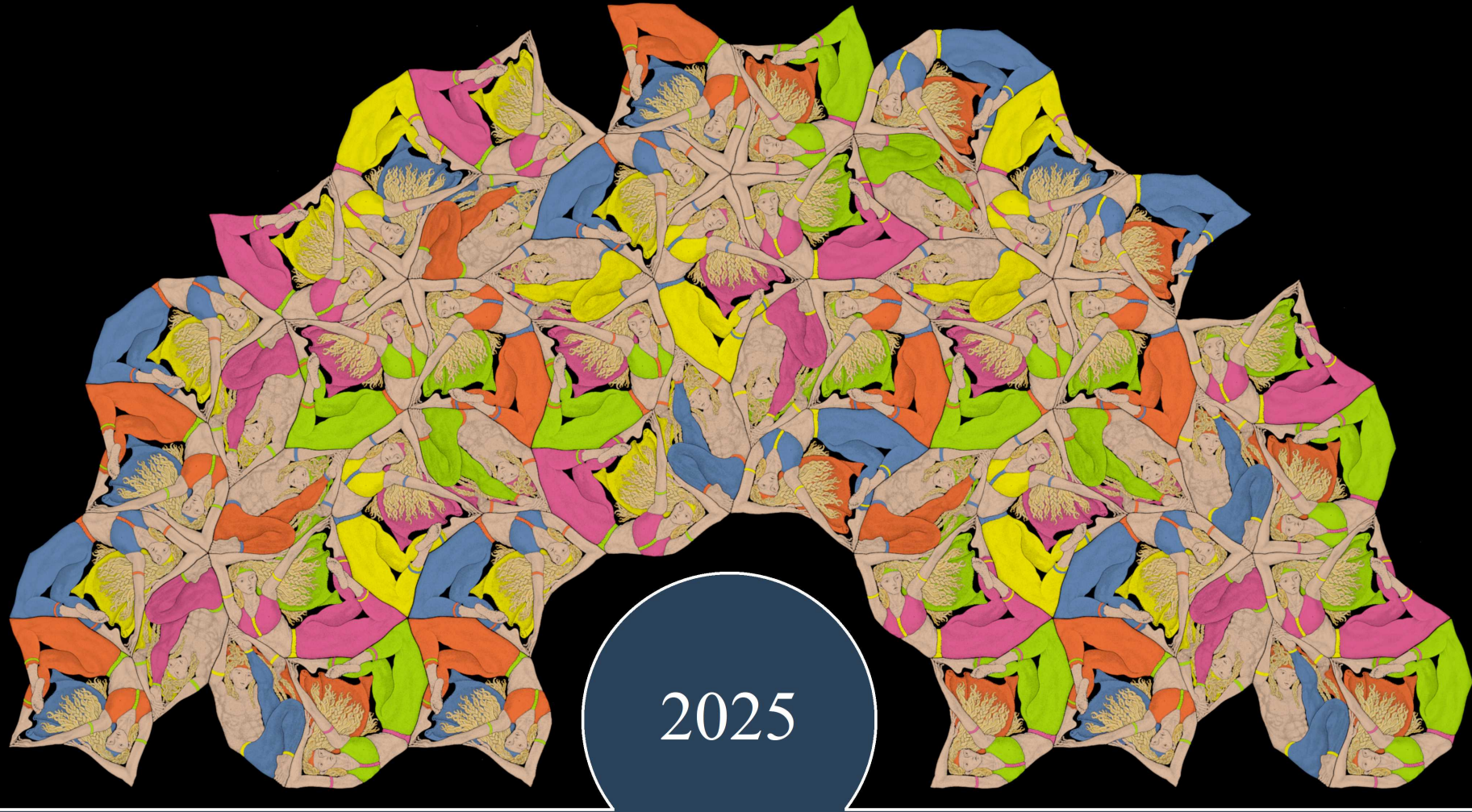


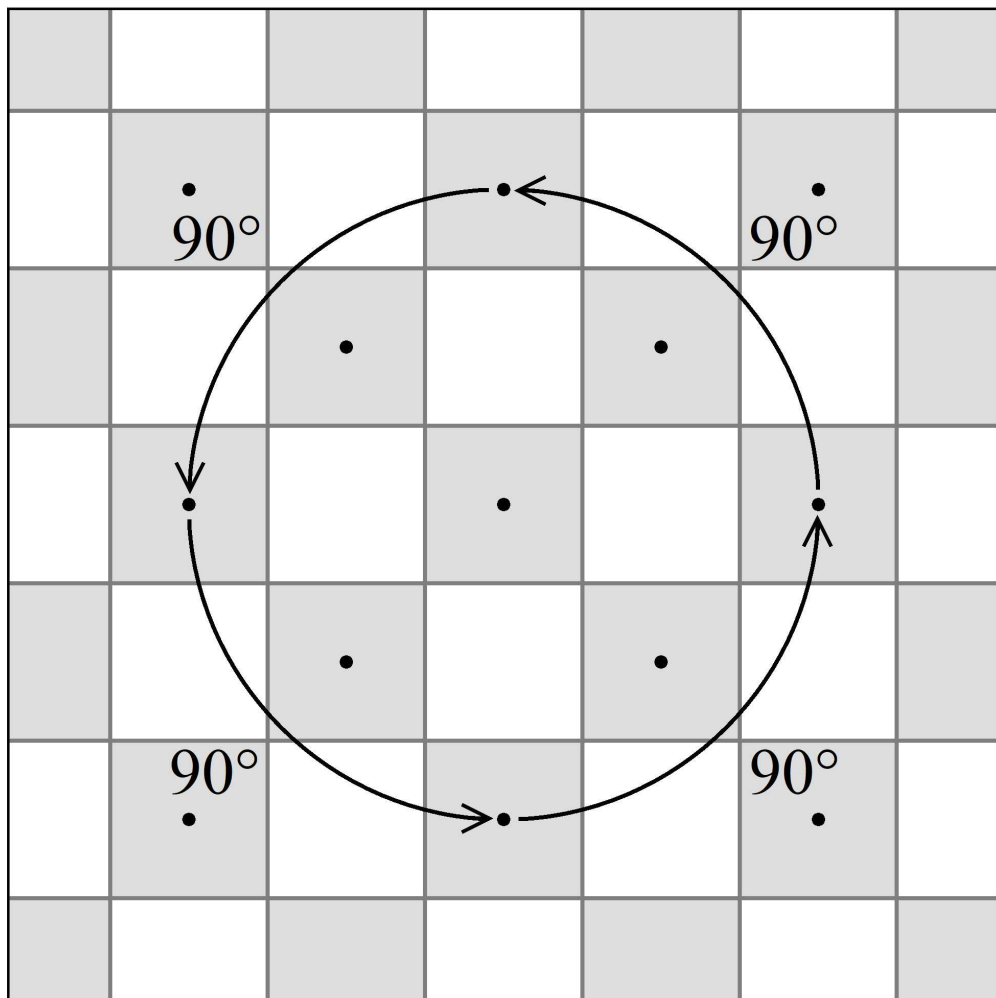
# Reflected Motifs in Quasiperiodic Escher-Penrose Tilings



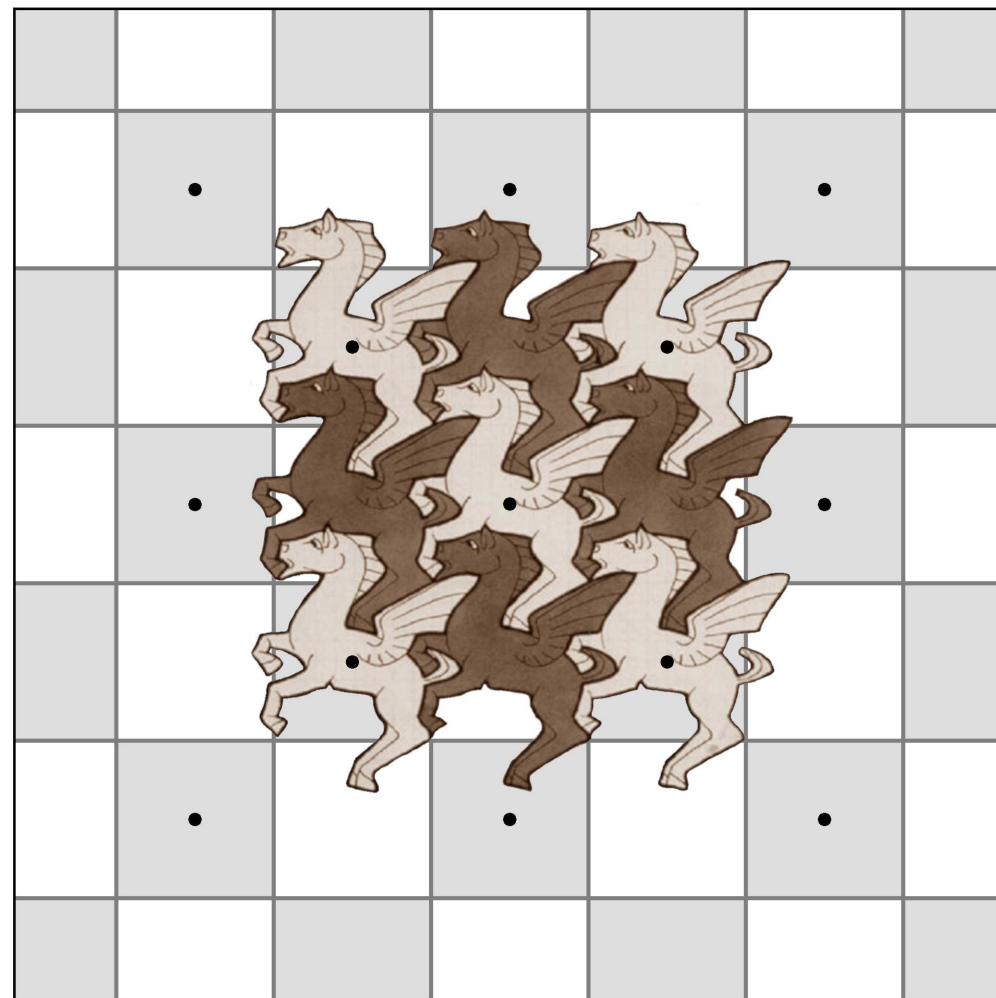
2025

Bridges Conference Eindhoven

Uli Gaenshirt / Amrita Acharyya

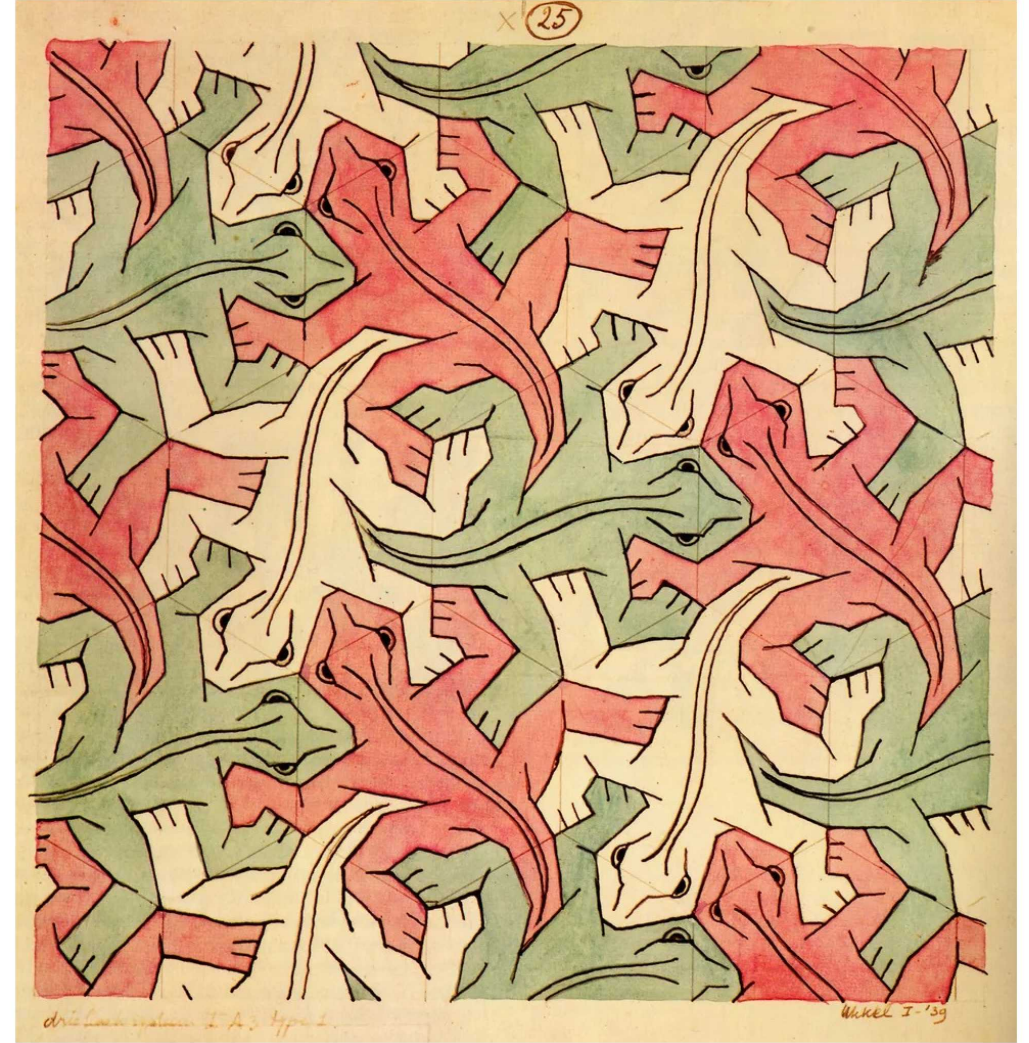
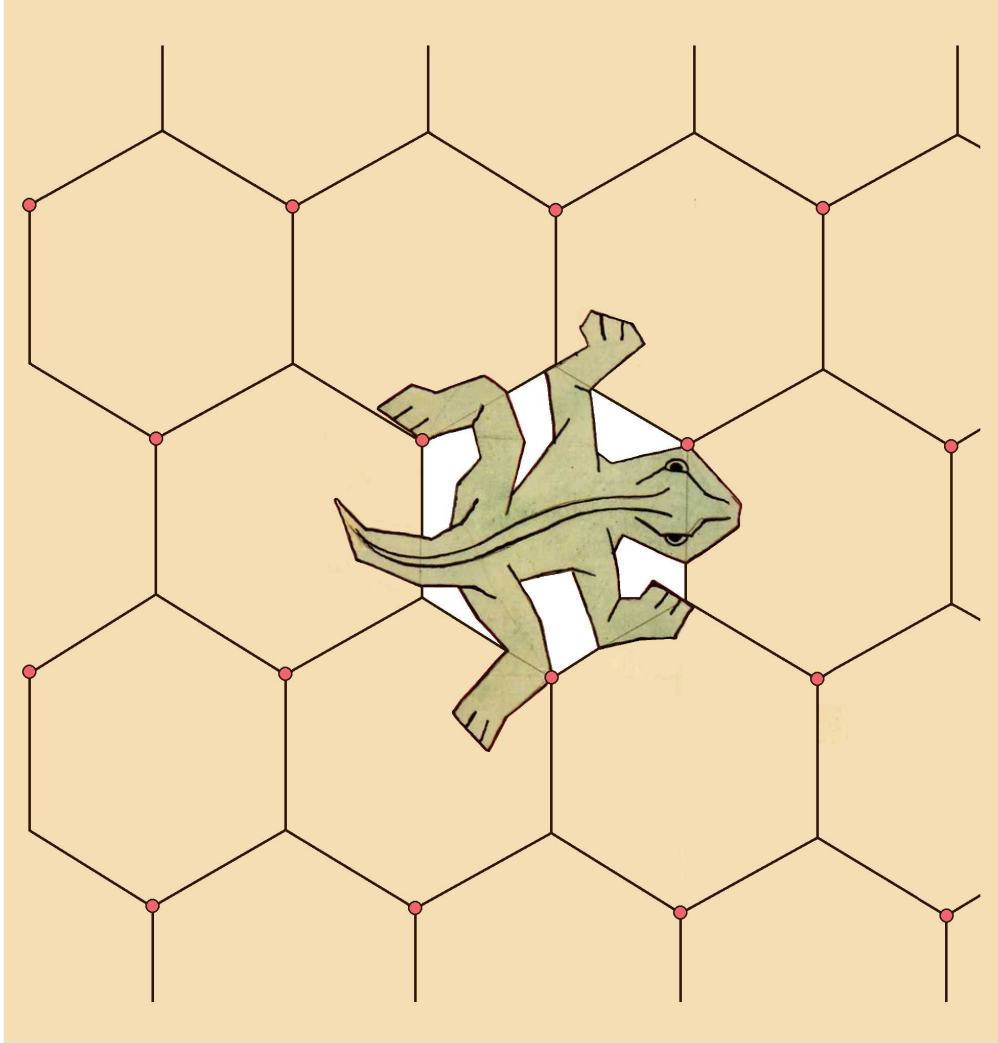


The symmetry group  $p4m$  has a fourfold rotational symmetry

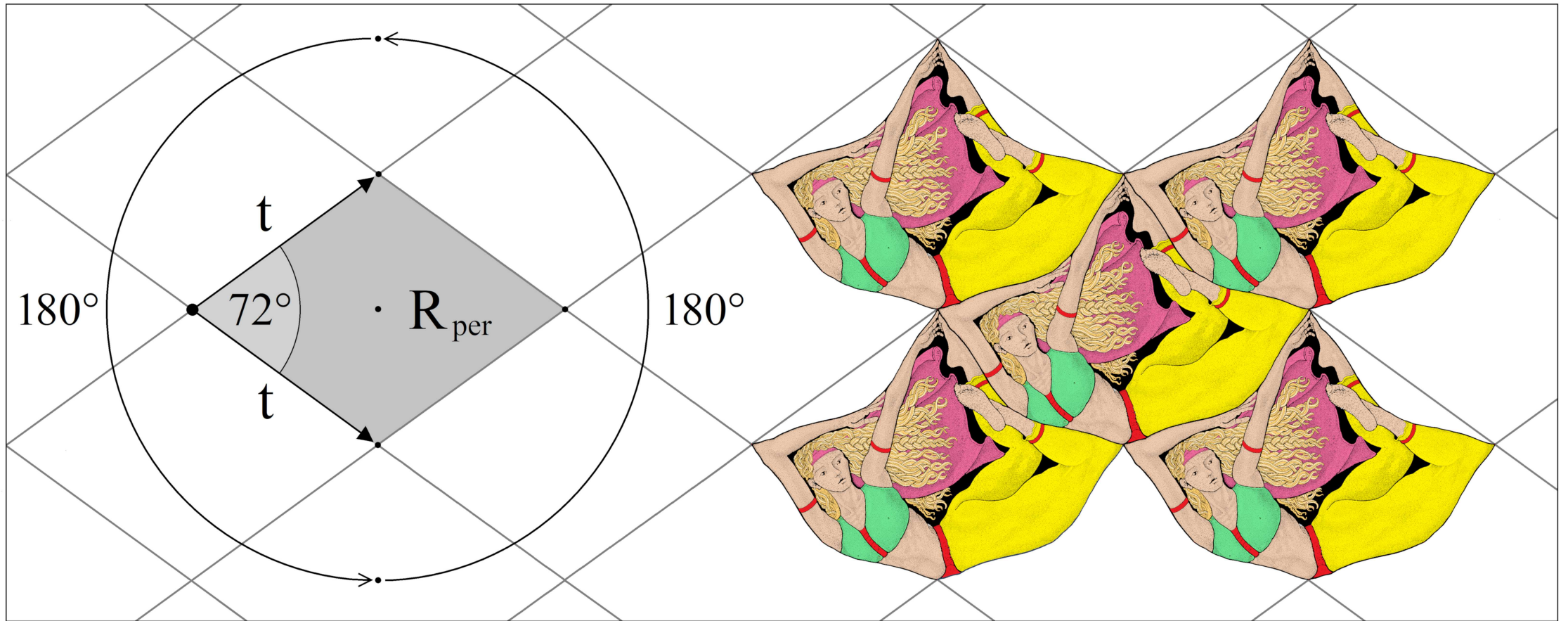


Escher's Pegasus pattern only has the translational symmetry of the group  $p1$





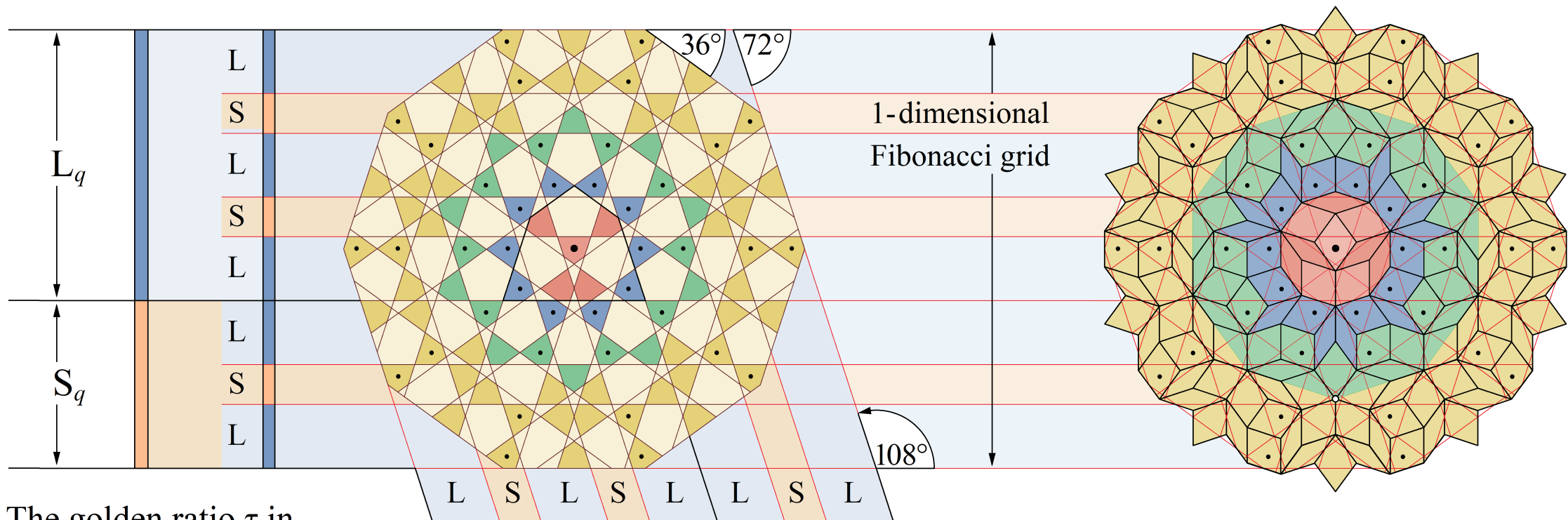
The sixfold rotational symmetry of group  $p6m$  is reduced to  $p3$  by Escher's lizards



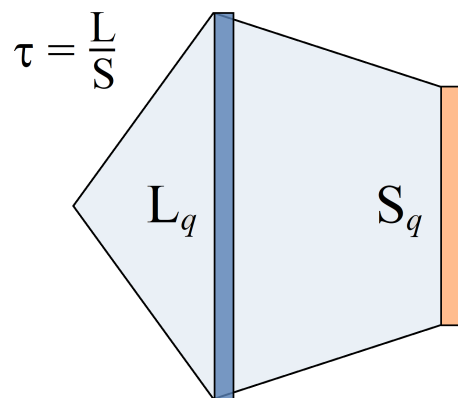
The group  $\text{cmm}$  of a sheared grid has a twofold rotational symmetry

Due to the dancers, the grid no longer has rotational symmetry and is part of group  $\text{p1}$



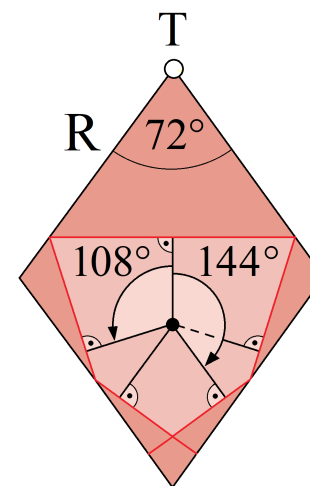


The golden ratio  $\tau$  in the regular pentagon

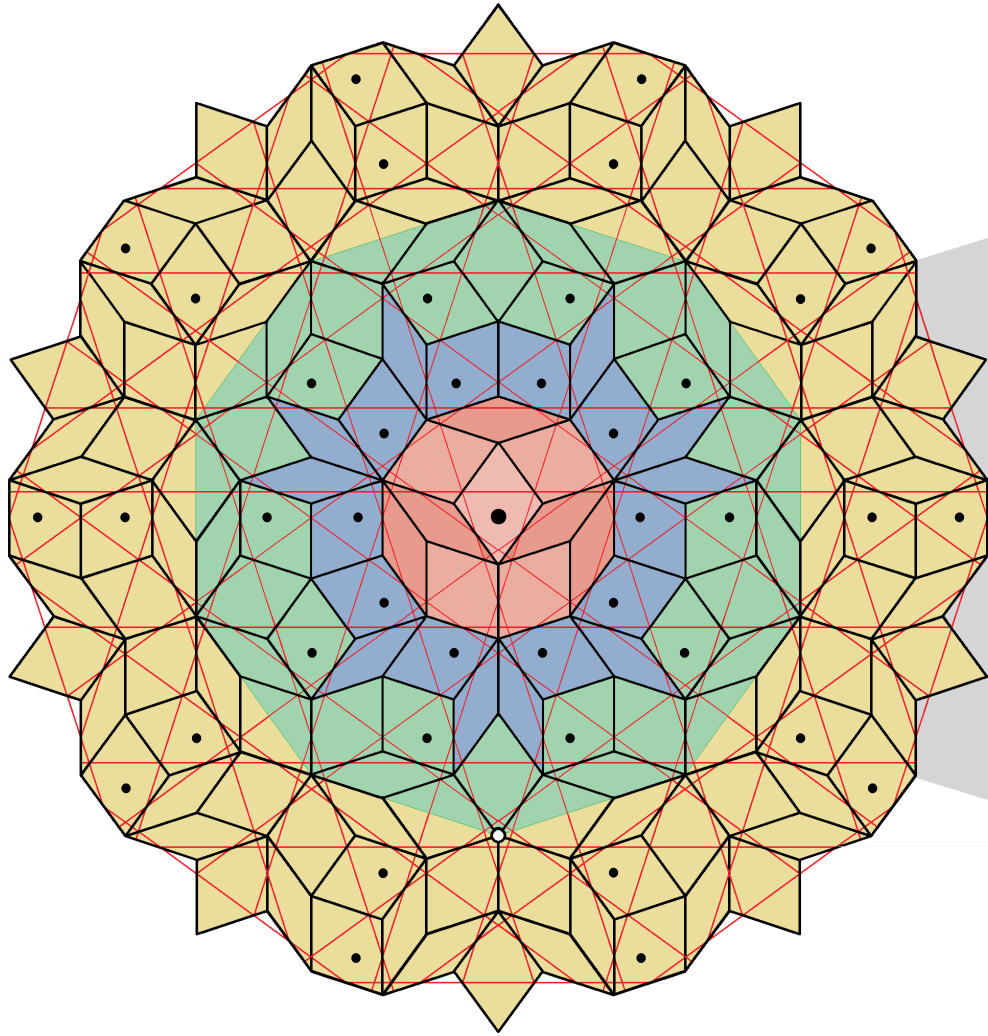


Four copies of the 1-dimensional LS Fibonacci grid are clockwise and counterclockwise rotated 108 and 144 degrees as shown in the red rhombus on the right.

Their superposition creates a 5D cartwheel-type Ammann grid.



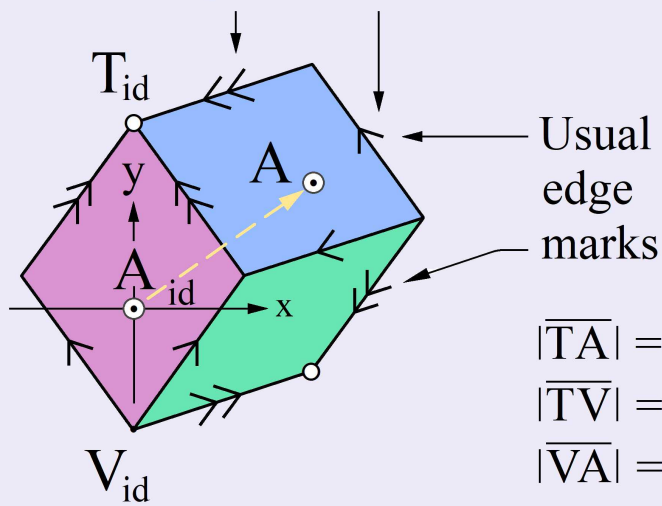
If you surround each irregular pentagonal grid mesh with a thick rhombus R as shown left and fill the gaps with skinny rhombs, you get the Penrose rhombus tiling above.



Penrose rhombus tiling with enlarged center that is decorated with reflected motifs



## De Bruijn arrows



Usual edge marks

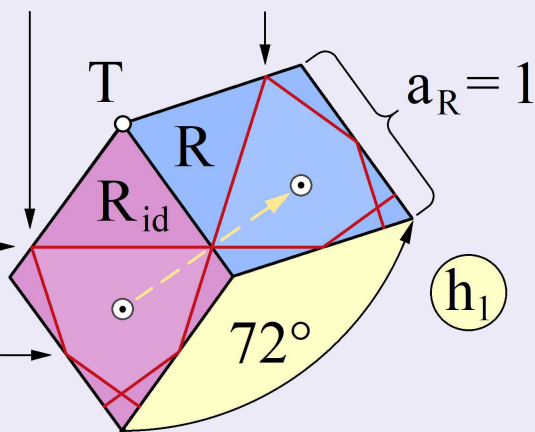
$$|\overline{TA}| = 1$$

$$|\overline{TV}| = \tau$$

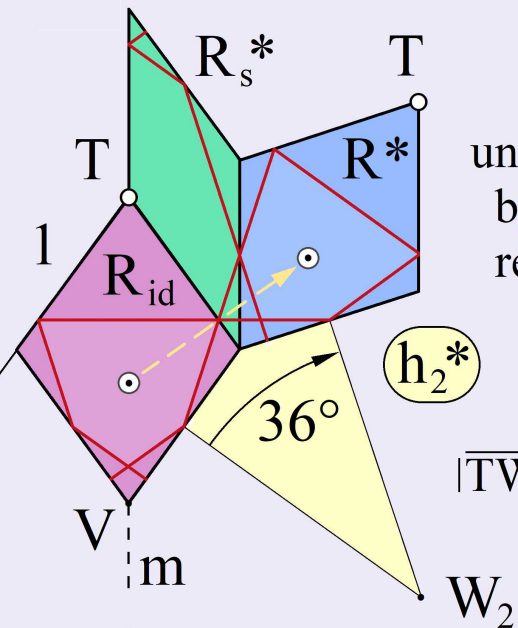
$$|\overline{VA}| = \tau^{-1}$$

$$\tau = (1 + \sqrt{5})/2$$

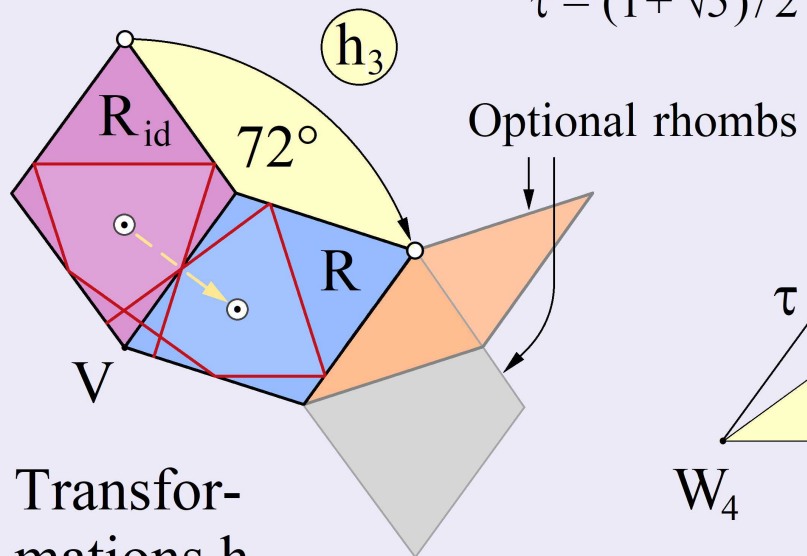
## Red Ammann line segments



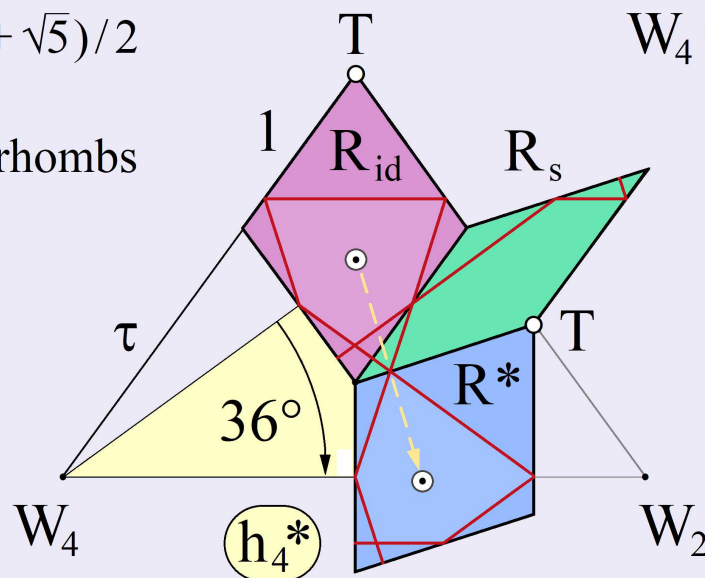
$h_1$



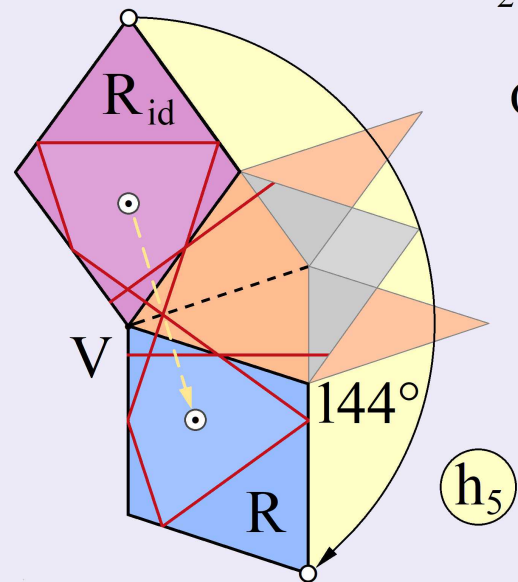
Marked rhombs unchanged by mirror reflection at  $m$



Transformations  $h$



Optional rhombs in the gap of  $h_5$





Glide  
reflec-  
tion  
of  $U_1$   
to  $U_1^*$   
at  $g$

$$h_2^*(R_{\text{id}})$$

# Glide path

## Reflected notch

 $36^\circ$ 
$$W_2$$
 $R_{id}$ 

## h<sub>2</sub>\* components:

## Decoration with animals

$$h_2^*(R_{\text{id}})$$
 $36^\circ$ 
$$h_2$$
 $h_2$ 
$$h_2^*$$
 $W_2$

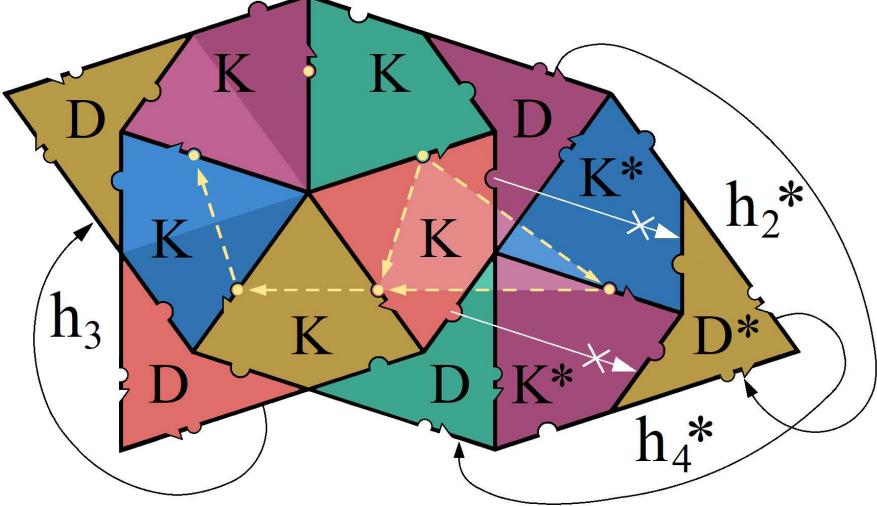


Equivalence  
relation  
between  
Penrose  
tilings

Kite & Dart coloring

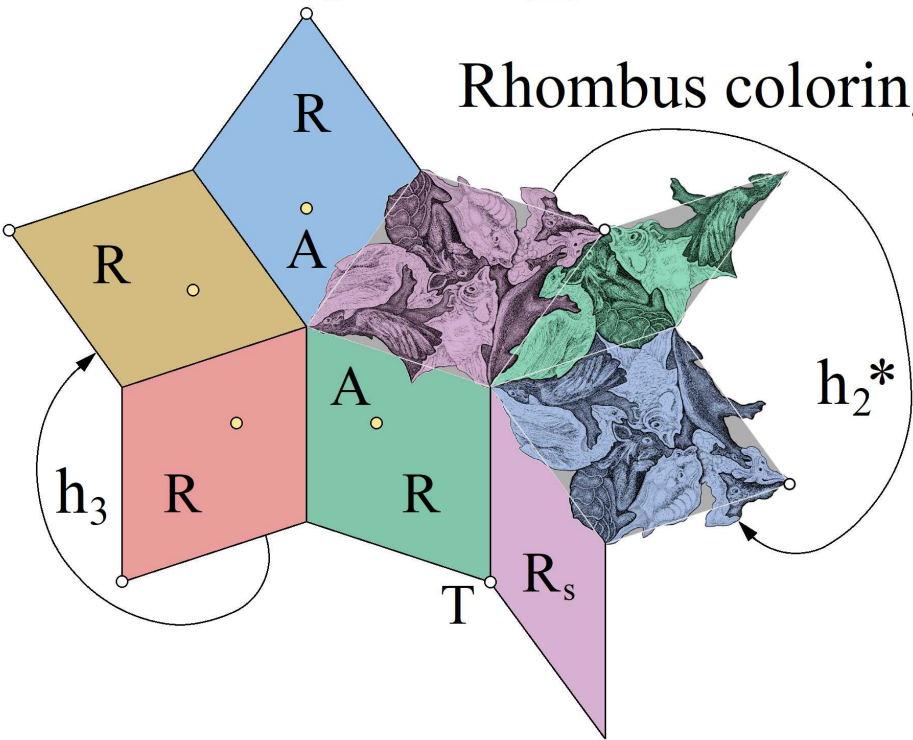


Kite & Dart tiling

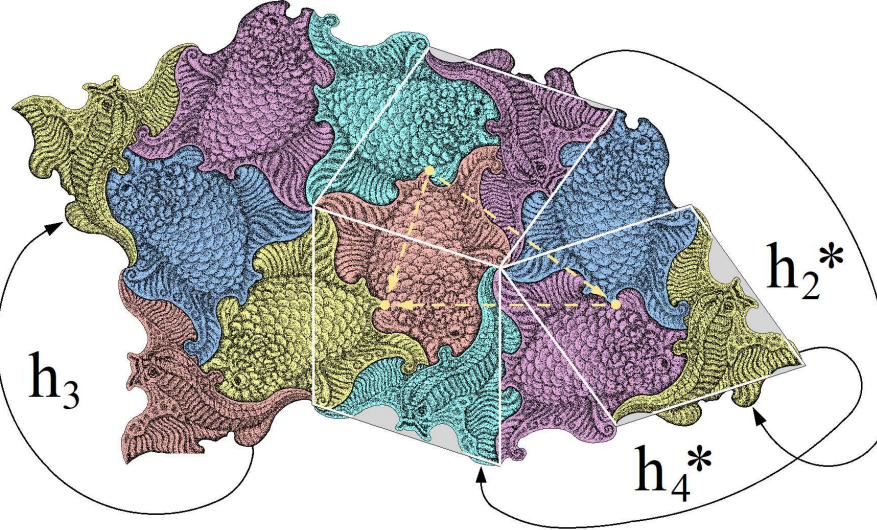


Universal  
animals  
for both  
tilings

Rhombus coloring

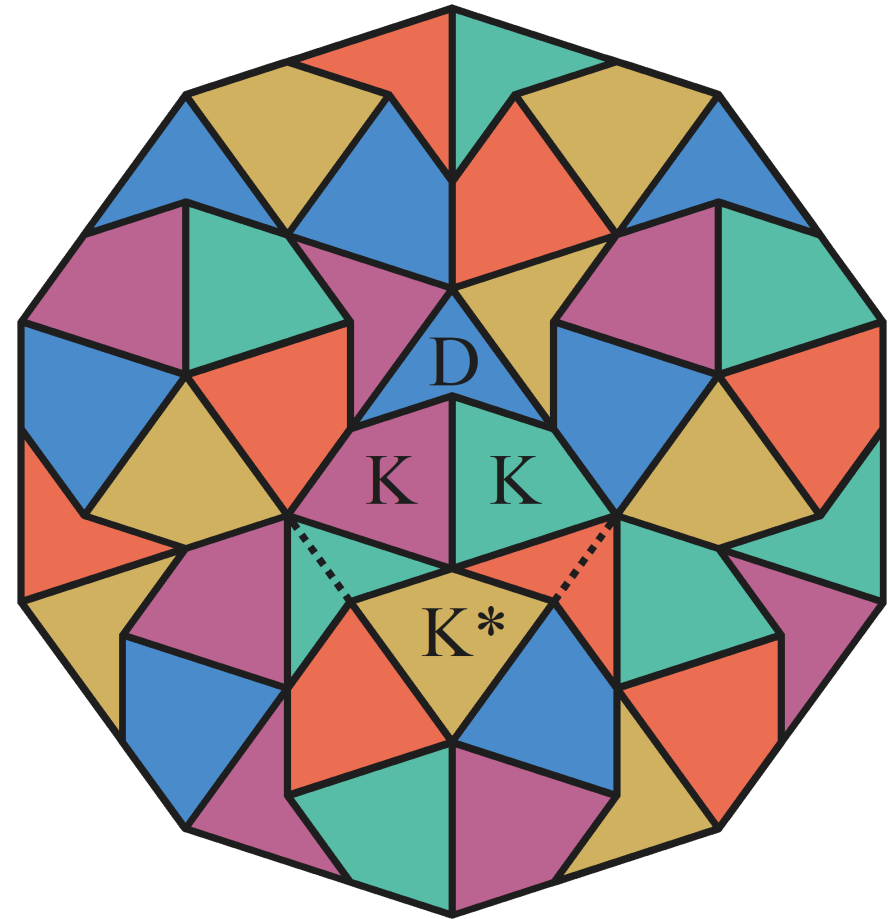
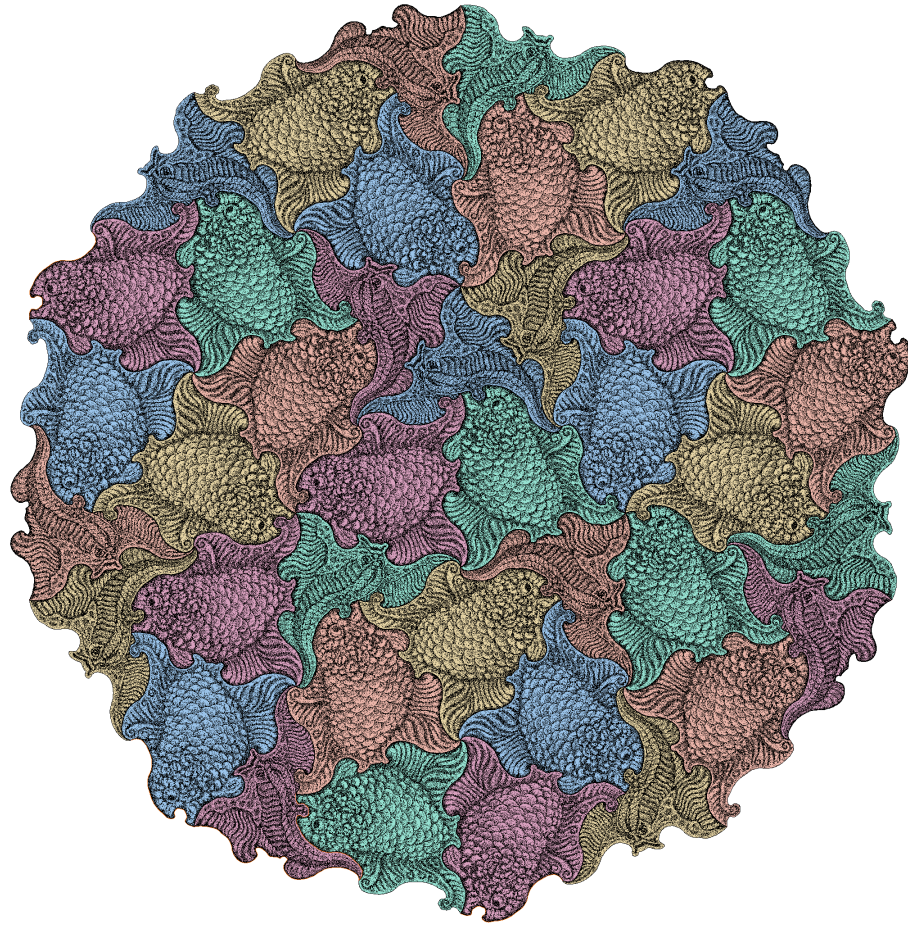


Kite Fish & Dart Rays



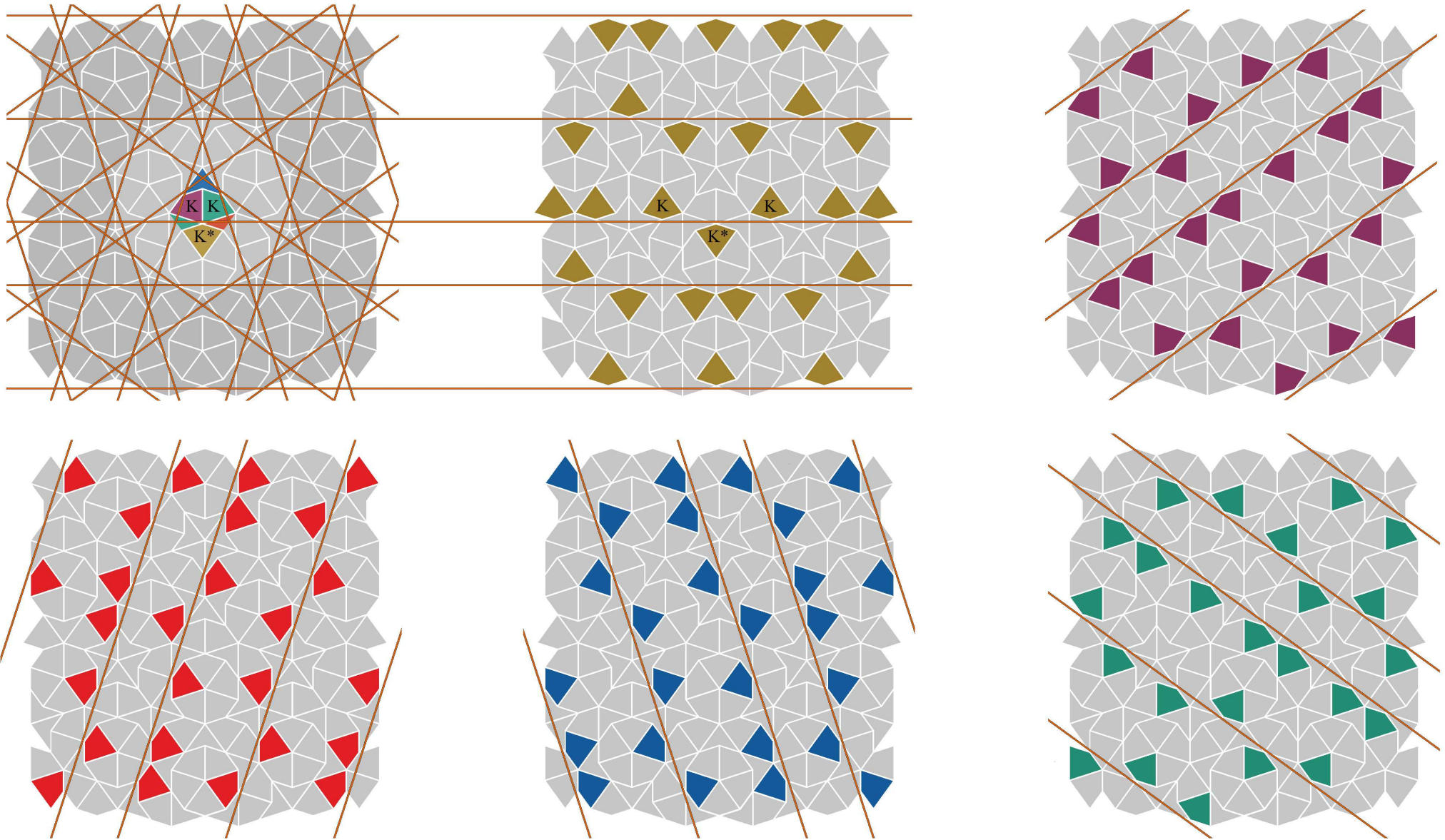
Rhombus  
tiling



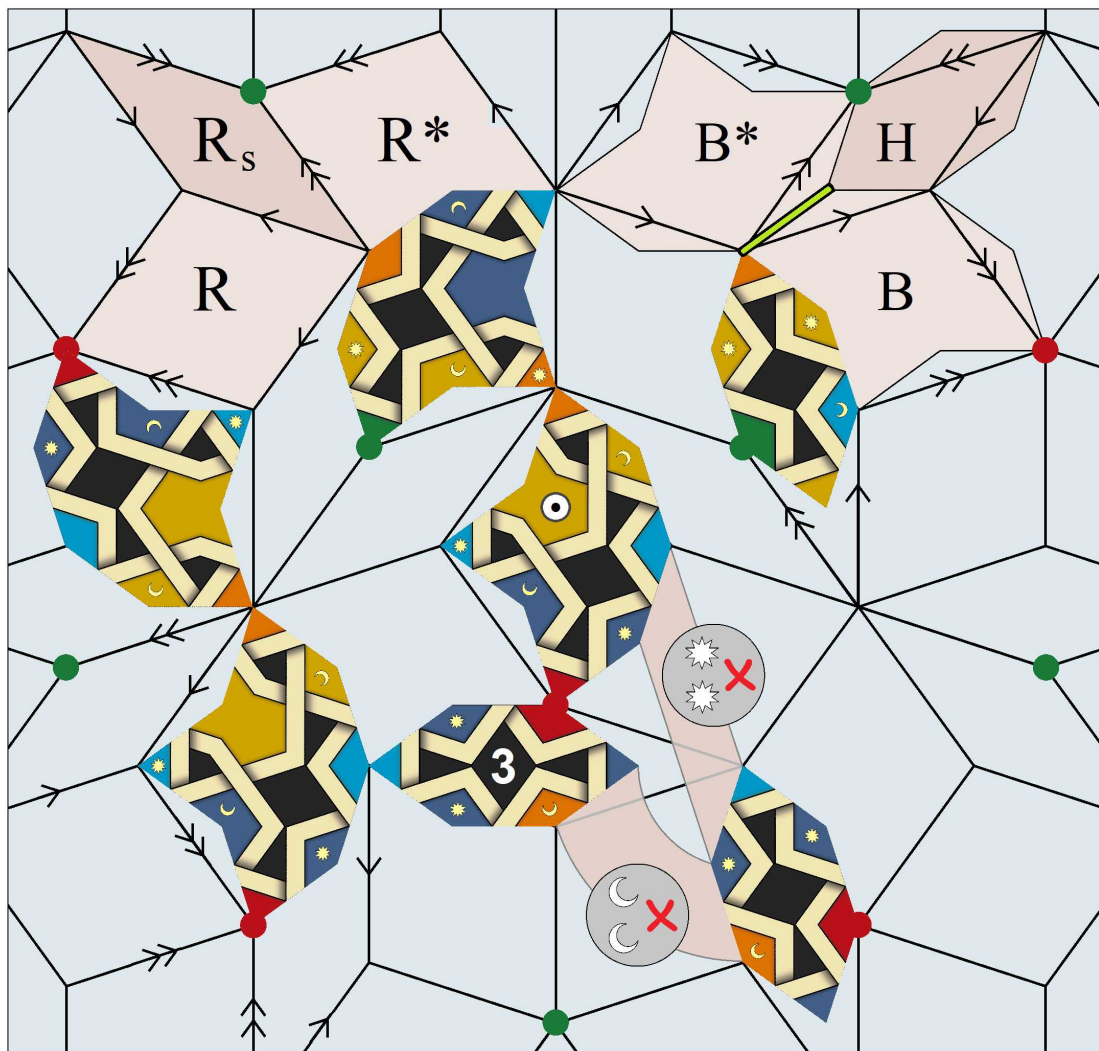


Cartwheel with kite fish and dart rays and corresponding geometric kite & dart tiling

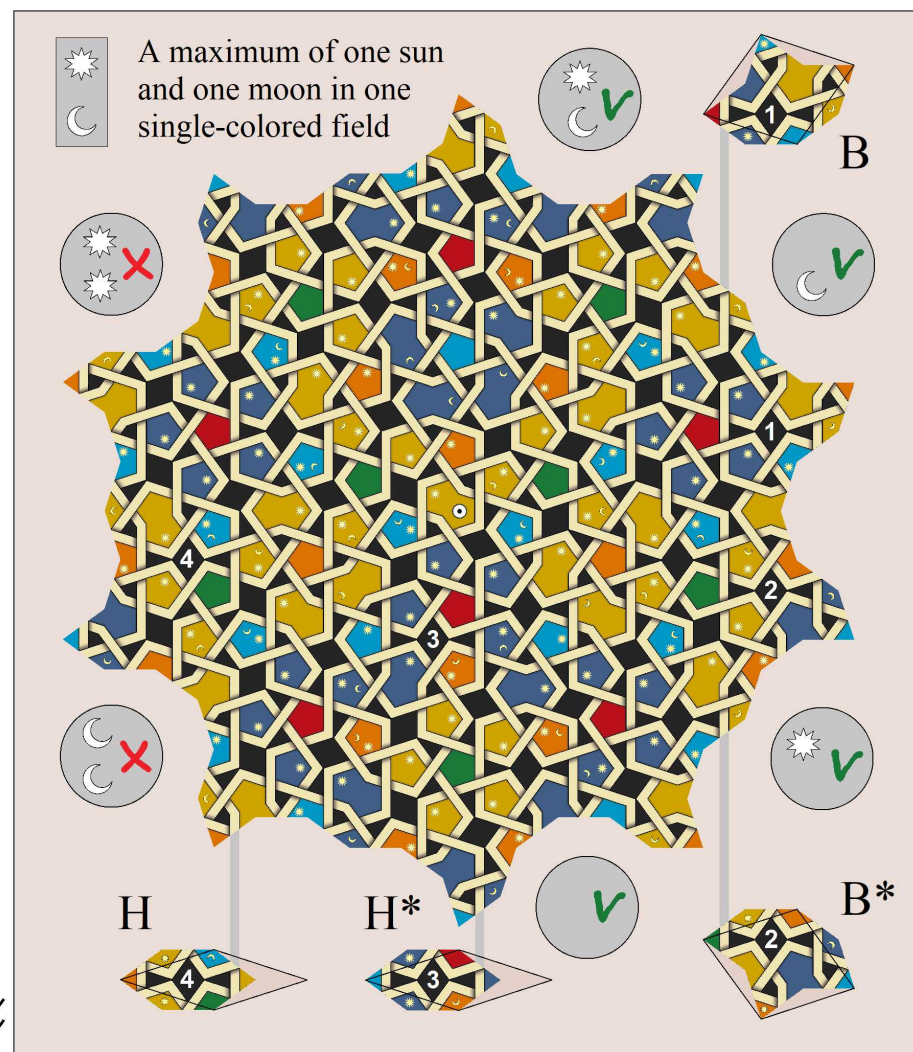
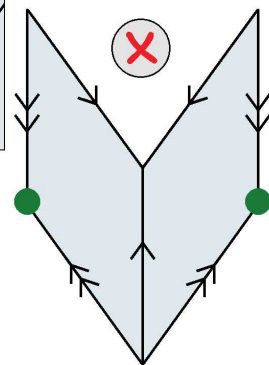




Kite & Dart tiling with Ammann lines and five individually colored 1D kite grids



Two  
suns  
or  
two  
moons  
in one  
color  
field  
lead  
to a  
dead  
end!



The color-coded matching rules result in a uniform Girih pattern





Girih Cartwheel  
18 Elements

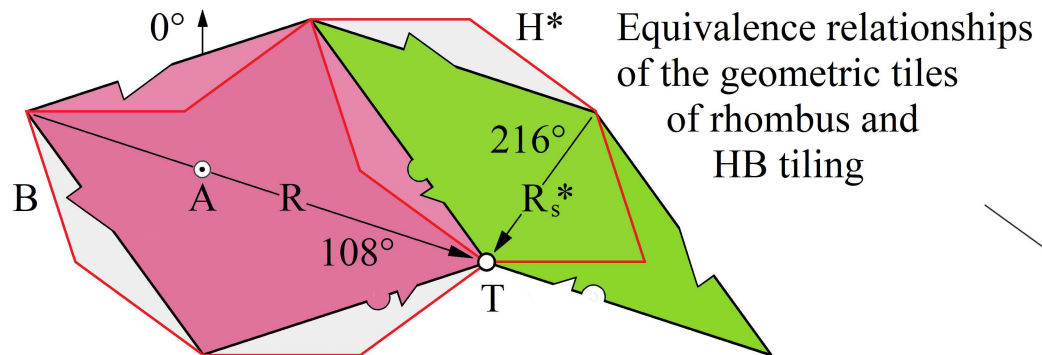


Girih Puzzle  
18 Elements

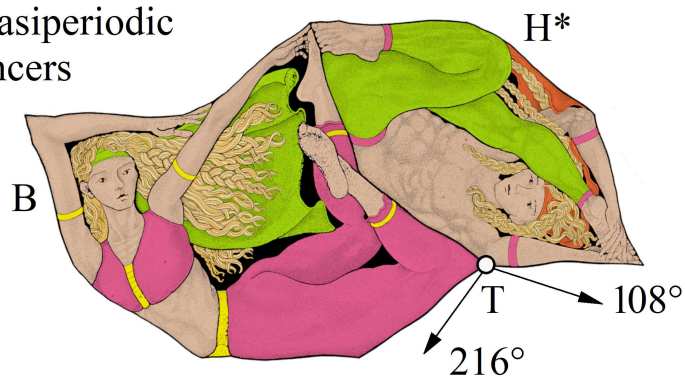


Magnetic  
Girih puzzle

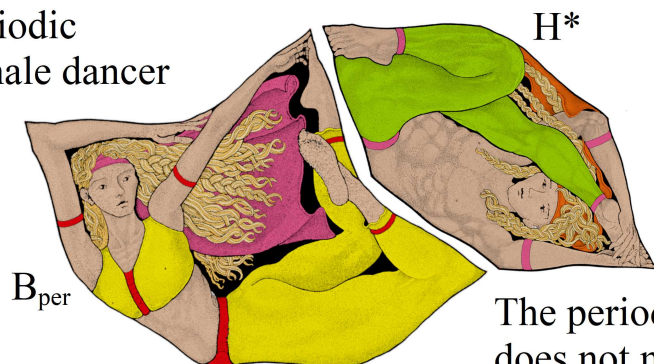




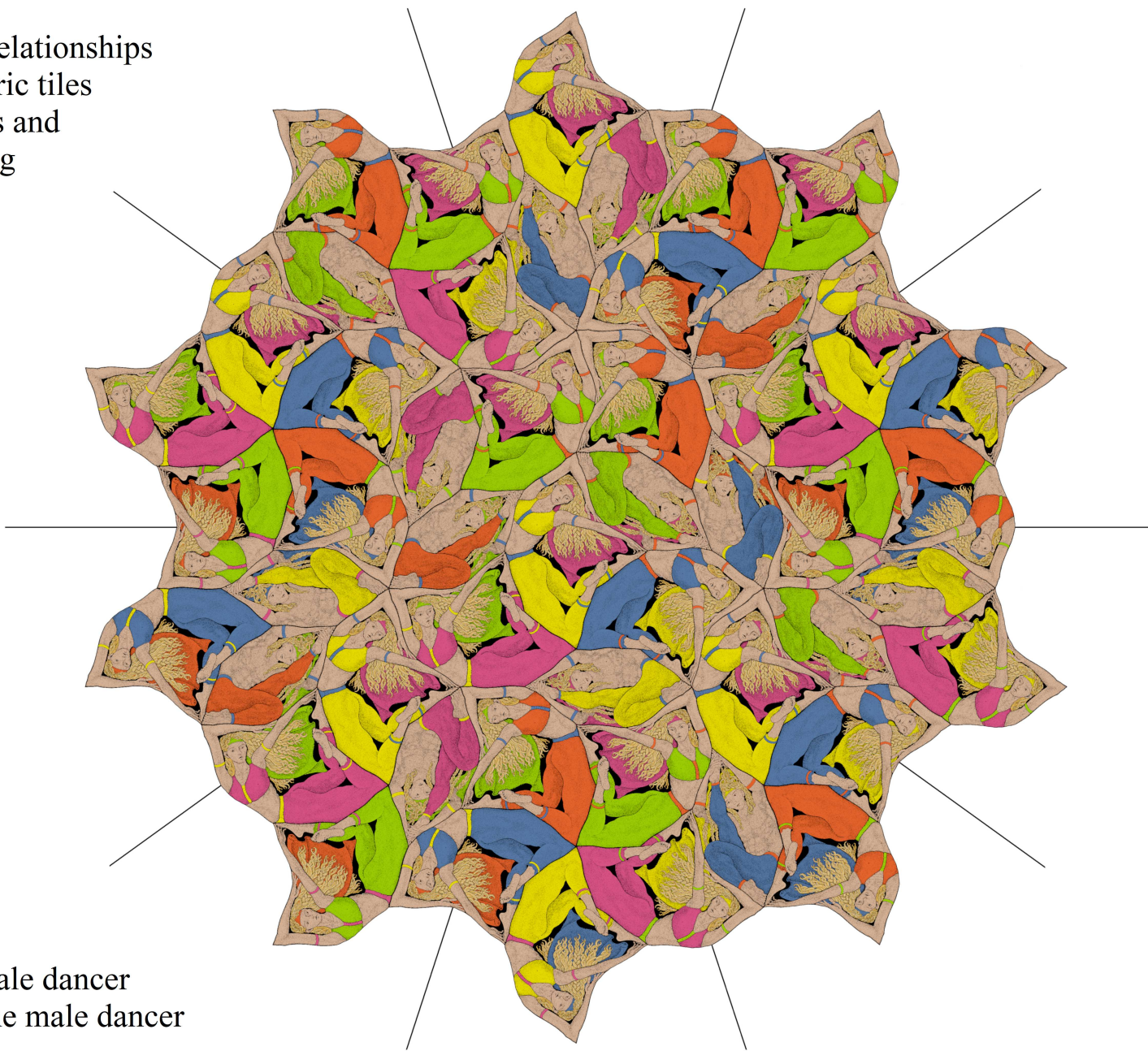
Quasiperiodic  
dancers



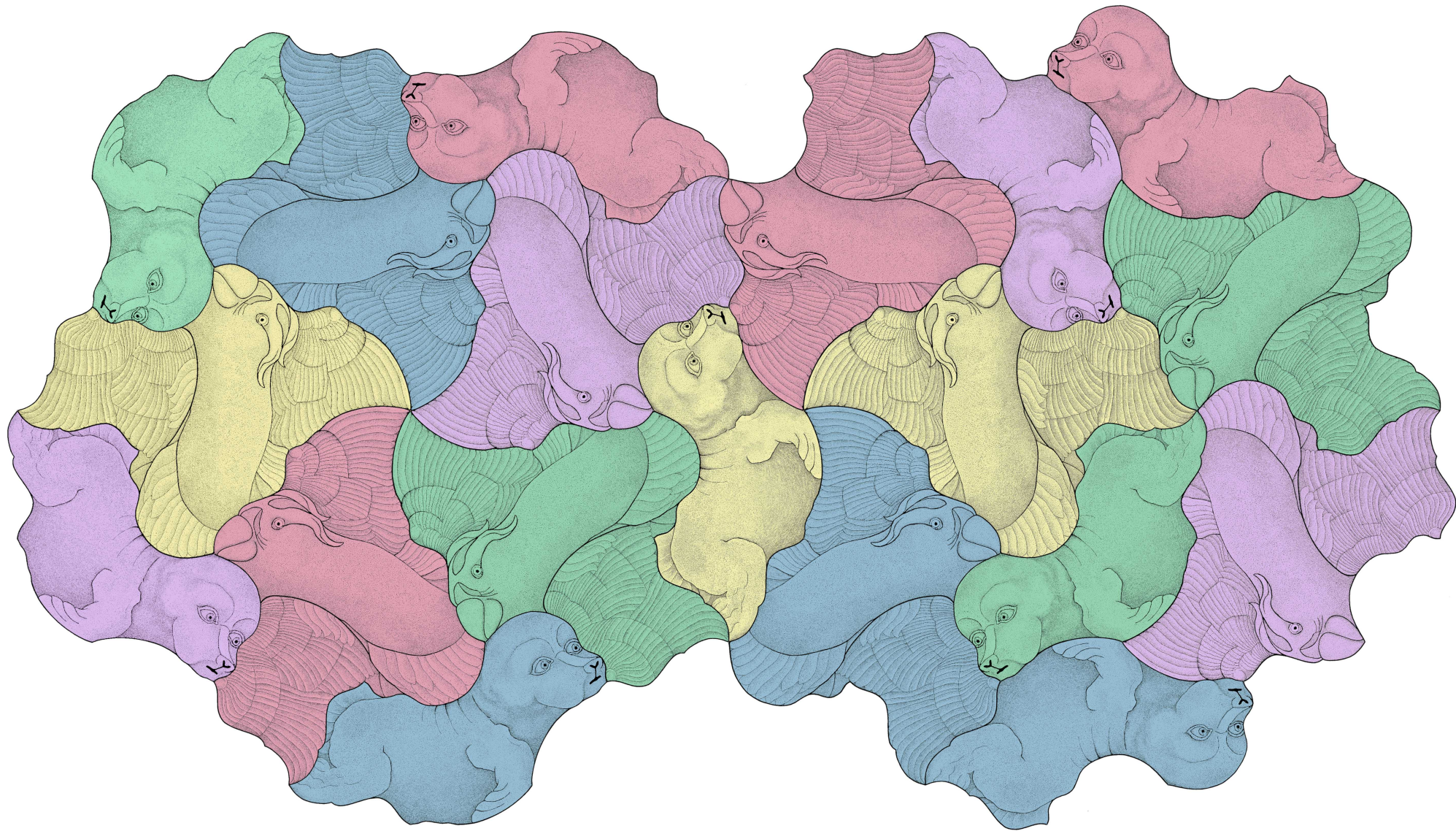
Periodic  
female dancer



The periodic female dancer  
does not match the male dancer

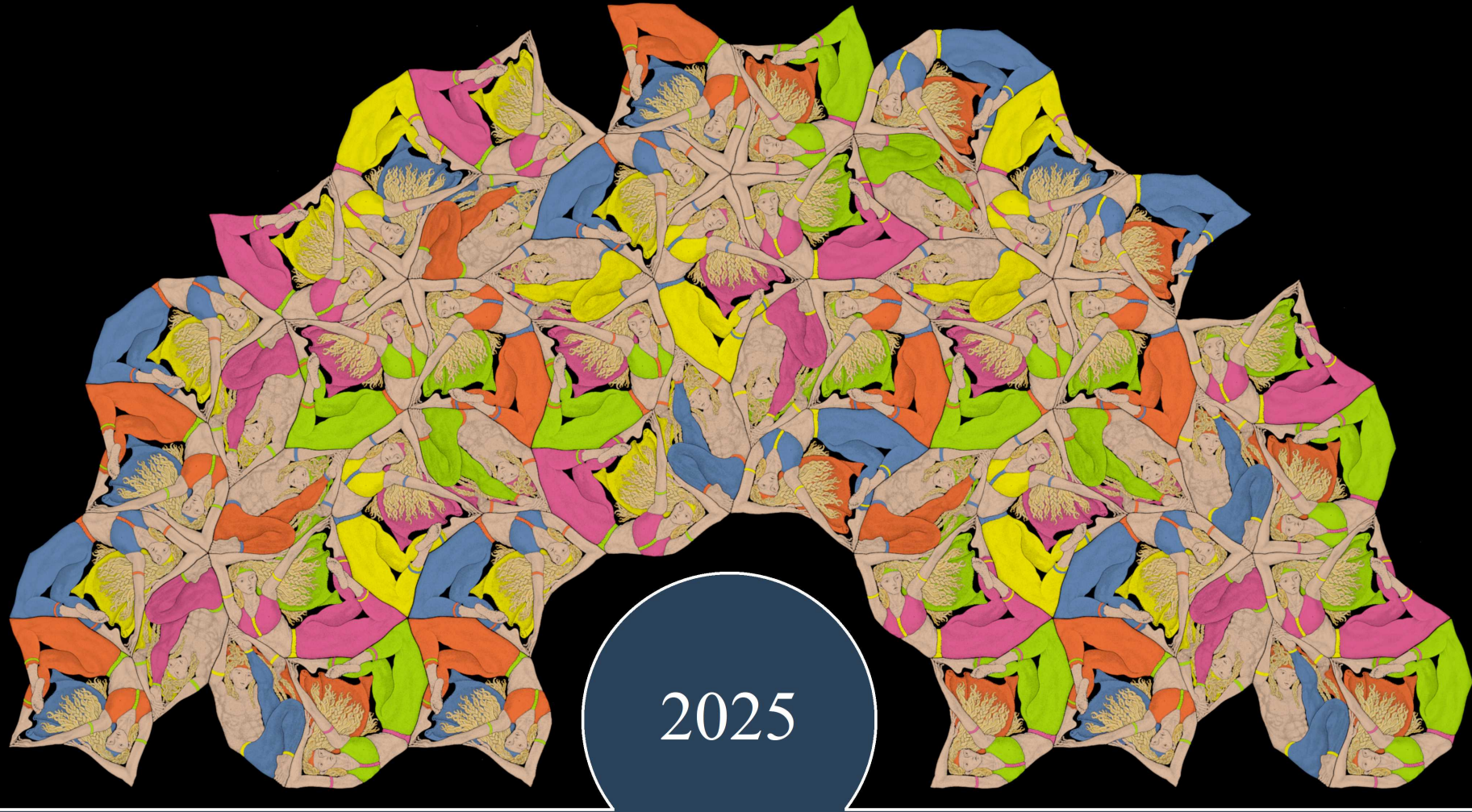








# Reflected Motifs in Quasiperiodic Escher-Penrose Tilings



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