IMAGINARY
open mathematics

IMAGE GALLERY GUIDE

7ECM
Exhibition
Berlin

1

2

3

4

5

6

7

8

9

10

11

12

13

14

15

16

1 Hecatonicosachoron

Authors: Aurélien Alvarez, Étienne Ghys, and Jos Leys

Also called the “120-cell”, this is a regular polytope in four dimensions. It is the four dimensional analogue of the three dimensional dodecahedron, that has 12 pentagonal faces, 20 vertices and 30 edges. The hecatonicosachoron has 120 ‘faces’, but they are in 4 dimensions, so they are in reality three-dimensional faces: they are all dodecahedrons! The two-dimensional faces of these dodecahedrons are of course pentagons, and there are 720 of them. There are 600 vertices and 1200 edges.

2 Wild Knot

Author: Aubin Arroyo

A Wild Knot is a circular curve in three-dimensional space which is infinitely knotted. It can be produced as follows:
If we string together a certain number of spheres into a knot we would have a true knotted necklace. If we also assume that all the spheres in the necklace are perfectly reflective, as if they were made of mirror, we could observe an image of the whole necklace in each one of the spheres, conforming a new necklace of smaller spheres. This new necklace is also knotted. In fact, it reproduces the original knot in each sphere, and so on. The repetition of this procedure infinitely many times yields a Wild Knot.

3 The Lorenz attractor

Author: Jos Leys

The Lorenz’ atmospheric model is what physicists use to call a toy model: although it is so oversimplified that it does not have much to do with reality anymore, Lorenz soon realized that this model was very interesting. If we consider two almost identical atmospheres (two points that are extremely close in Lorenz’ model), we tend to quickly see the separation of the two evolutions in a significative way: the two atmospheres become completely different. Lorenz saw on his model the sensitive dependence on initial conditions: chaos. Moreover, what is very interesting is that, starting from a large number of virtual atmospheres, even if they follow paths that seem a little bit crazy and unpredictable, they all accumulate on the same object shaped like a butterfly, a strange attractor.

4 Sea-Invader

Author: Torsten Stier

This image is based on the so-called Mandelbox, a fractal with a box-like shape found by Tom Lowe in 2010. It is defined in a similar way to the famous Mandelbrot set (see image of the Mandelbrot set by Aubin Arroyo) but, unlike the Mandelbrot set, can be defined in any number of dimensions.

5 Anosov Flow

Authors: Aurélien Alvarez, Étienne Ghys, and Jos Leys

Dynamics is the study of motion. Topology is the study of shapes. Arithmetic is the study of Numbers. Sometimes, concepts coming from arithmetic have a dynamical aspect and create interesting shapes! In this picture, you see shapes produced by the so-called “modular flow” which is fundamental to understand numbers, and in particular prime numbers.

6 The Mandelbrot Set

Author: Aubin Arroyo

The Mandelbrot Set is a well known set of complex numbers c for which the sequence $z_{n+1} \rightarrow z_n^2 + c$ remains bounded in absolute value.

The boundary of the Mandelbrot set is a fractal. You can find smaller versions of the main shape when increasing magnification. The dimension of a fractal does not have to be an integer, hence the word ‘fractal’ or latin ‘fractus’, which means ‘broken’. The definition and name of the Mandelbrot set are due to Adrian Douady, in tribute to Benoît Mandelbrot.

7 Slice of Boy

Author: Bianca Violet

Imagine sewing the edge of a Möbius strip, which is just one closed curve, to the boundary of a disk! The resulting closed surface still contains the Möbius strip and will therefore be non-orientable (you cannot tell what is the outside or the inside - it is all the same). That is why it must have self-intersections in our real three-dimensional space. But can it be done smoothly with no sharp or pointy edges? The famous mathematician David Hilbert assigned his student Werner Boy to prove that it is impossible. To the surprise of his teacher, Boy constructed such a smooth surface in 1901. He described it by giving the corresponding curves of its intersection with a family of parallel planes. Now in this picture, you see a slice of two Boy surfaces (one being the mirror image of the other), cut not by planes but by two spheres with the same center and of slightly different radii.

The equation used for this visualization of the Boy surface was found by François Apéry.

8 Assembly

Author: Arnoud Cheritat

It is possible to make a smooth model of Boy’s surface with simple primitives using Constructive Solid Geometry (CSG). Smooth means, there is a tangent everywhere, and it varies continuously with the point. All pieces are parts of either planes, cylinders, tori, or a sphere, that have been cut by intersecting with planes or cylinders. They can be assembled to make a model of Boy’s surface. The blue piece is part of a sphere, cut by cylinders and rectangles. It is quite a miracle that it smoothly fits to close the surface.

9 Animalistic Quasiperiodic Round Dance

Author: Uli Gaenshirt

In a quasiperiodical nuclear structure with a detectable fivefold rotational symmetry the centers of symmetry must be evenly distributed allover the structure and there must exist centers of higher order. Optimized quasiperiodical models - like the Penrose tilings - are hierarchical systems.
The beaks of the red and yellow colored turtles are exactly placed at the corners of a regular decagon. Three of them are centers of higher order. The directions of rotation alternate consequently.

10 Woman Teaching Aperiodic Geometry

Author: Uli Gaenshirt

The underlying structure of this image is a quasiperiodic (aperiodic) rhombic Penrose tiling, a geometric structure used for modeling decagonal quasicrystals.
The shape of the yellow colored area has a tenfold rotational symmetry although its center is dissymmetric. This array is commonly called a cartwheel. By taking up 35 different positions the young woman demonstrates what is meant by doing a cartwheel. As an allusion to the difficult order of a spatial rhombohedral Penrose tiling she turns 15 times her back on us.

11 Quasicrystalline Wickerwork

Author: Uli Gaenshirt

Even though the wickerwork decoration in the foreground of the large graphic was designed by medieval girih tiles (Persian girih, Eng. knot) it corresponds to the atomic structure of a decagonal quasicrystal.
Adapting the girih tiles to a modern rhombic Penrose tiling they are generating a girih wickerwork with a good approximation to a tenfold rotational symmetry. Surprisingly the created knots correspond to the underlying geometry of a today used covering model.

12 Girih Cartwheel

Author: Uli Gaenshirt

This wickerwork pattern is assembled only from two types of girih tiles, irregular hexagons and trapeziums (compare the tiles of the Girih Puzzle on the last page). Their aperiodic matching rules are defined by six different colors in their corner regions.
Inside the decagon with the center point C and the top corner T the same-color corner regions are

arranged to pentagons and twin pentagons with an equivalence relation to the order of a Penrose cartwheel.
The extension of this structure to the complete image area is made by four overlaps with copies of the central cartwheel. The rhombs PLVR clarify the positions of the five cartwheels.
If a red, blue or yellow colored pentagon or twin pentagon is reflected in one of the radial black lines, then the color of the mirror image is green, orange or violet.

13 Octic

Author: Torolf Sauermann

This is the central part of an algebraic surface of degree eight – an octic. It was visualized using the SURFER program available for free at www.imaginary.org/pogram/surfer. You can clearly see the octagonal symmetry in the picture.
Its formula is:
$(x^2-1)\cdot(y^2-1)\cdot(x^4-2\cdot x^2\cdot(y^2+2)+y^4-4\cdot y^2+4)-(z+1)\cdot(z^4-5\cdot z^3+6\cdot z^2+z-2)^2=0$

14 Togliatti Quintic

Author: Oliver Labs

Eugenio Giuseppe Togliatti proved in 1937 that an algebraic surface of degree 5 (quintic) with exactly 31 singularities exists - a world record at that time. In 1980 it was Arneau Beauville who used an interesting relation to coding theory in order to show the non-existence of a quintic with more singularities. This means that Togliatti's world record can never be improved!

The equation used for this visualization was found by Wolf Barth (1995).

15 Vis à Vis

Author: Herwig Hauser

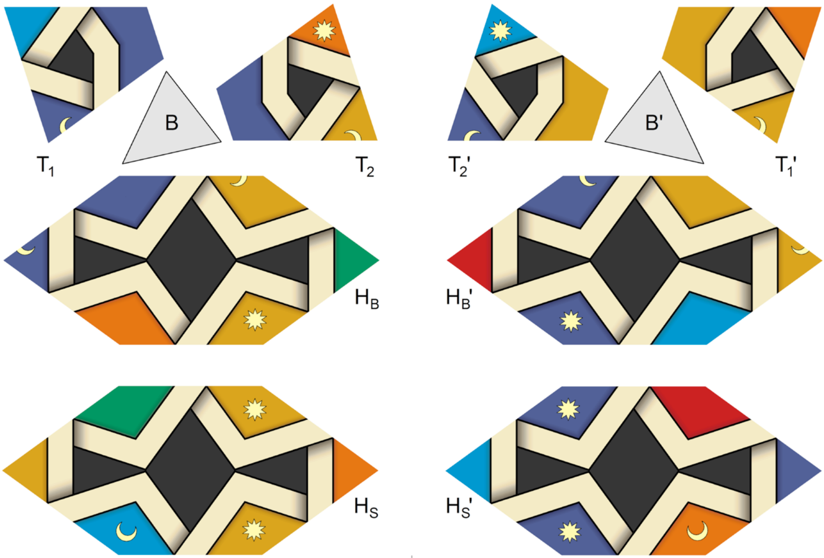
Vis à Vis means opposite – and, here, two essential phenomena of algebraic geometry stand opposite each other. The singular tip on the left looks at a curved but smooth hill on the right. This singularity is more exciting, because various changes to the equation can result in unpredictable changes to the figure, which does not happen at smooth points. The comparison of form and formula, i. e. of equation and corresponding surface, becomes an interactive experience which is intriguing to understand.

16 Ding Dong

Author: Herwig Hauser

This surface described by the equation $x^2+y^2+z^3 = z^2$ was one of the very first visualizations we tried. Equation and shape are simple: A vertical alphaloop rotates around the z-axis. But there was the problem with the colouring. Green is generally rather tricky in three-dimensional visualization of surfaces and, in addition, tends to be matt or yellowish. The lights and reflections must be well tested. Note the light blue hard shadow intensifying the spatial effect.

The image prints for this gallery are provided by KOMM-Bildungsbereich, Nuremberg, Germany.
Thank you!



Quasiperiodic Girih Tiling Puzzle

The girih tiles at the top correspond to historical Islamic archetypes. They give the eight prototiles of a geometric tiling puzzle (H = Hexagon, T = Trapezium). The internal angles are 72°, 108° and 144°.
Especially elaborated matching rules generate – if the tiles are adjoined in accordance to the rules – a quasiperiodical wickerwork decoration with an equivalent relation to a modern Penrose tiling.

Matching rules:

- *The edge-to-edge order of the tiles goes along with a corner-to-corner condition.*
- *Only same-color regions are allowed to be placed edge-to-edge.*
- *The half-devided crescents must always be completed to regular crescents.*
- *The single-colored pentagons and twin pentagons which occure in larger arrangements are restricted to contain at most one sun and one crescent each.*

Author	Uli Gaenshirt	email: uli.gaenshirt@yahoo.de

Mathematisches Forschungsinstitut Oberwolfach

Klaus Tschira Stiftung gemeinnützige GmbH

KTS