52 Grains of Sand - Geometry of Nature.

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Background

The following portfolio is a selection of some of the 150 visualizations available from the *"52 grains of sand – Geometry of Nature*" project [01] I started Jan 1st, 2017 and to be completed by December 31st. (365 visualizations by Dec 31s.t)

The idea for this project originated in a short educational program on the geometry of minerals published by the Space Foundation [02] that aimed to connect geometry, science and space exploration to investigate the mathematical properties of shapes, designs, and structures of existing minerals.

I have integrated geometry in my work for many years, both in the classroom and for my personal research and I was already familiar with some principles of symmetry used by gemologists to identify minerals from a prior project on symmetry [03]. I chose the particular - one image a day - format because I have conducted several mathematics and art projects this way and appreciate this strict framework that requires a modicum of logistical foresight and good planning.

Project Title:

The title of this series outlines the goal of this project: a visual tour of the geometry of 52 minerals over a 52-week period, one mineral a week, one image a day per mineral. Sand, or silicate, is a naturally occurring granular material that constitutes most of the Earth crust and is composed of finely divided quartz mineral particles. In recent years, sand has become a global planetary concern because man-made activities are quickly depleting this unique natural resource. In nature, all minerals are identified by a unique geometry that helps define them in two, three and higher dimensions [04]. Their unique system of classification provides an extensive base of mathematical exploration of three-dimensional shapes, platonic solids, and principles of symmetry.

- Goal

- Explore, investigate and get a better understanding of the synergy between abstract mathematical concepts and the real world,
- Gain insight and a new perspective on three-dimensional geometries in a unique context.
- Evaluate the dynamic of a scientific construct in a two-dimensional graphic environment
- Compile a knowledge-based resource that can lead to future educational and cultural output

Objective

- Test a 3D modeling program developed to study crystal morphologies.

- Test and compare the geometrical arrangements and the various outcomes to find analogies between scientific visualization and the graphic environment that can eventually enrich the discussion on perception and aesthetic in the field of visual communication.
- Compile written and visual information that can lead to future educational output.

A few years ago, I conducted a somehow similar project on a much smaller scale based on the 32 Crystallographic Point groups [05]. The outcome resulted on a successful science and art cursus I prepared for my Visual Communication classes, a book including the complete illustration series and several lectures on the topic of symmetry, technology and mathematics and the arts.

I am saving all the files produced for this project in large size, high-resolution format to allow for large scale wall display prints, multimedia HD presentations for deployment on the Internet or other public venues. In addition, a volume including the 365 illustrations and accompanying notes will be available for printing at the end of the project as well as potential educational material.

Resources

The Department of Geosciences at the University of Arizona provided me the resource to explore the various minerals according to their geometry. The department library catalogs and maintains an extensive archive of high-quality spectral data from well-characterized minerals. The data provides mineralogists, geoscientists, gemologists a standard for the identification of minerals both on earth and for planetary exploration. It contains at this day 4,714 individual mineral species descriptions [06] that have been published in professional journals such as Zeitschrift für Kristallographie, American Mineralogist, The Canadian Mineralogist, Journal of Materials Science, European Journal of Mineralogy, Journal of the American Chemical Society, and many other.

To visualize the mineral information in a scientific context, I use a program named VESTA, a 3D visualization program for structural models, volumetric data such as electron/nuclear densities, and crystal morphologies. VESTA originated from two windowing application programming interface. VICS and VEND, and was developed by R. A. Dilanian and F. Izumi in the early 2000s. Today, Koichi Momma of the Japanese National Museum of Nature and Science in the Ibaraki prefecture continues to maintain and upgrade the program [07].

Mr. Momma sent me his support when I submitted the outline of the project to the NIMS. I also contacted Professor Downs at the UofA to inform him I had selected the RRUFF library to conduct my research. I keep them both regularly informed of its development.

Finally, to complete the requirement of the project, I migrate the resulting images into what constitutes a Visual Communication lab and use various 2D and 3D image editing programs to develop visualizations of a more artistic nature. Different types of software create different types of graphics depending on the needs of the user and the intuitiveness of the interface.

Practices.

The overall logistic of the project came from the experience I had gathered conducting similar projects. In my Visual Communication program, I conducted several classes that linked scientific software and the graphic editing environment to improve learning practices [08]. In addition, as a researcher in mathematics and art collaborative projects, I learned about the challenges of good planning, time management, and the challenge of working in various time zones at the time as it happens with project conducted over a year time [09].

In Geometry, lines and curves are the elements of complex figures, such as circles, polygons, and angles. Similarly, graphic designers across the world use these elements to create images. Using this method results in a well-balanced, consistent piece of artwork. [10]

Visual communication uses the tools of the digital environment to convey non-verbal messages based on principle and techniques of art and graphic design. Furthermore, good design is based on sound geometry. It has been proven that non-verbal communication is improved when fundamental of 2D and 3D geometry are used to convey a message [11]

Nonverbal communication strategy supports the comprehending of mathematical concepts or problems to be solved. [12]

Development

All known geometrical solids can be found in minerals. The fact is that all minerals are classified into crystal systems defined by their polyhedral shape. Crystal systems contain complete geometric classes of space groups or symmetries. For instance, oblique, rectangular, square, hexagonal systems in two-dimension and in three-dimension, cubic, hexagonal, monoclinic, triclinic, orthorhombic, tetragonal, and trigonal systems.

My first problem was to define a methodology to include in a coherent manner a collection of minerals I could carry over 52 weeks. According to the Mineralogical Society of America, approximately 3,800 minerals have been recorded to this day and 30 to 50 get added annually. Following the 32 point groups symmetry order was my first choice but it would leave me short of 20 weeks. Provided that this project did not need to follow the rigorous format of a scientific experiment, I decided to insert in my selection process an additional mathematical element: randomness. It would be consistent with a large number of data available on the subject and be fitting with the title of the project as well. Sand is so numerous in its granular form how can we ever calculate the total number of grains on any chosen beach?

However, I was lucky to welcome onboard early in the process gemology experts Noreen Masaki and Hubert Heldner. They guided my mineral selection based on criteria that could meet my mathematical expectations and aided me to discover a fascinating side of our planet's geological make-up. Working close to them brought me the additional benefit of being able compare my results with the actual gems that they hold in their extensive collection. I am indebted to them for their patience, comprehension and good advice.

I did not always respect the physical size, dimension or positioning of the atoms, binding or polyhedral form of the crystal for reasons having to do with the dynamic of each particular design, sometimes preferring to privilege one over the other to highlight a particular point or an unexpected dynamic in the original shapes.

Future development

It is early to draw any conclusion in regards to the wealth of information I'm gathering as I progress on my task. With regards to the RRUFF resource and the VESTA software, both are adequate to extract inspiring geometry out of the original mineral structure and sustain the objective of the project.

However, I already found some strange but very real connection between the geometry of some crystals and the local culture associated with the mineral origin, its color scheme or geometry patterns.

Indeed, it would concur with ethnomathematician Paulus Gerdes findings in cataloging such similarities between Euclidian geometry and traditional textile pattern [13].

One unanticipated aspect of this project has been its artistic component that seems to resonate with many people that have been following my progress on the project's site and other social media platforms updated daily or weekly.

Inasmuch as Art has been associated with all the most significant expression of human spiritual and intellectual activities since the dawn of civilization, it highlights the positive aspect of art in communicating concerns consequential for all today. In the word of geoscientist E.C. Koster: *"Science alone cannot show us in the same manner: beauty, wonder, and unpredictability. Which is why we always need more Art in Science"* [14].

This project is also bringing to the forefront the educational potential of such an interdisciplinary approach to issues dealing with a scientific concept, social concerns, and esthetics explorations. Several multidisciplinary initiatives in that direction already attest of the shared interest in its potential to prepare students well to meet the challenges of tomorrow [15, 16, 17].

Conclusion

With the recent development of scientific visualization software, scientists and mathematicians access sophisticated design tools that provide the means to create good a visual composition. Many are already producing works of outstanding aesthetic quality. The new electronic environment also gives artists access to information they would not be otherwise trained to understand or able to compose with. Today, many among them integrate scientific information in their work, either as a source of inspiration or for practical and aesthetic purpose.

Trained as an artist and a designer, I do not claim to have the background that qualifies me to investigate in depth scientific phenomena of an abstract nature. However, because of the particular of my profession, I am able to discern the exceptional value of resources and patterns I would not otherwise be able to put into words or images.

Geometry is a large field of science into itself, so is mineralogy. Art and Visual Communication are means and techniques to communicate expertly to the largest possible audience. The first two originate in the observation of the world as it is, the latter communicate how we see the world. A joint effort between the two disciplines may be relevant and useful to a conversation that affects all interested in the betterment of our circumstances and will affect many future decisions relating to it.

References

[01] Jean Constant.*52 grains of sand*. In progress. http:jconstantblog.wordpress.com Retrieved on 06.2017.

[02] Space Foundation. https://www.spacefoundation.org/ Retrieved on 06.2017.

[03] Jean Constant. *Symmetry in Mathematics, Physics and Art*. Journals of Symmetry - Culture and Science. Volume 24. 2013.

[04] Geballe Laboratory for Advanced Materials. *Geometry of Crystals*. https://web.stanford.edu/group/glam/xlab/ Retrieved on 06.2017.

[05] Jean Constant. *Crystallographism*. BLB publishers. 2015.

[06] RRUFF Project, Department of Geosciences. University of Arizona http://rruff.geo.arizona.edu/AMS/amcsd.php Retrieved on 06.2017.

[07] VESTA. *Visualization for Electronic and structural analysis*. JP-Minerals http://jp-minerals.org/vesta/en/ Retrieved on 06.2017.

[08] Jean Constant. *Digital Sangaku*. Tessellation publishing .2012.

[09] Jean Constant *The 12-30 Project*. Blb publishers. 2015.

[10] Martin Kemp. *The Nature book of Art and Science*. U. of California Press. 2000.

[11] Haug Leuschner. *Microanalysis of nonverbal communication*. Koln University. 2010

[12] Daniel Clark. *Geometry in visual communication. PREZI.* 2014. https://prezi.com/o2vkvikwnxv_/copy-of-copy-of-geometry-in-visual-communication/

Retrieved on 06.2017.

[13] Paulus Gerdes. *Geometry from Africa: Mathematical and Educational Explorations*. The Mathematical Association of America. 1999.

[14] EC. Kosters. *The graphic artist M.C. Escher and his connections with geology*. Earth Science Society. 2014. https://earthsciencesociety.com Retrieved on 06.2017.

[15] Jawaan Burge. *Communicating through geometry*. UIC School of Design. 2013.

[16] EPSRC. *Centre for Doctoral Training in Science and Engineering in Arts, Heritage and Archaeology*. http://gow.epsrc.ac.uk/ Retrieved on 06.2017.

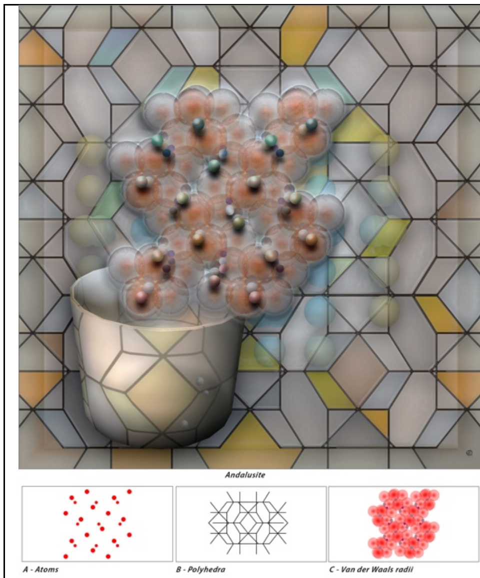
[17] MPE2013. *Mathematics of Planet Earth*. http://mpe.dimacs.rutgers.edu/ Retrieved on 06.2017.

Portfolio

52 Grains of Sand - Geometry of Nature.

https://jconstantblog.wordpress.com/

- Selection: Jan.-June, 2017 ***- File format: 2400x2400 wide, 300 DPI*** **The minerals have been selected* according to their 3D geometry, their class or the unique aspect of their symmetry.
Andalusite **Orthorhombic system. Class dipyramidal. Symmetry Pnnm.**

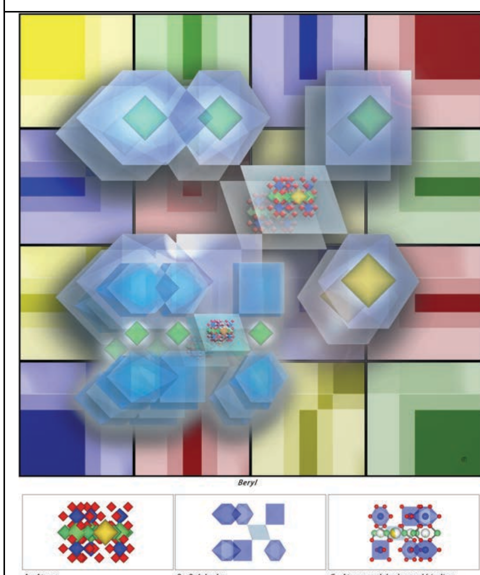


The mineral Andalusite belongs to a mathematically unique space group, centrosymmetric, and orthorhombic – a three-dimensional geometric arrangement having three unequal axes at right angles.
The symmetry elements of these operations all pass through a single point of the object. The rules that govern symmetry are found in the mathematics of group theory. Group theory addresses the way in which a certain collection of mathematical "objects" are related to each other. Such groups always have the following properties: identity, inverse, associativity, closure and commutativity. [1].
I used the grid-like pattern of the binding between vertices to make it look like a stainless window lead came.
The atoms are rendered at their default atomic radii
The larger spheres depict the outer surfaces of the atoms. Each sphere is represented as though it were a hollow shell with numerous dots. Their visualization is useful to gemologists to calculate distances and understand how atoms are combined with each other in a molecule.

3D model: 214 atoms/vertices, 288 bonds, 60 polyhedra.

[1] Robert B. Von Dreele. *X-Ray and Neutron Crystallography*. Los Alamos Science Summer. 1990.

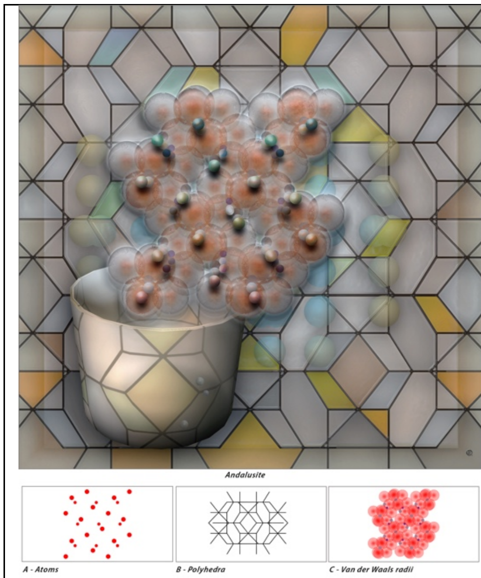
Beryl
Hexagonal –system. Class dihexagonal, dipyramidal. Symmetry *P6/mcc*



I created this visualization after the diagram of a beryl unit cell.
Beryl is a relatively rare hexagonal– a six sided polygon silicate mineral. The name Beryl comes from the ancient Greek *beryllos*. It means "precious blue-green" color of "sea water" stone.
Its structure is dihexagonal, dipyramidal and of symmetry P-P 6/m c c – two equal axes at an angle of 120° and a height perpendicular to the two base axes.
In Elements (proposition IV.15), Euclid showed how to inscribe a regular hexagon in a circle. To construct a regular hexagon with a compass and straightedge, draw an initial circle A. Picking any point on the circle as the center, draw another circle B of the same radius. From the two points of intersection, draw circles C and D. Finally, draw E centered on the intersection of circles A and C. The six circle-circle intersections then determine the vertices of a regular hexagon [1].
The most famous natural hexagonal structure on Earth is The Giant's Causeway on the northern coast of Ireland, an area of about 40,000 interlocking basalt columns, declared World Heritage Site by UNESCO in 1986.
I used a 4 color scheme after the famous 4 color theorem to define the structure of this elegant crystal coming from Val d'Ossola, Italy, and composed of 55 atoms, 54 bonds, 7 polyhedra.

[1] Eric W Weisstein. *"Hexagon."* From MathWorld--A Wolfram Web Resource.

Chrysoberyl
Orthorhombic system. Class dipyramidal. Symmetry Pbnm.



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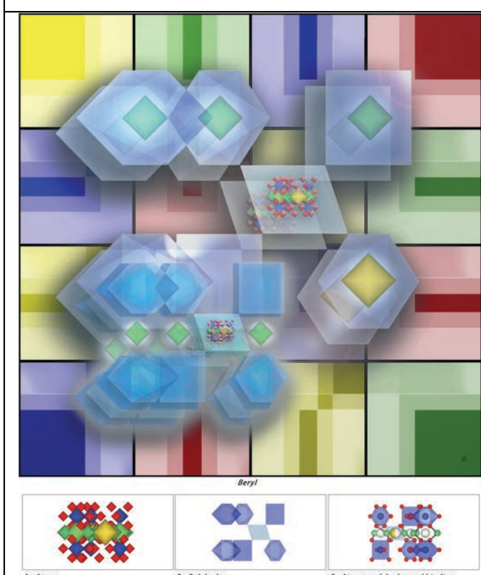
[1] Robert B. Von Dreele. *X-Ray and Neutron Crystallography*. Los Alamos Science Summer. 1990.

Beryl

Hexagonal –system. Class dihexagonal, dipyramidal. Symmetry *P6/mcc*



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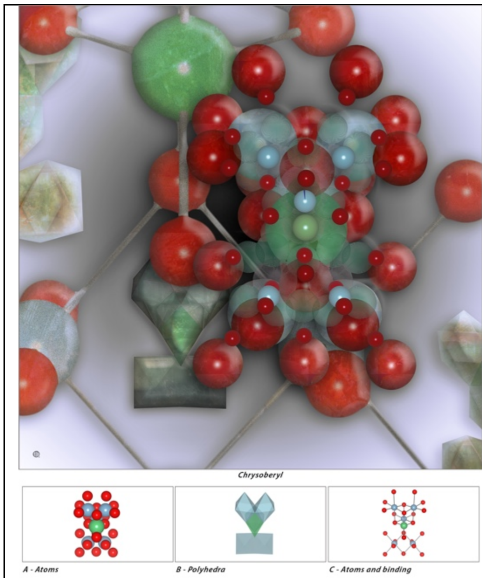
[1] Eric W Weisstein. *"Hexagon."* From MathWorld--A Wolfram Web Resource.

Chrysoberyl

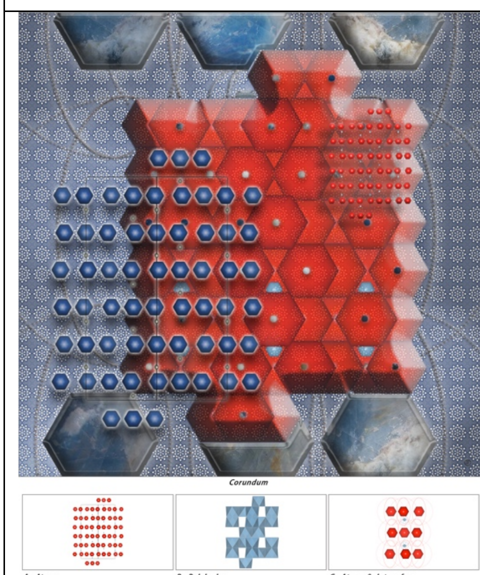
Orthorhombic system. Class dipyramidal. Symmetry Pbnm.



Chrysoberyl is an extremely hard, crystal that belongs to the orthorhombic system. Orthorhombic space groups are divided in 3 main groups because of the large number of ways in which twofold rotation, two-one screw axes, mirror and glide planes can interact together in three dimensions.
The crystal shape is dipyramidal and characterized by the hexagonal system having a vertical hexad axis, six horizontal diad axes, six vertical planes, and a horizontal plane of symmetry.
Chrysoberyl crystals are often distinctively twinned, either in V-shaped repeated twins or in snowflake-shaped "trillings" where three individual crystals join to form a unique pattern.

3D model: 29 atoms, 34 bonds, 6 polyhedra.

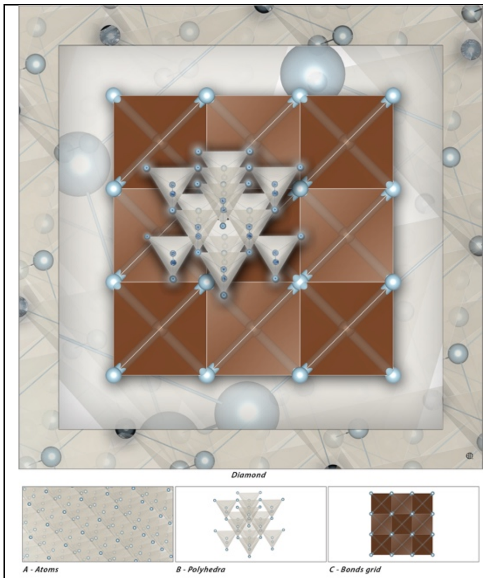
Corundum
Trigonal system. Class hexagonal, scalenohedral. Symmetry R3c.



Corundum is one of the hardest minerals in Nature.
Its hexagonal geometry makes for a simple well-balanced structure. The crystal looks like a bipyramidal hexagon wider in the center and tapers thinly on the ends. It belongs to the Trigonal system and has a symmetry of R -3 c, arranged in rhombohedral and hexagonal axes Its name derives from the Sanskrit word *kurundum* or *korund*, meaning "hard stone". Corundum gem varieties such as ruby and sapphire are present in many places from industrial bearings, scratch-resistant windows for electronic instruments, or wafers for circuit boards. Synthetic corundum is an essential part of many lasers.
The mineral can be found in many parts of the world in places like Myanmar, Cambodia, Sri Lanka, India, Afghanistan, and more recently in several parts of Africa, including Madagascar, Kenya, Tanzania, among other.

3D model: 102 atoms, 144 bonds, 24 polyhedra.

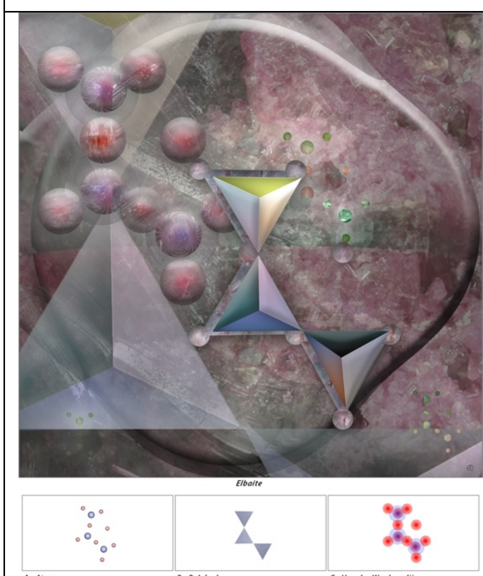
Diamond
Cubic isometric System. Class hexoctahedral. Symmetry Fd3m.



The mineral diamond is appreciated for many reasons.
Known and valued since antiquity, the diamond crystal belongs to the cubic system and has the shape of an octahedron–a polyhedron with eight faces, twelve edges, and six vertices.
A regular octahedron is a Platonic solid composed of eight equilateral triangles, four of which meet at each vertex. The octahedron is unique among the Platonic solids in having an even number of faces meeting at each vertex. It is the only member of that group to possess mirror planes that do not pass through any of the faces.
In mathematics, the rhombus is sometimes called a diamond, after the diamonds suit in playing cards which resembles the projection of an octahedral diamond, or a lozenge.

3D model: 865 vertices, 1460 bonds, 365 polyhedra.

Elbaite
Trigonal system. Class ditrigonal, pyramidal. Symmetry R3m.

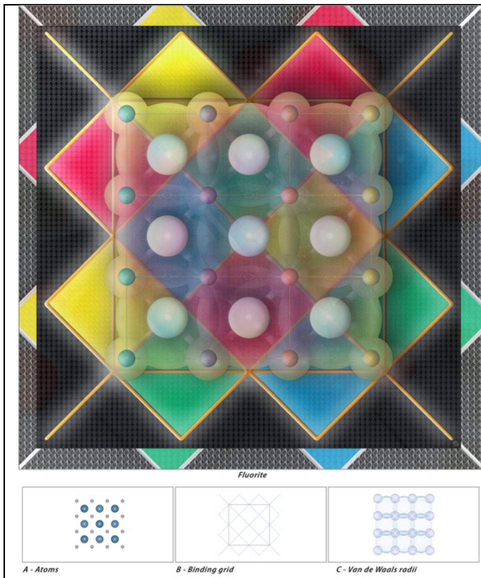


Elbaite is one of the 27 members of the tourmaline mineral group.
Elbaite and other tourmaline minerals crystallize in the trigonal system, a subsystem of the hexagonal system with three-fold symmetry and four crystal axes. Its six-sided geometry is modified in the cross section of the crystals, and three alternating prism edges are rounded to resemble a triangle [1]. Many green and blue specimens are strongly pleochroic. When viewed through their vertical axis, such specimens appear darker in color than when seen through their horizontal axis. Certain Elbaites exhibit a cat's eye effect when polished.
Tourmaline was first brought to Europe by Dutch traders in the 1700s. They gave it the name, Aschentrekker, or "ash puller," They used the crystals to pull ashes from their Meerschaum pipes. This ability to gain magnetic powers and become electrically "polar" by means of heat readily distinguished Tourmaline from any other gem. It is doubtful this custom is still practiced today considering its value.

3D model: 14 atoms, 12 bonds, 3 polyhedra.

[1] Stephen M. Voynick. *Colorado Rockhounding.* Mountain Press Publishing 1994.

Fluorite
Isometric system. Class hexoctahedral. Symmetry Fm3m.



Fluorite is an isometric, octahedral, colorless, soft, and almost translucent crystal.

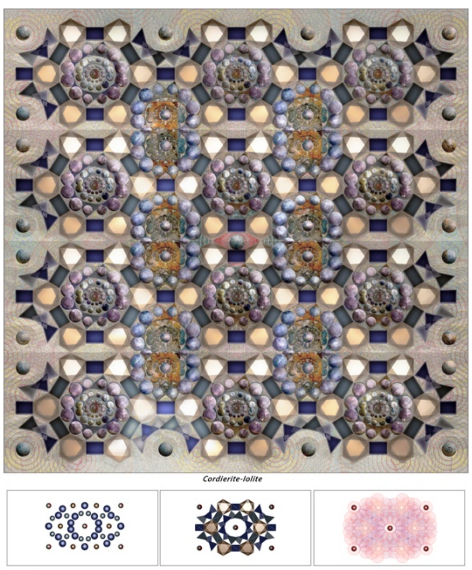
Its hexoctahedral structure–48 equal triangular faces, makes of this mineral one of the most symmetrical geometric solid found in three-dimensional space.

The geometry of the fluorite has been known since antiquity. Archaeologists have uncovered artifacts of carved fluorite in the ruins of Pompeii. A fluorite carving from the Mississippi Mound Builders era found in a burial pit in Illinois near the Ohio River was dated between 900 and 1650 A.D.

Today fluorite is used in some high-performance telescopes and camera lens elements due to its strong hexagonal crystal structure that refracts light evenly. Fluorite also serves to correct optical aberrations in lenses.

3D model: 78 atoms, 112 bonds, 14 polyhedra.

Iolite-Cordierite
Orthorhombic system. Class dipyramidal. Symmetry Cccm.



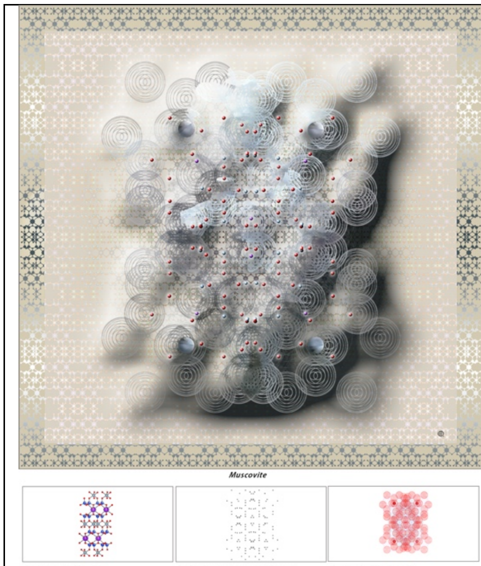
Cordierite is a tetrahedral framework silicate that belongs to the dipyramidal, orthorhombic family. It is pseudohexagonal, meaning its geometry is slightly deformed when compared to higher symmetry. Its mesh has a geometry more regular than required by the symmetry of the pattern [1]

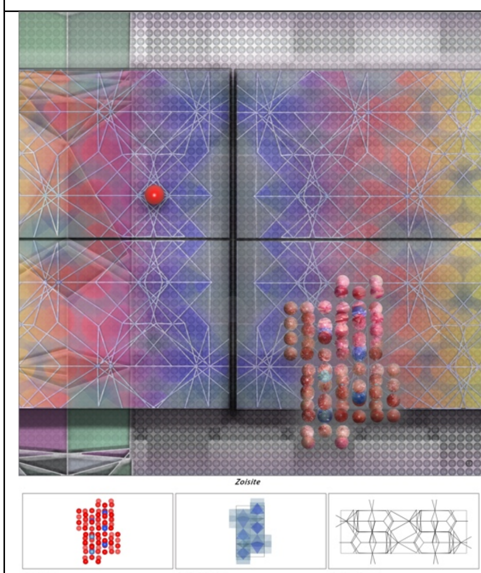
When the mineral cordierite is transparent and of gem quality, it is known as "iolite". The name "iolite" comes from the Greek word for violet and it comes in blue, smoky blue, bluish violet; greenish, yellowish brown, gray; colorless to very pale blue in thin section. It is also called "water-sapphire" or "Vikings' Compass". Leif Eriksson and other Viking explorers used it to determine the direction of the sun on overcast days. It may qualify as the world's first polarizing filter.

226 atoms, 280 bonds, 64 polyhedra.

[1] István Hargittai (Editor). *Symmetry: Unifying Human Understanding*. Pergamon Press. 1986.

Muscovite
Monoclinic system. Class sphenoidal, prismatic. Symmetry B2, C2/c.

<table>
<tr>
<td>
Muscovite
A - Atoms and binding B - Binding tips, top positioning C - Van der Waals radii</td>
<td>The name muscovite comes from Muscovy-glass, a name given to this mineral in Elizabethan England due to its use in medieval Russia as an alternative to glass in windows.
It is a transparent monoclinic crystal where two vectors are perpendicular to each other, and the third vector meets the other two at an angle other than 90°.
The layer structure repeats itself and can come in enormous crystal groupings that can weigh several hundred pounds. Today it is still used as an insulator for various electrical products and semiconductors.

3D model: 130 atoms, 148 bonds, 30 polyhedra.</td>
</tr>
<tr>
<td colspan="2">Zoisite-Tanzanite
Orthorhombic System. Class dipyramidal. Symmetry Pnma.</td>
</tr>
<tr>
<td>
Zoisite
A - Atoms B - Polyhedra C - Wireframe</td>
<td>Tanzanite is a subset of the mineral zoisite. It is part of the orthorhombic group–a three-dimensional geometric arrangement having three unequal axes at right angle.
The Tanzanite crystal has a dipyramidal, Pnma symmetry. It is centrosymmetric with 8 points of inversion per unit cell. Along each of the unit-cell directions, there are 4 two-one screw axes per unit cell. Perpendicular to each of the unit cell axes there are two planes per unit cell.
The vertices in this composition show the atomic arrangement of the crystal. The mesh in the background reflects the binding between the vertices in a 3 dimensional model.

3D model: 124 atoms, 176 bonds, 32 polyhedra.</td>
</tr>
</table>

* Minerals technical descriptive:**Mindat.org.**

Mindat.org is an outreach project of the Hudson Institute of Mineralogy. The Hudson Institute of Mineralogy is a not-for-profit research, cultural and educational entity chartered by the Board of Regents of the University of the State of New York in 2003. The institute is dedicated to the discovery, study and preservation of mineral species and their history, and to increasing public awareness and appreciation of the mineral kingdom through outreach and education.