to use in practical situations. Other flood models simplify the equations to 2D or 1D solutions. However the flood model that is actually used by the environment agency in the UK does not solve the Navier-Stokes equations at all but applies a simple model that splits the floodplain into a grid and models the flow between neighbouring cells.

Simplifications make mathematical modelling possible but they also introduce confusion as to what the parameters of the model really mean. In a perfect model where every process of the flood is modelled we might expect that the amount of flood for a given inflow would be dependent on some friction which varies across the floodplain. We could measure that friction across the whole channel and the whole floodplain and supply this spatial parameter to our model. In reality friction is hard to measure and in simplified models a few scalar parameters to account for friction across the whole floodplain.

Let z be a past flood extent that has been observed without error. Let w be the amount of inflow associated with that event. Let θ be the unknown parameters that we need to calibrate and let us assume that these parameters are the same between flood events. Let z'be a future event that we want to predict and let w' be the amount of inflow associated with that event. Then a Directed Acyclic Graph for model calibration is:



From this the posterior probability for the parameters is:

$$p(\theta|z) \propto \int p(z|w,\theta)p(w)p(\theta)\,dw$$

But if we assume no error in the inflow input then this simplifies to:

$$p(\theta|z) \propto p(z|\theta)p(\theta)$$

Given the posterior probability for the parameters in the model we can calculate the probability of a future event given an observation of a past event:

$$p(z'|z) = \int p(z'|\theta)p(\theta|z)\,d\theta$$

This model of calibration has been drastically simplified so we can focus on the calibration of parameters without concerning ourselves with all the other uncertainties inherent in prediction.

However in this simplified form we can see the pattern of calibration:

1. Select some values to try for the parameters (the prior $p(\theta)$).

2. Judge how well the model predicts some past observed event z using different parameter values θ (using the likelihood $p(z|\theta)$).

3. Predict some future event z' using the model with the value of the parameters determined by steps 1 and 2 (the posterior $p(\theta|z)$).

In the activity itself I have not introduced this system for Bayesian calibration but just mentioned it at the end. I think it is better to be able to try a manual calibration exercise by selecting parameter values yourself and comparing the model predictions by eye to the observed event.

Whilst this activity covers flood model calibration is a very important topic throughout environmental modelling.

Explanatory Text

The following text is provided within the interactive program itself, but included here for reference.

On the left is a floodplain. On the floodplain are two villages, one on the hilltop and one in the valley. At the moment the level of the river is normal but there is a flood coming!

In Drag Mode click and drag to look around the floodplain.

A mathematical model has been built to predict the flood. It is a very simple model. As inputs it takes the amount of extra water *w*, and some parameters *a*, *b* and *c*. The output is a prediction of the extent of the flood, it is shown on the floodplain as a red line.

The trouble is we don't know the values of these parameters! In reality the flood will depend on friction which will vary across the whole floodplain. If we tried to model this accurately we would have 1000s of parameters which are too complicated to measure directly.

In our simple model we approximate friction, so our three parameters need to account for all the friction across the floodplain. We have no idea what the values of *a*, *b* and *c* could be, so how can be approximate them?

We can calibrate the model!

Calibrating the model means that we use the model to predict something we already know the answer to. We already know where the river is when the water level is normal so we can adjust the parameters of our model so it predicts this better.

Open the Constant Controller and select *a* from the drop-down menu. Use the arrow buttons to adjust the value of *a*.

Adjust *b* and *c* in the same way until you are happy with the model.

Notice how sometimes the red line goes into the river and sometimes it is outside the river. These correspond to two types of error in prediction. What are they?

Which error do you think it is more important to avoid?

Now that we have calibrated our model we can use these values of *a*, *b* and *c* when we predict the flood.

In the Constant Controller, select *w* from the drop-down menu. Press Play.

The amount of extra water *w* will slowly increase and our model prediction will adjust. In practice we would normally create the prediction before the flood arrives but it is nice to see the two together so we can see how our model performs during the flood.

How did your model perform during the flood?

In the Constant Controller, select *w* from the drop-down menu. Press Stop. Change the value to 0 and adjust the values of your parameters again to see if you can improve your prediction.
Select *w* from the drop-down menu. Press Play.

Is your prediction any better?

You may find that your prediction is better in some areas of the floodplain and worse in others. There is not one correct value for each of the parameters.

In this activity we have covered many different aspects of calibration. In Bayesian calibration we can see the uncertainty in our predictions by making the following changes:

1. We said we had no knowledge of the parameters before using the model to predict an observed event. In practice experts may have some subjective knowledge of what the values should be. This knowledge can be provided as a probability distribution for the parameters called the **Prior**.

2. We simply compared the model prediction to the observed river edge by eye. For rigorous calibration we need to provide a probability distribution which is the **Likelihood** of the observed data given our model output.

3. We used single values of the parameters for prediction. In practice we can use the prior and the likelihood to calculate the probability distribution for the parameters called the **Posterior**. Then we can provide probabilities of flooding across the floodplain.